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Angelo Corelli

Analytical Corporate Finance

Second Edition

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Analytical Corporate Finance

Second Edition

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A Ettore, Lollo,
la Famiglia
e gli Amici

Preface to the First Edition

A Modern Approach

The book offers an innovative view of the most important areas of study for a newcomer in finance. The book aims to be a guide through the most common and interesting topics of corporate finance, supported by a rich mathematical foundation (from which the term "analytical" in the book title derives).

The focus of the book is on the life of the corporation, from the start to the implementation of the most complex projects, and the many different ways financing can be achieved. The new approach proposed by this book involves a strong analytical insight into each topic to give students the tools to really understand the effects of managerial decisions on the life of the corporation.

The vision of the book is to offer a guide to corporate finance, capable of helping students and practitioners gain an introduction to the area, as well as giving intermediate users useful insights into and a deeper understanding of the analytics behind each part.

The literature of books in corporate finance spans over a wide range of topics and theories. The market is already full of (even good) textbooks facing the issue of the corporation from many points of view.

However, the overall offer of corporate finance textbook lacks a rigorous mathematical approach. All these books share the same business-related approach to the topic, focusing more on the corporate aspects rather than the financial substance.

The aim of *Analytical Corporate Finance* is to turn the attention to the financial aspects of the daily life of the corporation, with a robust mathematical setting, and the explanation and derivation of the mostly popular models of the firm.

The potential of filling such a niche is evident if one thinks that a course in corporate finance nowadays makes only sense if held at the very beginning of an undergraduate course program in basic finance.

There is a demand for high-quality education at master's level, including a more advanced level of corporate finance courses, which is not fulfilled nowadays and definitely requires a good textbook as a guide for such advanced education in the field.

Corporate finance is the first area of knowledge in an ideal sequence of studies in finance. It contains, in fact, reference to many topics in finance, which are subject to deeper understanding in subsequent modules: risk management, derivatives pricing, asset pricing, etc.

It is therefore very important to address each single topic in the most comprehensive way, giving a complete description of the main aspects of the topic and leaving the need for further knowledge to the curiosity of the student.

As an area, corporate finance can be thought of as an introductory aggregation of all the main topics that constitute the object of a deeper knowledge when studying other areas of finance.

It does not mean knowledge can be superficial or merely introductory. The purpose of a good textbook in corporate finance is therefore to include all necessary knowledge without missing any crucial information, but also avoiding useless widening of the explanation beyond what can really satisfy students' needs.

The structure of the book corresponds to the above rationale, giving a complete view of the various topics that make up the corporate finance area, and offering a good quantitative basis, which helps students understand the topics.

An Innovative Pedagogy

The book stands on a double pillar of theory and analytics, which merge in a way that makes it easy for students to understand the exact meaning of the concepts and their representation and applicability in real-world contexts. Examples are given throughout the chapters in order to clarify the most intricate aspects; where needed, there are appendices at the end of chapters that give more mathematical insights about specific topics.

The duality stands at the basis of a complete learning experience, and students may get the most out of the book if they have a prior solid background in economic theory and an introductory level of financial mathematics. Indeed, students facing financial topics for the first time may benefit from using the book as a medium-level introduction to some aspects of financial theory and practice.

Due to the recent growth in knowledge demand by the private sector, practitioners can also benefit from the chapters in the book to fill a gap between university and industry, which stood prior to the crisis. The book provides useful information for managers who want to increase their knowledge about risk management and understand what may have been lacking in their own systems.

A Selected Audience

The book is meant for third-year undergraduate students of business finance, quantitative finance, and financial mathematics, as well as first-year postgraduate (master) students. Most universities offer the type of training in mathematics and statistics that would be prerequisite for the successful completion of a course using *Analytical*

Corporate Finance. Potential users include students of universities, technical schools, and business schools offering courses in financial risk management.

The book represents a unique and innovative approach to the field of corporate finance. Competitors, in fact, focus on the business side of the story, centering the discussion on corporate aspects only. There is no challenge or criticism of the markets side, and there is no drive to understand the mathematical foundations of theory in a critical sense. That is exactly what *Analytical Corporate Finance* wants to offer instead.

A quantitative approach incorporates a more critical view, contributing to a description of theory that does not blindly rely on numbers, indices, and ratios, but takes into account the variety of (sometimes unpredictable) situations that characterize financial markets.

Certainly, it is not the typical corporate finance book, but it is a book that never gives up on the reader. Even in the most complicated parts, which are anyway at an intermediate level, students are guided through the processes and given the tools they need; nothing is cryptic.

Although the market for books in corporate finance is heavily crowded, *Analytical Corporate Finance* aims to distinguish itself through the higher level of mathematics involved, compared to the competition.

Moreover, the links to real industry examples are much more updated than other texts can offer at the moment. Therefore, a niche in the market can be envisioned for a book that challenges students on a more analytical than business-related level.

The book is totally comprehensive (at least for a large part of the relevant topics) and represents a standard in introductory and mid-level study of corporate finance. In a single book, the features that singularly belong to the most challenging competitors now on the market are summarized and critiqued.

A Reliable Partner for Instructors

Analytic Corporate Finance is mostly tailored for in-class lectures, yielding the best learning experience when combined with good quality lectures. The overall flexibility of the book, in 12 chapters, and the straightforward structure also make it a good reference for online learning. However, the medium-high level of difficulty of the book suggests the need for a closer relation with the instructor and the chance of in-person explanations.

The structure of the book is such that a typical module of six ECTS and approximately 30 h of front teaching would suit it. The 14 chapters fit a course design of about 14–16 lectures of 1.5 hours of effective teaching. The structure also fits the international standard of a course with two lectures per week spanned over a 2-month teaching term. The overall contents of the book can fill approximately 40–60 hours of teaching.

Every chapter follows a precise structure, with the full-text body of most sections complemented by snapshots relating to cutting-edge research and up-to-date news. At the end of each chapter, there is an exercise section consisting of different types of tasks. Each chapter is supplemented with a list of references and appendices which are meant to analyze, in a deeper form, some of the mathematical issues presented in the chapter.

Value for Students and Instructors

The rigorous mathematical approach is supported by an appropriate amount of graphical and explanatory support, in the parts where this is needed. Overall, the book is rich in contents of every kind.

In particular, it is convenient to summarize some statistics. The book comprises:

- **14** chapters
- **70** major learning outcomes
- **126** specific learning outcomes
- **83** numerical examples
- **203** in-text examples
- **45** figures
- **11** tables

The above contents offer the support to the body of the text and the equations that help to clarify all the major aspects of the topics touched in the book chapters. The work is a valid partner for instructors and students, who can find a lot of material to learn the theory about corporate finance and practice about it with problems and exercises.

Dubai, UAE Angelo Corelli

Addendum to the Preface

This second Edition of *Analytical Corporate Finance* introduces meaningful and significant additions to the previous edition, making the textbook a more reliable source for students and instructors of courses like corporate finance, financial management, and principles of finance.

The addition of case studies and the thorough revisiting of all the text in order to correct mistakes and improve the narration of some topics enrich the learning experience by shedding a clearer light on the major points of discussion.

The innovations in this edition can be summarized, by chapter, as:

Chapter 1:
- Improved Analysis of Financial Ratios
- Pro-Forma Analysis of Financial Statement
- Case Study: Pro-Forma Statements

Chapter 2:
- Case Study: Time Value of Money

Chapter 3:
- Snapshot: Risk and Return in Excel
- Case Study: Risk and Return

Chapter 4:
- Improved Coverage of Capital Budgeting Decision Rules
- Case Study: Net Present Value

Chapter 6:
- Relative Valuation Models

Chapter 7:
- Case Study: Capital Structure

Chapter 8:
- Case Study: Company Valuation

Chapter 10:
- IPO Underpricing

Chapter 11:
- Conversion Cycles
- Cash and Liquidity Management
- Case Study: Conversion Cycles

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Angelo Corelli

Contents

1 Basic Concepts . . . 1
- 1.1 The Corporation . . . 2
  - 1.1.1 Organizational Forms . . . 2
  - 1.1.2 The Corporate Objective . . . 6
- 1.2 Financial Statement Analysis . . . 10
  - 1.2.1 Balance Sheet . . . 10
  - 1.2.2 Income Statement . . . 15
  - 1.2.3 Cash Flow Statement . . . 22
  - 1.2.4 Pro Forma Statements . . . 25
- 1.3 Arbitrage . . . 30
  - 1.3.1 The Law of One Price . . . 30
  - 1.3.2 Forms of Arbitrage . . . 33
- 1.4 Summary . . . 37
- Problems . . . 38
- Case Study: Pro Forma Statements . . . 38
- References . . . 39

2 Valuation Tools . . . 41
- 2.1 The Time Value of Money . . . 42
  - 2.1.1 The Rules of Time Travel . . . 42
  - 2.1.2 Valuation of Cash Flow Streams . . . 47
  - 2.1.3 Annuities and Perpetuities . . . 48
- 2.2 Interest Rates . . . 52
  - 2.2.1 Types of Interest Rates . . . 52
  - 2.2.2 Compounding Frequencies . . . 55
  - 2.2.3 The Drivers of Interest Rates . . . 56
- 2.3 Present Value Calculation . . . 59
  - 2.3.1 Net Present Value . . . 59
  - 2.3.2 Internal Rate of Return . . . 60
  - 2.3.3 Growing Cash Flows . . . 62
- 2.4 Summary . . . 66
- Problems . . . 67

Case Study: Time Value of Money . . . 69
References . . . 71

3 The Relationship Between Risk and Return . . . 73
3.1 Expected Return and Volatility . . . 74
3.1.1 The Portfolio Return . . . 74
3.1.2 Volatility and Correlation . . . 77
3.1.3 Maximum Likelihood Methods . . . 83
3.2 Modern Portfolio Theory . . . 87
3.2.1 The Risk/Return Trade-Off . . . 87
3.2.2 Optimal Portfolios . . . 92
3.2.3 The Market Price of Risk . . . 99
3.3 The Capital Asset Pricing Model . . . 102
3.3.1 Model Assumptions . . . 102
3.3.2 The Security Market Line . . . 106
3.3.3 Beyond CAPM . . . 110
3.4 Summary . . . 112
Problems . . . 113
Appendix: Liquidity CAPM . . . 116
Case Study: Risk and Return . . . 117
References . . . 119

4 Business Analysis . . . 121
4.1 Capital Budgeting Decision Rules . . . 122
4.1.1 The Net Present Value Rule . . . 122
4.1.2 The Internal Rate of Return Rule . . . 125
4.1.3 The Payback Rule . . . 126
4.1.4 The Profitability Index Rule . . . 127
4.2 Project Valuation . . . 130
4.2.1 Scenario Analysis . . . 130
4.2.2 Decision Trees . . . 132
4.2.3 Simulations . . . 134
4.3 Sub-disciplines of Business Analysis . . . 137
4.3.1 Enterprise Analysis . . . 137
4.3.2 Business Requirements Analysis . . . 140
4.3.3 Analysis Techniques . . . 143
4.4 Summary . . . 144
Problems . . . 145
Case Study: Net Present Value . . . 146
References . . . 148

5 Debt Valuation . . . 149
5.1 Bond Markets . . . 150
5.1.1 Bond Fundamentals . . . 150
5.1.2 Types of Bonds . . . 151
5.1.3 Knowing the Risks . . . 153
5.2 The Price/Yield Relationship . . . 156
5.2.1 Bond Prices and Yields . . . 156
5.2.2 The Yield Curve . . . 162
5.2.3 Duration and Convexity . . . 165
5.3 The Term Structure of Interest Rates . . . 169
5.3.1 The Brownian Motion . . . 169
5.3.2 Single-Factor Models . . . 172
5.3.3 Multifactor Models . . . 177
5.4 Summary . . . 179
Problems . . . 179
Appendix: Principal Component Analysis of the Term Structure . . . 181
References . . . 183

6 Equity Valuation . . . 185
6.1 The Stock Market . . . 186
6.1.1 Market Microstructure . . . 186
6.1.2 Price Formation . . . 190
6.1.3 Funding vs. Market Liquidity . . . 192
6.2 Common Equity . . . 194
6.2.1 Dividend Discount Models . . . 194
6.2.2 Discounted Free Cash Flow Models . . . 202
6.2.3 Relative Valuation Models . . . 206
6.3 Special Issuances . . . 208
6.3.1 Preferred Stocks . . . 208
6.3.2 Warrants . . . 213
6.3.3 Convertibles . . . 216
6.4 Summary . . . 218
Problems . . . 219
References . . . 220

7 Capital Structure . . . 223
7.1 The Modigliani–Miller Theorems . . . 224
7.1.1 The Irrelevance of Capital Structure . . . 224
7.1.2 The Weighted Average Cost of Capital . . . 228
7.2 Payout Policy . . . 234
7.2.1 Dividends vs. Share Repurchase . . . 234
7.2.2 Modigliani–Miller Revisited . . . 239
7.3 Modern Theories of Capital Structure . . . 241
7.3.1 Static Trade-Off Theory and the Pecking Order Hypothesis . . . 241
7.3.2 The Adjusted Present Value . . . 244
7.3.3 Tax-Based Theories of Capital Structure . . . 246

7.4 Summary . . . 249
Problems . . . 250
Appendix: Risk-Adjusted Return on Capital . . . 252
Case Study: Capital Structure . . . 253
References . . . 254

8 Company Valuation . . . 257
8.1 CAPM Parameters Estimation . . . 258
8.1.1 The Risk-Free Rate . . . 258
8.1.2 The Risk Premium . . . 260
8.1.3 The Beta . . . 262
8.2 Valuation of Private Firms . . . 265
8.2.1 The Cost of Capital . . . 265
8.2.2 Beta Estimation . . . 270
8.2.3 Growth and Terminal Value . . . 273
8.3 A Real Case: The IPO of Facebook™ . . . 281
8.3.1 The Road to the IPO . . . 281
8.3.2 The Firm's Fundamentals . . . 284
8.3.3 After the IPO . . . 288
8.4 Summary . . . 291
Problems . . . 292
Case Study: Company Valuation . . . 293
References . . . 294

9 Financial and Real Options . . . 297
9.1 Basic Concepts . . . 298
9.1.1 Financial Derivatives . . . 298
9.1.2 Option Structure and Payout . . . 304
9.1.3 Strategies and Combinations . . . 306
9.2 Option Valuation . . . 310
9.2.1 Binomial Trees . . . 310
9.2.2 Black-Scholes-Merton Model . . . 314
9.2.3 The Greeks . . . 317
9.3 Real Options . . . 320
9.3.1 Types of Real Options . . . 320
9.3.2 Valuation Methods . . . 327
9.4 Summary . . . 333
Problems . . . 333
References . . . 336

10 Long-Term Financing . . . 339
10.1 Early-Stage Funding . . . 340
10.1.1 Angel Investors . . . 340
10.1.2 Venture Capital . . . 342
10.1.3 Private Equity . . . 346

10.2 Initial Public Offering . . . 348
10.2.1 Types of Offering . . . 348
10.2.2 IPO Underpricing . . . 351
10.3 Debt Financing . . . 353
10.3.1 Types of Debt . . . 353
10.3.2 Loans Vs. Bonds . . . 361
10.3.3 Leasing . . . 364
10.4 Summary . . . 368
Problems . . . 369
References . . . 369

11 Working Capital Management . . . 371
11.1 Structure of Working Capital . . . 372
11.1.1 Current Assets and Liabilities . . . 372
11.1.2 Conversion Cycles . . . 376
11.1.3 Significance of Short-Term Financing . . . 380
11.2 Receivables and Payables . . . 383
11.2.1 Accounts Receivable . . . 384
11.2.2 Accounts Payable . . . 387
11.3 Inventory Management . . . 390
11.3.1 Definition and Purpose . . . 391
11.3.2 Inventory Models . . . 394
11.4 Cash and Liquidity Management . . . 397
11.4.1 The Management of Cash . . . 397
11.4.2 Float Management and Collection . . . 400
11.5 Summary . . . 405
Problems . . . 406
Case Study: Conversion Cycles . . . 406
References . . . 408

12 Financial Planning . . . 409
12.1 The Birth of a Business . . . 410
12.1.1 Business Start-Up . . . 410
12.1.2 Rights . . . 413
12.2 Growth Management . . . 415
12.2.1 The Life Cycle of the Firm . . . 416
12.2.2 Short-Term Financial Planning . . . 418
12.3 Planning and Forecasting . . . 422
12.3.1 Basic Planning Rules . . . 422
12.3.2 The Process of Strategic Planning . . . 423
12.4 Summary . . . 427
Problems . . . 428
References . . . 428

13 International Corporate Finance . . . 431
13.1 Internationalization . . . 432
13.1.1 International Trade . . . 432
13.1.2 Multinational Corporations . . . 433
13.1.3 Globalization and the Cost of Capital . . . 438
13.2 The Variables Involved . . . 442
13.2.1 Exchange Rate . . . 442
13.2.2 Purchasing Power Parity . . . 445
13.2.3 Interest Rate Parity . . . 447
13.3 Foreign Exchange Markets . . . 450
13.3.1 Background . . . 450
13.3.2 Foreign Exchange Risk . . . 453
13.3.3 International Capital Budgeting . . . 455
13.4 Summary . . . 460
Problems . . . 460
References . . . 461

14 Special Topics . . . 463
14.1 Mergers and Acquisitions . . . 464
14.1.1 Reasons for a Merger . . . 464
14.1.2 The Process of Merging and Acquiring . . . 469
14.1.3 Hostile Takeovers and Defense Strategies . . . 470
14.2 Corporate Governance . . . 478
14.2.1 Management Failures . . . 478
14.2.2 Remuneration and Incentive Systems . . . 481
14.2.3 Postcrisis Perspectives . . . 483
14.3 Risk Management . . . 484
14.3.1 Corporate Risk . . . 484
14.3.2 From Identification to Mitigation . . . 488
14.3.3 Response Strategies . . . 490
14.4 Summary . . . 492
Problems . . . 492
References . . . 493

Index . . . 495

Basic Concepts

1

The core of corporate finance is the company and its life cycle. The focus is on the analysis of how the company finances itself, how it spends the money acquired, and what the sources of income derived from projects are.

The objective of every company should be the maximization of its value through processing the capital raised by employing it in projects related to the core business, in view of capitalizing a positive outcome.

The road to profitability includes an understanding of the tools and processes that govern the daily life of the corporation and is based on the analysis of the opportunities offered by the market and the appropriate choice of the relevant ones.

Knowledge about the financials of the company must be crossed with the understanding of the dynamics of financial markets, in order to give a fair and robust valuation of the business, as a whole, as well as of single projects.

Before going into detail, it is necessary to give a robust introduction about the corporation. A company can take many organizational forms, and there are several issues involved in proper financial management.

Moreover, there are basic concepts of financial theory that cannot be ignored when being introduced to the world of corporate finance and company valuation, since they have a crucial impact overall learning experience.

After studying this chapter, you will be able to answer the following questions, among others:

1. What is a corporation?
2. What legal forms can a business take?
3. What is the importance of the financial statement of the firm?
4. How can pro forma financial statements be used to forecast the financing needs of the company?
5. How can arbitrage be used to understand financial markets?

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The first section of the chapter is an introduction to the organizational forms a company can take, their main features, and the differences among them. The second section focuses on the analysis of the financial statement of the firm, which is made of the balance sheet, the income statement, and the cash flow statement. The third section deals with introducing the concept of arbitrage on financial markets, a foundation of modern financial economics. The law of one price is described, as well as the Brownian motion for stock prices and the market price of risk, in its analytical form.

1.1 The Corporation

Learning Outcomes

- Analyze and comment on the corporate organizational forms.
- Understand the objectives of a business.
- Become familiar with the role of financial management.

1.1.1 Organizational Forms

A business can be organized in several ways. The main issue is about legal liability associated with the business, which will change according to what corporate form is chosen.

What follows is a comprehensive analysis of the most common forms of corporate organization, and the impact of their structure on the responsibilities assigned to the ownership and management.

Sole proprietorship is represented by a company owned by just one person, and it is the simplest type of business organization and consequently the least regulated by commercial and financial laws (Allen and Sherer 1995).

In many places in the world, the start of a sole proprietorship is subject to getting a business license to begin operations. Given the ease of start-up, this form of organization is the most developed.

In general, most of the world's businesses that have become big enterprises had a start as small sole proprietorships, evolving during time. These companies benefited from an easy start while growing progressively to become big and fully regulated corporations.

Example 1.1 Bookkeeping, home healthcare, landscaper, computer repair services, catering company, and housecleaning service are typical examples of business that are normally structured in the form of sole proprietorship.

All profits of a sole proprietorship go to the owner, with the disadvantage that the owner is unlimitedly liable for the business obligations. Creditors can bypass the business and get to the personal wealth of the owner.

The distinction between personal and business income is not defined, and all income is taxed through personal income tax. The life of the business coincides with the life of the owner, with a limit in equity that can be raised equal to the personal wealth of the sole owner.

That limitation hinders the possibility to exploit business opportunities and limits the growth of the business, due to insufficient capital. That is why the ownership of a sole proprietorship is difficult to transfer, given that it represents a transfer of the whole business to another person.

The option of being a sole proprietor involves much less paperwork than other organizational forms, and the business is structured so that legal documents are not required to determine the allocation of profit sharing from business operations.

The sole proprietorship links the owner and the business very strictly, with no distinction. However, it does not preclude the use of a business name, brand, and denomination that is different from the owner's name. The income of the proprietor is taxed as self-employment income.

Sole proprietorship is not the right organizational form for high-risk business in that personal assets are put at risk. The need for a high amount of debt for business start-ups and a history of trouble with personal debt in the past may be a reason to choose a different legal structure, in order to protect the personal assets.

Examples of risky business are those involving human care, the manufacturing of food, the repairing of valuable items, and the alcohol-related business. In addition to these examples, many other activities can make your business high risk.

In case there are some risks involved in the business, but they are not very high, it is still feasible to choose the sole proprietorship; insurance policies are right to cover the basic risks associated with the business operations.

Another form of business is partnership. It is a simple organizational form, which allows two or more partners to be owners of the company. The partners share gains and losses from the business, with unlimited liabilities on partnership obligations.

A partnership agreement describes all the conditions of the enterprise including how profits and losses are divided among partners. Such an agreement can be informal and oral, or a formal written document, depending on jurisdictions (Arrow 1974).

In summary, there are three possible forms of partnership, namely, the general partnership (GP), the limited partnerships (LP), and limited liability partnerships (LLP).

A limited partnership is characterized by the presence of both general partners and limited partners. General partners have unlimited liability and run the company on behalf of limited partners, whose liability is limited to the amount that they contribute to the partnership. Maybe arrange these with limited partnerships first, so they match the order in which they are introduced.

The advantages and disadvantages of a partnership are the same as those of a proprietorship. Partnerships based on a relatively informal agreement are easy and inexpensive to form.

Example 1.2 Companies like medical, legal, real estate, and creative are normally structured as partnerships.

The partnership for general partners terminates at the death of a partner or transfer of quotes. The partners are taxed by personal income tax, and the amount of equity that can be raised is limited to the partners' combined wealth.

The ownership of a general partnership is particularly hard to transfer, because it involves the formation of a new partnership. The limited partners can instead transfer the property of their shares without dissolving the partnership, but finding a buyer may be very difficult.

It is very important to get a written agreement when forming a partnership. In fact, a partner in general partnership can be held responsible for all business liabilities, making it very uncomfortable to not have all rights and duties spelt out from the beginning.

The limited partners are by logic not deeply involved in important decisions for the company, given that they are not personally liable for those. When things go bad, limited partnerships act as a shield against creditors, unless the partner has turned into a general partner for some reason.

The disadvantages of the partnership are the same as sole proprietorship and can be summarized as unlimited liability for business debts, limited life of the business, and difficulty of transferring the shares.

The growth of the business is seriously hindered by its own intrinsic features and can be seriously limited by an inability to raise cash for investment, due to the legal structure.

Limited partnerships are widely used by professionals such as doctors and lawyers, and the structure protects partners in these types of jobs that involve serious responsibilities. Different countries have varying regulations regarding these establishments of which business owners must take note.

Partnerships must file information returns with the fiscal agencies, but they do not file separate tax returns. For tax purposes, the partnership's profits or losses pass through to its owners, so a partnership's income is taxed at the individual level.

The limited liability company structure allows for separation between a company's liabilities and the owner's personal assets. In this way, the owner is not responsible for the company debt with his own wealth.

Special cases also entail personal responsibilities for the limited partnership owner, but they are limited to those cases where the partner intentionally does something fraudulent, reckless, or illegal or fails to adequately separate the activities of the LLC from personal affairs.

Depending on the location of the business, the rules governing the LLC can change considerably. Some countries or states, for example, do not allow banks, insurance companies, or nonprofit organizations to be LLCs.

In some federate countries like the United States, for example, the LLC is a state structure, with no special federal tax duties. It can take the form of individual, partnership, or corporation, and once the paperwork is filed and all fees are paid, the business can be started.

Some regulatory systems require simple documents to be filed, like articles of organization and the operating agreement containing information about each owner's percentage interest in the business, responsibilities, and voting power, as well, as how profits and losses will be shared and what happens if an owner wants to sell her interest in the business. In some cases, a notice in some local newspaper stating that you are forming an LLC should be published as well.

Similar to the LLC, the corporation form of organization separates the business entity from its owner, thus reducing the exposure to liabilities. The main difference resides in the taxation, accounting standards, record keeping, and paperwork requirements.

Corporation is not the most logical form of business for a start-up, unless required by circumstances, and the LLC is a much better choice in terms of accounting standards and regulatory issues.

In order to establish a corporation, many of the same steps of founding a LLC must be followed, including choosing a business name, appointing directors, filing articles of incorporation, paying filing fees, and following any other specific requirements.

Corporations may take different subforms. For example, in the United States, there are two types of corporations, namely, the C corporations and the S corporations. They differ in terms of management and legal and fiscal duties.

The C corporations have separate tax duties and file their own tax and income papers, with the income being retained as equity in the corporation until paid out as salaries and other benefits to the officers and employees.

Given that corporate tax is usually lower than personal income tax, in most cases it is preferable to leave the income in the company rather than take it out for personal wealth increase.

When the corporate income is just enough to support the owner's private expenses is of course a separate issue, and in this case, there is no way that money can be left in the company, since the owner needs it as cash.

In case of shareholders, the issue of double taxations arises in that the income earned by the corporation is taxed and dividends distributed to shareholders are taxed. However, provisions stand so that the owner of a one-person corporation does not have to worry about double taxation.

As opposed to C corporations, the S corporations are intermediary entities, and their income, losses, deductions, and credits pass through the company and become the direct responsibility of the company's shareholders.

It is the responsibility of the shareholders to report all the items associated with the corporate activities on their personal income tax returns; thus, S corps avoid the income double taxation that is associated with C corps.

In some countries, all shareholders must sign specific forms to make the business an S corporation for tax purposes. The authorities often require S corporations to meet the following requirements:

- Be a domestic corporation.
- Have only allowable shareholders, including individuals, certain trusts, and estates.
- Not include partnerships, corporations, or nonresident alien shareholders.
- Have no more than a certain number of shareholders.
- Have one class of stock.
- Be an eligible corporation.

Regardless of the way a business is structured, its owners will have the same overarching goals when it comes to the company's financial management.

1.1.2 The Corporate Objective

Corporate finance is the discipline that studies the relationship between the business and financial decision of the management, and their impact on the value of the business, by looking at how the capital is raised and how the value is generated.

The theory of corporate finance is led by a unifying objective, so that all models of the theory are built around it. The objective in conventional corporate financial theory when making decisions is to maximize the value of the business or firm (Coase 1937).

The main issue with the theory is that it is entirely built on that one objective, making the connection so strong that to the degree that this objective is flawed, it can be argued that the theory built on it is flawed as well.

The main disagreement between corporate finance theorists and practitioners is in fact the view they have about the real objective of the firm. Some critics of corporate finance argue that firms should have multiple objectives, in order to satisfy multiple interests (stockholders, labor, and customers).

Other experts suggest that companies should focus on what they view as simpler and more direct objectives, such as market share or profitability, regardless of the actual value the company has.

In order to fully understand the reasons to choose one objective among the abovementioned many, it is crucial to understand the reasons behind the choice of corporate finance to focus on corporate value.

Claiming that the objective of the firm is profit and value maximization is equivalent to saying that what stockholders do in their own self-interest is also in the best interests of the firm.

On the other hand, criticism points out that the goal is considered dependent on the existence of efficient markets, and it is often blind to the social costs associated with value maximization.

In this framework, the role of the financial managers in making decisions on behalf of the shareholders is crucial. The list of possible goals for a financial manager can be very long and diversified.

What really matters is the definition of a good financial management decision from the point of view of the shareholders. If we assume that stockholders buy stock because they seek to gain financially, then the obvious definition is that good decisions increase the value of the stock, and poor decisions decrease the value of the stock.

It follows that the appropriate goal for the financial manager is to make decisions that increase the value of the stock, therefore maximizing the current value per share of the existing shares.

Having such a target helps in overcoming the difficulties implicit in assuming a complicated list of many different goals. The criterion is not ambiguous, and there is no short-run versus long-run issue.

The concept of value maximization as the core objective of the firm is justified by considering the shareholder as residual owner. This means that after having paid for the employees, suppliers, creditors, and taxes, what is left belongs to the shareholder.

If any of the above obligations (or any other arising from the corporate activities) are not met, the shareholder gets nothing. Therefore, if the shareholders are getting a positive income from the corporation, it means that everyone else has also been satisfied.

The core of the financial manager's activity is therefore to identify the investments and financing arrangements that have a favorable impact on the profitability of the company and its overall value consequently.

Although the following chapters are dedicated to the corporation, the basic idea applies to all forms of business, with slight differences. Some considerations even apply to nonprofit organizations, which are beyond the scope of the book.

It is also true that the objective should be reached by avoiding illegal or unethical actions in the hope of increasing the value of the equity in the firm. Therefore, the financial manager best serves the owners of the business by identifying goods and services that add value to the firm because they are valued in the free marketplace.

In corporations where there are many shareholders, and the property is heavily diluted, the manager gets effective control of the firm and may not necessarily act in the best interest of the shareholders.

Put another way, the management might be tempted to pursue its own objectives, of salary maximization through incentives, and job securing, at the stockholders' expense. These considerations must be taken into account when exploring financial management.

The relationship between stockholders and management is called an agency relationship. It is established when an owner or group of owners hires an agent to manage the corporation.

As mentioned above, there is an agency problem whenever the interest of the company's manager differs from that of the shareholders. For example, consider a new investment providing a high value at the cost of relatively high risk.

The owners of the firm will wish to take the investment (because the stock value will rise), but management may not because there is the possibility that things will turn out badly and managerial jobs will be lost.

So one choice is to not pursue the project, but on the other hand, the shareholders may lose value from that decision, involving an agency costs. Agency models are the focus of Chap. 7.

The scope of this section is to introduce the topic, in light of managerial issues. The agency costs can be indirect, when they translate in a lost opportunity for the business, or they can be direct.

Direct costs get two possible forms. The first type is a corporate expenditure that benefits management but costs the stockholders. The second type of direct agency cost is an expense that arises from the need to monitor management actions.

Some experts argue that letting managers decide by themselves would result in them trying to maximize the amount of resources over which they have control or, more generally, corporate power or wealth.

Managers typically will act in the interest of their job position first, and the risk is that they will try to demonstrate corporate power or even increase the nominal value of the business by, for example, overbuying other companies.

There are two factors at the basis of the behavior of the management toward the interest of the shareholders. First, the owners of the company should make sure that their goals are aligned to those of the managers. The alignment of goals can be done in various ways, for example, through a well-structured compensation plan.

Managerial compensation management acts in defense of shareholder rights by tying incentives to the financial performance of the firm, thus aligning the objective of the manager to that of the ownership.

For example, managers are frequently given the option to buy stock at a bargain price. The more the stock is worth, the more valuable is this option. In fact, options are increasingly being used to motivate employees of all types, not just top management.

Another factor is the job prospect, in the sense that better performances are rewarded with promotions and longevity of managerial contracts. Managers who are successful in pursuing stockholder goals will be in greater demand in the labor market and thus command higher salaries (Fama and Jensen 1983).

Executive compensations are public information and can be easily retrieved on the websites or papers of any public company (see Table 1.1), together with all the other pieces of financial information.

A particular form of defense for shareholders against an unwanted manager is called a proxy fight, which allows some of the shareholders to vote on behalf of someone else’s stock.

This particular situation happens when a group solicits proxies in order to replace the existing board and thereby replace existing management. The mechanism is that of shareholders trying to put their nominees on the board and let them fire the management on their behalf.

One of the main tools companies use for value maximization is cost control. By controlling costs, they can manage and reduce their business expenses in order to maximize the profit from operations.

A crucial duty of a financial manager is to identify and evaluate all of the business’s expenses, to determine whether those costs are reasonable and affordable, and reject them if useless or superfluous (Winton 1993).

Financial management deals with costs on a daily basis, by looking at ways to reduce them through various methods. Cutting back, moving expenses in time, or

Table 1.1 10 Highest paid CEOs in the United States in 2016 (total compensation)

Name	Company	Approx. salary ($)
Tom Rutledge	Charter Communications	98,000,000
Les Moonves	CBS Corp	68,600,000
Bob Iger	Walt Disney Co.	41,000,000
David Zaslav	Discovery Communications	37,200,000
Robert Kotick	Activision Blizzard Inc.	33,100,000
Brian Roberts	Comcast Corp.	33,000,000
Jeff Bewkes	Time Warner Inc.	32,600,000
Ginni Rometty	IBM	32,300,000
Leonard Schleifer	Regeneron Pharmaceuticals	28,300,000
Steve Wynn	Wynn Resorts	28,200,000

Source: Business Insider: http://www.businessinsider.com/highest-paid-ceos-2016-2017-5

changing suppliers are some of the tricks a manager can adopt in order to reduce costs.

The task seems to be very complicated if one thinks about how many different expenses a company incurs during daily activities, from energy to phone, Internet and various utilities, employees' salaries, outsourced services, and more.

When costs are too high, profit margins shrink, and it becomes difficult for a company to be successful against competitors. For a public company, high costs involve a decrease in the shares' value and difficulty in attracting investors.

Another possible goal of the corporation is the market share maximization. Market share is the ratio between the total sales of the company and the total sales of the industry over the same period.

The measure gives the size of the company relative to the market and competitors, defining the position of the firm in the industry sector of reference. That is why companies are always looking to expand their share of the market. On top of that, it is common practice to grow the size of the total market by appealing to larger demographics and lowering prices or through advertising.

When market share increases, a company gets the opportunity to expand its business by growing the scale of the operations and improving profitability. The market size itself is not fixed, but it varies according to it being mature or growing.

The market share of a firm is a good indicator of the competitiveness of its products and services in the industry sector of pertinence. As the total market for a product or service grows, a company that is maintaining its market share is growing revenues at the same rate as the total market (Williamson 1981).

A consistent growth in market share will be reflected in a growth of the firm's revenues, at a rate much faster than nongrowing competitors. Growth markets are typical of technology sectors; consumer goods companies generally operate in a mature market.

Start-ups and small companies usually experience higher growth rates than well-established mature ones. This is because a company that has already achieved a large market share will grow its sales more difficultly as the potential customers decrease in number.

It seems appropriate at this stage to dedicate a final consideration to companies that are not publicly traded. In this case, the objective has to be defined as a more general goal, given that corporations are not the only type of business in the economy.

A difference can be made between companies that have traded stocks and companies that do not. Corporations are certainly not the only type of business, and the stock in many corporations rarely changes hands, so it is difficult to say what the value per share is at any given time.

Anyway, no big modifications are needed when dealing with for-profit organizations. The total value of the company can be seen in fact as the value of the equity in the hands of the owners.

The corporate objective can then be stated in more general terms as to maximize the market value of the existing owners' equity. With this in mind, it does not matter whether the business is a proprietorship, a partnership, or a corporation.

1.2 Financial Statement Analysis

Learning Outcomes

- Learn how to analyze the financial statements of the corporation.
- Learn how to calculate and interpret financial ratios.
- Learn how to build and analyze pro forma financial statements.

1.2.1 Balance Sheet

Financial accounting takes care of the papers of the company and the numbers generated by the business. In the very general case, the papers take the form of a balance sheet, an income statement, and a cash flow statement.

The simplest forms of business are allowed by law to keep record in a very simplified form, thus not recognizable in the above definition of financial accounting, applying instead to limited liability ventures and corporations.

A standard balance sheet is the only part that applies to a single point in time of a business' calendar year. It is structured in three parts: assets, liabilities, and ownership equity.

Assets are listed in order of liquidity, with main categories being first in the list. After the assets, the liabilities are listed. The difference between the assets and the liabilities is known as equity.

The equity of a firm is also known as the net assets or the net worth or capital of the company, and it is straightforward from the accounting equation that the net worth must equal assets minus liabilities.

The sense of the balance sheet equation is that equity and liabilities finance the assets of the corporation. The financing happens either by borrowing money (liability) or by using the owner's money (owner or shareholders' equity).

This is why the layout of the balance sheet is such that it is usually presented with assets in one section and liabilities and equity in the other section, so that the algebraic sum of the two sections equates.

The simplest forms of business that operate only in cash can simply check the bank balance at the end of the observation period and sum up the cash in hand, in order to get the financial picture of the business.

However, most businesses, and corporations especially, operate through debit and credit, meaning that they do not pay their obligations immediately, nor are paid for the goods and services they supply, immediately.

Corporations build up inventories of goods, and they acquire buildings and equipment, so that they have assets that they cannot, even if they want to, immediately turn into cash at the end of each period.

As opposed to the other parts of the financial statement, a balance sheet is a static snapshot of the firm at the end of each fiscal year. It organizes and summarizes the assets and liabilities to create a picture of what the firm owns and what it owes.

Assets usually are put on the right-hand side of the balance sheet, and their first classification of assets is between current and noncurrent. Current assets are cash, receivables, and any other assets that can be easily converted to cash within a short time (usually 1 year) or whose life is shorter than that time.

The current assets convert to cash in less than 12 months, thus including the inventories that are processed into final products and sold in a short time, or accounts receivable (credits) to be cashed in a short time. Examples of current assets are:

– Cash: it includes the accounts used by the business to receive customer payments and pay business expenses, or any quick-access account, which keeps a fixed amount of cash in it.
– Accounts receivable: it shows all credits that the firm has toward customers to get to a completed sales transaction. It is money not yet received, but expected supposed to flow in the near future.
– Inventory: it includes the goods available for sale, after completing the production process, or also products on shelf for a distribution store. For a manufacturing company, the definition also includes the raw materials used for production.
– Prepaid expenses: these are expenses paid for in advance, such as rent, insurance, office supplies, postage, travel expense, or advances to employees. If they carry benefits in the 12-month period, they qualify for inclusion in current assets.

Noncurrent assets have a relatively long life and cannot be easily turned into cash in a reasonably short time. There can be tangible noncurrent assets (usually buildings, equipment, vehicles, and more) and non-tangible assets (trademark, patents, brand, and more) which are not physically consistent but represent a value to the company. Examples of noncurrent assets are:

1. Long-term assets: these are supposed to be held by the company for more than 1 year. The category includes investments and property, plant, and equipment currently in use by the company in day-to-day operations.
2. Fixed assets: it includes the company's physical assets like property, plant, and equipment. The account also includes long-lived assets, such as a car, land, buildings, office equipment, and computers.
3. Long-term investments: these investments are financial assets held by the company, such as bonds, stocks, or notes.
4. Intangible assets: the category includes assets that are not physical but still have a value to the company, like patents, trademarks, and goodwill. They all classify as noncurrent assets.

The right-hand side of the balance sheet incorporates liabilities and shareholders' equity.

Liabilities are obligations of the firm that have to be paid in cash within a stipulated period. In most cases, the obligation implies the payment of some capital plus the interest accrued over time.

First, liabilities are listed, and as for the assets, it is possible to differentiate between current liabilities and noncurrent liabilities.

Current liabilities are listed before the noncurrent liabilities in the balance sheet. They are obligations due for payment within 12 months, with the same life span of current assets.

The settlement of current liabilities is done by using current assets, mostly cash, or current sales of inventory. In some cases, it is also possible to swap out one current liability for another.

Examples of current liabilities are:

- Short-term notes payable: these are notes due in less than 12 months after being registered in the balance sheet. A working capital loan, issued by banks with the expectation that it will be paid back from collecting accounts receivable or the sale of inventory, is an example.
- Accounts payable: this account shows the amount of money the company owes to its vendors, for having acquired supplies or services.
- Dividends payable: payments due to shareholders of record after the date of dividend declaration.
- Payroll liabilities: as it is typical for a company to owe the payroll and related taxes, having not paid them yet, it is typical for companies to accrue them in the balance sheet.
- Current portion of long-term notes payable: when the company has a long-term note on payment (due after a 12-month period). The portion of that note that is paid at registration must be showed on the balance sheet as a current liability.
- Unearned revenue: this category includes money the company collects from customers that it hasn't yet earned by doing the complete job for the customers but that it anticipates earning within 12 months of the date of the balance sheet.

Debts with maturity longer than 12 months are commonly classified as noncurrent liabilities. Firms borrow long-term funds from a variety of sources. Common terminology use in finance is to use the terms debt and debt holders to refer to the amount owed and to whom it is owed, respectively.

Examples of noncurrent liabilities are:

- Bonds payable: the term indicates long-term lending agreements between borrowers and lenders. It is an alternative way to raise capital for a company, besides selling stocks.
- Long-term leases: capital lease is recorded on the balance sheet as an asset, rather than in the income statement as an expense. The related lease obligation must be recorded as a liability.
- Product warranties: the repairing and replacing obligations on products sold by the company that extend beyond 12 months are registered in the balance sheet as long-term liability.

As mentioned above, the shareholders' equity, which closes the right-hand side of the balance sheet, is calculated as the difference between the total value of assets and the total value of liabilities.

The equity of shareholders is a claim against the firm's asset, residual after payment of all the debt. The debt holder has the first claim on the company value, when borrowing is put in place.

Thus, the equity can be calculated as a difference between the total value of corporate assets and the amount of liabilities in the balance sheet, with the formula

$$\text{Assets} - \text{Liabilities} = \text{Equity}$$

This feature of the balance sheet is intended to reflect the fact that, in case the firm would sell all of its assets and use the proceedings to pay off its debts, the eventual residual value remaining belongs to the shareholders. Therefore, the value of the left-hand side always equals the value of the right-hand side, giving the balance (Table 1.2).

The accounting value (also named book value) of the equity is then dependent on the level of assets and liabilities and increases when earnings are partly retained, instead of being distributed as dividends.

The accounting value of a firm is generally recorded on a cost value base, meaning they do not reflect the actual value of the assets, so to make almost meaningful to call it book value.

Market value on the other hand is the price at which buyers would be willing to trade the asset at present time. This value is usually different from accounting value, and the management's job is to create a value for the firm that is higher than its cost.

There are two major accounting standards that are used worldwide, namely, Generally Accepted Accounting Principles (GAAP) and the International Financial Reporting Standards (IFRS).

Table 1.2 A balance sheet for a hypothetical company AAA

Balance sheet (€ 000s) Company AAA			
Current assets		*Current liabilities*	
Cash	1,330,000	Accounts payable	382,000
Short-term investments	250,000	Accrued interest	8000
Accounts receivable	320,000	Other current liabilities	350,000
Inventory	225,000	Total current liabilities	740,000
Other current assets	52,000		
Total current assets	2,177,000		
Fixed assets		*Long-term liabilities*	
Property, plant, and equipment	11,250,000	Notes payable	118,000
Accumulated depreciation	(2,125,000)	Mortgages	3,600,000
Total net fixed assets	9,125,000	Other long-term debt	1,224,000
		Total long-term liabilities	4,942,000
		Equity	
		Stocks	5,300,000
		Retained earnings	320,000
		Total equity	5,620,000
Total assets	11,302,000	Total liabilities and equity	11,302,000

The GAAP is the accounting standard used in the United States, while IFRS is used in over 110 countries around the world. GAAP is considered more rule oriented, while IFRS is more principles based.

The two systems have been progressively converging in the last years to a similar standard, and the US Securities and Exchange Commission is looking to switch to IFRS by 2015, meaning that GAAP will no longer be used in the future.

There are some key differences between the two standards. In general, due to its principle-based nature, IFRS is supposed to represent and capture the economics of a transaction better than US GAAP.

The main differences between GAAP and IFRS can be summarized as follows:

- Intangibles: GAAP recognizes intangibles at their fair value, while IFRS only recognizes them if the asset will have a future economic benefit and has measured reliability.
- Inventory costs: IFRS does not allow for the last-in, first-out (LIFO) method to be used, while GAAP allows both LIFO and first-in, first-out (FIFO) standards to be used for inventory accounting. Moreover, the IRFS allows revision of the write-down of inventory in the future, according to some criteria being met, while GAAP does not allow for any revision once the inventory has been written down.
- Consolidation: in the IRFS, a control model is preferred, while in GAAP a risk-reward model is used.
- Extraordinary items: according to IFRS, extraordinary items are not segregated in the income statement, while they are shown below net income in the GAAP.

- Earnings per share: under IFRS, the earning-per-share calculation does not average the individual interim period calculations, whereas under US GAAP, the computation averages the individual interim period incremental shares.
- Development costs: these are considered expenses under GAAP standards, while they are capitalized under IFRS only if they meet some specific criteria.
- Write-downs: IFRS uses a single-step method for impairment write-downs rather than the two-step method used in US GAAP, making write-downs more likely.

In general, IFRS involves less paperwork, and it gives a detailed picture of the corporate financials in a more synthetic way, making it a desirable standard worldwide. That is probably the reason behind the shift to IFRS all over the world.

At present, most liabilities show up on the balance sheet at historic cost rather than fair value. Moreover, as long as they are properly classified as current, there are no requirements from GAAP for the order in which they appear on the balance sheet.

1.2.2 Income Statement

The income statement measures a company's financial performance over a specific accounting period. It summarizes the incomes and expenses incurred by the business in the period considered, including both operating and nonoperating activities.

As a final result, it shows the profit or loss incurred in the specified period, so that it can also be called profit and loss statement. The income statement is divided into two parts: the operating items section and the nonoperating items section.

In the operating items, revenues and expenses coming as a result of the regular business operations are listed, while the revenue and expense information about activities that are not tied directly to a company's regular operations are included in the nonoperating section.

If a car producer sells cars, this is an operating activity that generates operating revenue. In addition, when buying raw material for car construction, the producer incurs an operating expense.

Assume instead that the car seller is selling some of the equipment previously employed for production. In this case the operation is not related to the core business and therefore leads to a nonoperating profit. The same reasoning works for buying and losses (Needles and Crosson 2002).

The income statement is made of many entries that contribute in calculating the profitability of the corporation. What follows is a list of the main entries with an explanation:

Operating incomes can be listed as:

- Revenues: income earned or asset enhancement from delivering or producing goods, rendering services, or other activities that constitute the entity's ongoing major operations, over the period considered. It is net of all discounts and allowances and usually referred to as gross revenue or sales revenue.

- Expenses: any incurrence in liabilities, as cash outflows or using up of assets or incurrence of liabilities from the main activities for production and delivering of goods, in the specified period.
- Cost of goods sold (COGS): all the direct cost related to the production of goods and services, during the specified period, including material, labor, and overhead costs. Operating costs, such as selling, administrative, advertising, or R&D, are excluded.
- Operating expenses or selling, general, and administrative expenses (SG&A or SGA): these include all payroll costs, and it usually represents a large share of total costs for production. The category includes:
- Selling expenses: these are the costs incurred to sell products and include salaries for sales agents, commissions, shipping, advertising, and depreciation of sales-related assets, during the specified period.
- General and administrative (G&A) expenses relate to the management of the business, including compensation of executives, professional fees, insurance, utilities, and rent.
- Depreciation and amortization: the ratio of asset value capitalized on the balance sheet for a specific accounting period. It is a systematic and rational allocation of cost rather than the recognition of market value decrement. This section treats the two separately, because normally depreciation refers to tangible assets, while amortization refers to intangible assets. However, please note that in some cases the two terms are considered equivalent in the literature.
- Research and development (R&D) expenses represent expenses included in research and development.

Nonoperating incomes can be listed as:

– Other revenues or gains include the outcomes from activities that are not primary business, like gained rents, income from patents, and goodwill. It also includes unusual or infrequent gains, like the ones coming from sale of securities or from disposal of fixed assets.
– Other expenses or losses are also not related to primary business operations and include, for example, foreign exchange loss.
– Finance costs are the expenses incurred for borrowing from various creditors and include interest paid and bank charges.
– Income tax expense is the sum of the amount of tax payable (or receivable) to (from) tax authorities in the current reporting period and deferred tax liabilities (or assets).

Incomes from irregular assets come from discontinued operations, which are the most common type of irregular items. It refers to a segment of a company's business that has been sold, disposed of, or abandoned. It can range from a certain product line to an entire line of business (Jensen and Meckling 1976).

Table 1.3 A single-step income statement for a hypothetical company ABC

Income statement: single step Company ABC	
Sales	1,015,000 €
Interest earned	15,000
Total revenues	*1,030,000*
Cost of sales	(500,000)
Depreciation	(40,000)
Amortization	(10,000)
Commissions paid	(40,000)
Office supplies	(8000)
Office equipment	(10,000)
Advertising expenses	(12,000)
Interest paid	(3000)
Other operating expenses	(2000)
Taxes	(100,000)
Total expenses	*(725,000)*
Net income	*305,000 €*

First of all, the operating income is calculated as difference between all revenues and net of discounts, minus the expenses related to the production of the revenues. The costs deducted from revenues are typically the cost of goods and the operating expenses.

From the operating income, it is possible to calculate the recurring income by summing up with the other income that is not operative but investment-related. To be included in this category, these items must be recurring in nature.

As mentioned previously, the income statement can be presented in either a single-step (see Table 1.3) form or a multiple-step form (see Table 1.4). Both forms yield the same net income, besides showing different levels of details in the calculation.

Single-step income statement is very common and easy given that it uses only one mathematical passage to calculate net income. The formula is

$$\text{Net Income} = (\text{Revenue} + \text{Gains}) - (\text{Expenses} + \text{Losses})$$

Based on this, and recalling the definitions for the statement entries given above, a single-step income statement looks like

The multi-step statement is an alternative that slightly complicates the accounting of revenues and costs because it uses multiple subtractions in computing the net income shown on the bottom line.

It does by separating the operating revenues and operating expenses from the nonoperating revenues, nonoperating expenses, gains, and losses. Gross profit is also shown as net sales minus the cost of goods sold. Here is a sample income statement in the multiple-step format:

Table 1.4 A multiple-step income statement for a hypothetical company ABC

Income statement: multiple steps Company ABC	
Sales	1,015,000 €
Cost of sales	(500,000)
Gross profit	*515,000*
Advertising expenses	(12,000)
Commissions paid	(40,000)
Total selling expenses	*(52,000)*
Office supplies	(8000)
Office equipment	(10,000)
Total administrative expenses	*(18,000)*
Depreciation	(40,000)
Amortization	(10,000)
Other operating expenses	(2000)
Total operating expenses	*(52,000)*
Total expenses	*(122,000)*
Operating income	*393,000*
Taxes	(100,000)
Interest earned	15,000
Interest paid	(3000)
Total nonoperating expenses	*(88,000)*
Net income	*305,000*

The multiple-step income statement entails three fundamental steps for net income calculation. First of all the gross profit is calculated, then operating expenses are subtracted to get to the operating income, and finally the net position on nonoperating items (separately calculated) is added or subtracted in order to get the net income.

The use of multiple-step statement implies several benefits. First, it clearly states and isolates the gross profit amount, which is very important due to many investors being interested in the company's gross margin.

The second benefit is that also the subtotal of operating income is separated, which gives an indication of the profit earned by the company on its primary activities. Finally, as the bottom line, the statement reports the net amount for all the items on the income statement. If the net amount is positive, it is labeled as net income. If the net amount is negative, it is labeled as net loss (Williams et al. 2002).

In corporate finance, when it comes to the calculation of cash flows, it is very important to distinguish between expenses related to production and administration and those due to the depreciation of assets and payment of interest.

This distinction is crucial for the right calculation of the free cash flows to the firm and free cash flow to equity, as the basis for discounted cash flow valuation of the company, which is the focus of Chap. 8 of the book. An example of corporate finance income statement is shown in Table 1.5.

Table 1.5 A corporate finance income statement for a hypothetical company ABC

Income statement: Corporate Finance (€ 000s) Company ABC	
Sales	1,015,000
Cost of sales	(500,000)
Advertising expenses	(12,000)
Commissions paid	(40,000)
Office supplies	(8000)
Office equipment	(10,000)
Other operating expenses	(2000)
EBITDA	*443,000*
Depreciation	(40,000)
Amortization	(10,000)
EBIT	*393,000*
Interest earned	15,000
Interest paid	(3000)
Taxes paid	(100,000)
Net income	*305,000*

It is clear from the above statements that EBIT and operating margin are the same, and they are normally used as equivalent, for example, for calculating the profitability ratios associated with them.

A very interesting feature of the combined use of income statement and balance sheet, which is very appreciated by financial analysts, is the possibility it gives to calculate ratios and margins that give a deeper sense of the financial strength of the company.

In terms of importance and usage, profitability ratios are prominent in ratio and investment analysis. These ratios indicate the margins realized by the company on the sales over some specific period (quarter or year).

Bottom line, the profitability ratios give a measure of whether the company is able to generate adequate return for the stakeholders. As for many other ratios, in order to grasp the most of the informative power, they have to be compared with the industry averages.

Margin and ratios in fact normally vary across different industries, with some industries being typically low-margin types of business (easily available commodities, replicable products, and services, among others) and other industries being high-margin types of business (i.e., non-replicable and exclusive products and services, among others).

The gross profit margin (GPM), for example, indicates what profit a company makes on the cost of goods so to uncover how efficiently the various resources (included labor) are used in the production process.

The margin is calculated as

$$\text{Gross Profit Margin} = \frac{(\text{Sales} - \text{Cost of sales})}{\text{Sales}}$$

Example 1.3 Suppose that a company has 1,000,000 € in sales and the cost of its labor and materials amounts to 600,000 €. Its gross margin rate would be

$$\text{Gross Profit Margin} = \frac{(1,000,000 - 600,000)}{1,000,000} = 0.40 = 40\%$$

The GPM indicates how efficiently raw materials, labor force, and production-related fixed assets are used to generate income. A higher margin percentage is a favorable profit indicator.

There are differences in the GPM according to the type of business the company is running and what industry it belongs to. For instance, the airline industry has a gross margin of about 5%, while the software industry has a gross margin of about 90%.

The operating profit margin (OPM) is another interesting metrics, showing how successful a company's management has been at generating income from the operation of the business. It is calculated as

$$\text{Operating Profit Margin} = \frac{\text{Operating income}}{\text{Sales}} = \frac{\text{EBIT}}{\text{Sales}}$$

Example 1.4 If EBIT amounted to 200,000 € and sales were 1,000,000 €, the operating profit margin would be

$$\text{Operating Profit Margin} = \frac{200,000}{1,000,000} = 0.20 = 20\%$$

Measuring how much EBIT is generated for every euro of sales, the OPM measures roughly what operating leverage the company can achieve by running its core business (operational).

The higher the margin, the better, indicating that sales growth is faster than costs. Moreover, positive and negative trends in this ratio are, for the most part, directly attributable to management decisions.

The net profit margin (NPM) is a measure of the margins generated by the business, when taxes are also taken into account. It compares net income with sales in order to give a measure of how effectively managers run the business.

It is calculated as

$$\text{Net Profit Margin} = \frac{\text{Net income}}{\text{Sales}}$$

Example 1.5 If a company generates after-tax earnings of 100,000 € on its 1,000,000 € of sales, then its net margin amounts to

$$\text{Net Profit Margin} = \frac{100{,}000}{1{,}000{,}000} = 0.10 = 10\%$$

The NPM is the key index for understanding company profitability, so it is widely used in financial management and for investment analysis. It also varies between industries, and by comparing a company's gross and net margins, one can see the non-production and non-direct costs like administration, finance, and marketing costs.

A higher NPM means the company has an advantage compared to its competitors, and it is capable of increasing its market share during hard times, in order to benefit when the upturn is back.

Other ratios based on the profitability of the company (called profitability ratios) are (Table 1.6):

Activity ratios measure the efficiency of usage of the corporate assets, by comparing them (current and fixed assets) to different financial measures, so to describe the overall operational efficiency of the business.

All the activity ratios come in a form of turnover ratio, and they are calculated based on financial entries that are calculated on average over the reference period. All of the activity ratios measure the rate of turnover of the corporate assets and liabilities.

As a number, they can be reinterpreted as the number of times that the reference indicators (inventory, receivables, payables) are replenished over the reference period, in order to cover the other relevant reference financial sales, COGS, etc. (Table 1.7).

Table 1.6 Profitability ratios

Profitability ratios	
Return on asset (ROA)	$= \frac{\text{Net income}}{\text{Total assets}}$
Return on equity (ROE)	$= \frac{\text{Net income}}{\text{Equity}}$

Table 1.7 Activity ratios

Activity ratios	
Inventory turnover	$= \frac{\text{Cost of goods sold}}{\text{Average inventory}}$
Receivables turnover	$= \frac{\text{Net income}}{\text{Average receivables}}$
Payables turnover	$= \frac{\text{Cost of goods sold}}{\text{Average payables}}$
Asset turnover	$= \frac{\text{Net income}}{\text{Total assets}}$

Table 1.8 Liquidity ratios

Liquidity ratios	
Current ratio	$= \frac{\text{Current assets}}{\text{Current liabilities}}$
Quick ratio	$= \frac{\text{Current assets} - \text{Inventory}}{\text{Average receivables}}$
Cash ratio	$= \frac{\text{Cash}}{\text{Current liabilities}}$

Table 1.9 Solvency ratios

Solvency ratios	
Debt to asset ratio	$= \frac{\text{Total liabilities}}{\text{Total assets}}$
Debt to capital ratio	$= \frac{\text{Long-term liabilities}}{\text{Long-term liabilities} + \text{Equity}}$
Cash ratio	$= \frac{\text{Long-term liabilities}}{\text{Equity}}$
Interest coverage ratio	$= \frac{\text{EBIT}}{\text{Interest paid}}$

Liquidity ratios are a reference measure to creditors in that they reveal the level of liquidity generated and saved by the business. Consequently, they measure the ability of the company to meet the short-term obligations and the ability to make payments.

As for other ratios, liquidity ratios give full information when compared to competitors and the industry, in that some types of business normally run on high levels of liquidity, while others are not very liquid in assets. In addition, the dynamics (trend) of the ratios over time are important (Table 1.8).

As oppose as liquidity ratios, solvency ratios measure the company's ability to meet the long-term obligations, by providing information on the corporate capital structure, leverage, and level of cash flows generated.

It is therefore possible to understand, among other things, whether a company is able to repay the interest on debt. Lack of cash flows may hinder the payments, and the company could be at risk of default (Table 1.9).

1.2.3 Cash Flow Statement

The statement of cash flows is a report of the cash flows generated by the company's operations, in terms of operating, investing, and financial activities on cash flows over an accounting period (see Table 1.10).

It is a synthetic method to deduce interesting information about the following issues:

- How the firm obtains and spends cash
- Reasons for differences between net income and cash flows
- If the company generates enough cash from operation to sustain the business
- If the company generates enough cash to pay off existing debts as they mature
- If the company has enough cash to take advantage of new investment opportunities

Table 1.10 A cash flow statement for e hypothetical company XYZ

Cash flow statement (€ 000s) Company XYZ	
Net income	295,000 €
Depreciation	13,500
Adjustments	(7200)
Changes in account receivable	(4300)
Changes in liabilities	2600
Changes in inventories	68,000
Changes in other operating activities	(65,000)
Net cash flow from operating expenses	*7600*
Capital expenditures	(4500)
Investments	(125,500)
Other cash flows from investing activities	2350
Net cash flow from financing expenses	*(127,650)*
Dividends paid	(10,250)
Change in shares outstanding	(4750)
Change in debt	63,500
Other cash flows from financing activities	59,900
Net cash flow from financing expenses	*108,400*
Net change in cash and cash equivalents	*(11,650)*

The statement of cash flows is made of three sections:

- Operating activities
- Investing activities
- Financing activities

Each section leads to the corresponding free cash flow through calculation. Cash flow from operating activities (CFO) arises from normal operations such as revenues and cash operating expenses net of taxes. Operative activities range from production to sales and payments collected from customers. On the cost side, the list includes production and sales costs, inventory, and advertising.

The cash flows related to production activities therefore can be listed as:

- Income from the sale of goods or services
- Income for the sale of financial assets in a trading portfolio
- Interest received
- Payments to suppliers
- Payments to employees
- Other payments
- Interest payments (can be reported under financing activities depending on the accounting standard)
- Merchandise

In order to go from income to cash flows, it is then important to add (or subtract) back to the income some figures that have been subtracted (or added) previously to the gross income but do not constitute a cash flow. These items are:

- Depreciation
- Deferred tax
- Amortization
- Income or loss from trading noncurrent assets
- Dividends
- Revenue from certain investing activities

Obviously some items are summed up, and others are subtracted as can be described by the following example:

- Cash inflow
  - Income from the sale of goods or services
  - Interest received
  - Dividends
- Cash outflow
  - Payments to suppliers
  - Payments to employees
  - Payments to government
  - Payments to lenders
  - Payments for other expenses

The cash flow from investing activities (CFI) arises from investment activities such as the acquisition or disposition of current and fixed assets:

- Purchase or sale of an asset (land, building, equipment, marketable, etc.)
- Loans made to suppliers or received from customers
- Payments related to mergers and acquisition

Items subtracted as described by the following example:

- Cash inflow
  - Sale of property, plant, and equipment
  - Sale of debt or equity securities (other entities)
  - Collection of principal on loans to other entities
- Cash outflow
  - Purchase of property, plant, and equipment
  - Purchase of debt or equity securities (other entities)
  - Lending to other entities

The cash flow from financing activities (CFF) is built on the changes of cash due to the modification of the company's shares number or through short-term or long-term debt for the company's operations.

Financing activities are those generating inflow of cash from investors and outflow of cash to shareholders, in the form of dividends, as the company generates income. In general, all the activities which impact the long-term liabilities and equity of the company are included, and the most important are:

- Dividends paid
- Sale or repurchase of the company's stock
- Net borrowings
- Payment of dividend tax
- Repayment of debt principal, including capital leases
- Cash inflow
  - Sale of equity securities
  - Issuance of debt securities
- Cash outflow
  - Dividends to shareholders
  - Redemption of long-term debt
  - Redemption of capital stock

1.2.4 Pro Forma Statements

Besides official documents following the international standards, sometimes companies also prepare some statements that are adjusted to take into account the forecasted change in the profitability of the business.

The so-called pro forma statements allow investors to have a clearer picture of the operating results of the company, by setting some scenario under which the financial outcome of the business is rather manipulated.

The core of pro forma calculation of earnings and balance sheet entries is based on the concept of relevant cash flows. Given the variable set to create the desired scenario, only the entries in the statements that are related to that variable are adjusted in the pro forma version.

So onetime expenses are cut from the calculation in that they are normally not included in the accounting of ordinary corporate operations. Onetime items do not contribute to the company's representative valuation.

Pro forma statements are somehow controversial in that the company may choose to exclude selected financial entries in order to improve what is believed to be the correct financial representation of the business, sometimes at the expenses of accuracy.

While historical statements, based on either GAAP or IFRS statements, are an official documentation that should drive the investor's decision toward investing or not in the company, some analysts may find it interesting to read and interpret the forecasts the company gives in a pro forma manner.

Pro forma statements are normally based on the so-called percentage of sales (POS) approach. The rationale of such approach is that, as mentioned above, sales are normally considered to drive most changes in the balance sheet and income statement.

It therefore makes sense to use sales as the key factor to build up the pro forma statements.

By doing this, the accounting identity can be checked on pro forma basis from forecasted assets, liabilities, and equity. When the two sides of the forecasted balance sheet are not even, the external financing needed (EFN) can be calculated as difference.

Normally a plug variable will be chosen, in order to rebalance the statement, by acting on it. For example, an excess of assets compared to liabilities and equity could be addressed by increasing the amount of long-term debt.

One important point is that not all items in the financial statements vary with sales. It is therefore important to learn what financials increase or decrease when transferred to the pro forma statements and what others stay unchanged.

Items that normally vary with sales are:

- Costs generally do change with sales in that the profit margin is supposed to stay constant.
- Current assets generally do vary with sales, assuming that the increased costs are related to additional inventory needed for extra production to support the increase of sales. Same logic works for receivables and cash.
- Net fixed assets vary with sales only if the company is operating at full capacity. If the company is operating below full capacity in fact, the extra assets are considered to be already in place, to support the increase in sales.
- Current liabilities also vary with sales for reasons similar to those applied to current assets.

Items that normally vary with sales are:

- Depreciation does not vary with sales in that it is related to fixed assets.
- Interests paid are also not related in that they represent financial costs.
- Dividends depend on managerial decision. It is therefore opportune to not let them vary with the sales.
- Long-term debt and notes payable do not vary with sales because it relates to the capital structure decision.
- Equity does not vary for the same reasons as long-term debt.

This approach uses the cost of goods sold (COGS) as the overall cost related to production before EBIT is calculated, so that only interest paid and taxes are left out. The subtraction of the latter leads then to the net income.

Example 1.6 Assume company LTD is willing to perform a pro forma analysis on its financials, projecting figures forward by 1 year. The company has the following financial statements:

Company LTD
Current income statement (€ 000s)

	Current	% of sales
Sales	25,000	100
COGS	(15,000)	60
Interest paid	(0)	0
EBT	10,000	40
Taxes	(3750)	15
Net income	6250	25
Dividends (payout ratio, 60%)	3750	
Addition to retained earnings	2500	

Company LTD
Current balance sheet (€ 000s)

	Current		Current
Assets		*Liabilities and equity*	
Cash	3000	Accounts payable	2700
Accounts receivable	5300	Notes payable	8000
Inventory	3700	Long-term debt	9200
Other current assets	7500	*Total liabilities*	*19,900*
Total current assets	*19,500*		
Net PPE	22,000	Common stocks	17,300
Net fixed assets	*22,000*	Retained earnings	4300
		Total equity	*21,600*
Total assets	*41,500*	*Total liabilities and equity*	*41,500*

Sales are projected to increase by 10% next year, and the company has a current payout ratio of 60%, which the company is planning to keep constant. The pro forma income statement therefore looks like (with figures in dollar amounts and as percentage of sales):

Company LTD
Pro forma income statement at 10% projected sales growth (€ 000s)

	Current	% of sales	Pro forma	% of sales
Sales	25,000	100	27,500	100
COGS	(15,000)	60	(16,500)	60
Interest paid	(0)	0	(0)	0
EBT	10,000	40	11,000	40
Taxes	(3750)	15	(4125)	15
Net income	6250	25	6875	25
Dividends (payout ratio, 60%)	3750		4125	
Addition to retained earnings	2500		2750	

It is also possible to build the pro forma balance sheet in case the company is operating at full capacity. As already mentioned, some items vary, and some others do not. Pro forma retained earnings are obtained by summing up to the current retained earnings, the additional retained earnings, the latter being calculated as the product of the retention ratio (1 minus payout ratio) and the projected net income, as from the pro forma income statement.

$$\begin{aligned}\text{Retained earnings}_{\text{Pro-Forma}} = & \ \text{Retained earnings}_{\text{Current}} \\ & + \text{Retained earnings}_{\text{Projected Addition}}\end{aligned}$$

The current and pro forma balance sheets then look like:

Company LTD
Pro forma balance sheet (full capacity) at 10% projected sales growth (€ 000s)

	Current	Pro forma		Current	Pro forma
Assets			*Liabilities and equity*		
Cash	3000	3300	Accounts payable	2700	2920
Accounts receivable	5300	5830	Notes payable	8000	8000
Inventory	7500	8250	Long-term debt	9200	9200
Other current assets	3700	4070	*Total liabilities*	*19,900*	*20,120*
Total current assets	*19,500*	*21,450*			
Net PPE	22,000	24,200	Common stocks	17,300	17,300
Net fixed assets	*22,000*	*24,200*	Retained earnings	4300	7050
			Total equity	*21,600*	*24,350*
Total assets	*41,500*	*45,650*	*Total L and E*	*41,500*	*44,470*

According to the pro forma balance sheet, the external financing needed is

$$\text{EFN} = 45,650,000 - 44,470,000 = 1,180,000 \ €$$

meaning that the company will have to increase the right-hand side of the balance sheet by an amount of 1,180,000 €. This could be done by increasing the long-term debt or issuing more shares to increase the equity. Alternatively, the amount of planned retained earnings could be increased by reducing the payout ratio.

As mentioned before, in case the company is not operating at full capacity, the current and forecasted sales are based on partial usage of the potential of the corporate assets. It is therefore opportune to not grow them in the pro forma balance sheet in that the company is ready to face increase in production and sales without additional fixed assets required.

Example 1.7 Assume company LTD is operating below at full capacity. The pro forma balance sheet will then look as follows:

Company LTD
Pro forma balance sheet below full capacity (€ 000s)

	Current	Pro forma		Current	Pro forma
Assets			*Liabilities and equity*		
Cash	3000	3300	Accounts payable	2700	2920
Accounts	5300	5830	Notes payable	8000	8000
Receivable	3700	4070	Long-term debt	9200	9200
Inventory	7500	8250	*Total liabilities*	*19,900*	*20,620*
Total current assets	*19,500*	*21,450*			
Net PPE	22,000	22,000	Common stocks	17,300	17,300
Net fixed assets	*22,000*	*22,000*	Retained earnings	4300	7050
			Total equity	*21,600*	*24,350*
Total assets	*41,500*	*43,450*	*Total L and E*	*41,500*	*44,470*

As can be noted, things change due to the invariance of the net fixed assets, and the EFN becomes negative

$$\text{EFN} = 43,450,000 - 44,470,000 = -1,020,000 \ \text{€}$$

meaning that in this case the management should try to reduce the liabilities and equity by, for example, repaying back some of the debt or repurchasing some shares. Another way to do it is to increase the payout ratio so to reduce the amount of additional retained earnings.

Compared to GAAP and IFRS strict rules on reporting earnings, the pro forma approach allows the freedom to guess hypothetical figures, obtained following an estimation of the sales growth and adjusting the financial figures accordingly.

The pro forma earnings calculation is therefore something that is up to the standards and criteria of the company and may not include all costs that by default must be included when calculating current (effective) earnings as per accounting standards.

The pro forma analysis of financial statements contributes to the thorough understanding of the potential of the company and by the investors. Depending on the industry of relevance, sometimes pro forma statement may end up being more significant to the investors than current (official) ones.

Some of the ways a pro forma statement can differ from a standard one are the absence of noncash expenses, thus giving an idea of the cash profitability of the company and the absence of extraordinary expenses, when deemed irrelevant to the potential future earnings power of the company.

After important operations like mergers and acquisitions, abnormal onetime expenses are normally registered, but they do not affect the ordinary daily process of the business, thus pushing down the figures of profitability compared to those that can be recorded when the business runs smoothly.

One should also consider that pro forma statements, even if not publicly accepted and valid, are very often used as explanatory internal documents. They can be used by the management of the company and other stakeholders to assess the quality of the business and the potential future developments.

Analysts should pay attention when comparing pro forma statements of different companies, from the same or different sectors. It is very possible in fact that the discretion of the standards for compiling them can mislead the reader about the actual comparison between different companies.

1.3 Arbitrage

Learning Outcomes

- Learn the law of one price and how to describe it.
- Describe the various forms of arbitrage.
- Give examples of arbitrage and deviations from equilibrium.

1.3.1 The Law of One Price

The law of one price (LOP) states that the same good or service must have the same price, when traded in different markets, if it has the same features. In case of price disparity in fact, one could take advantage of the mispricing.

Usually the price disparity would not last because people would start buying on the cheaper market, and the prices on both markets would then adjust to match supply and demand, causing the convergence of prices.

The structure imposed on prices by economic incentives is the same in financial markets. Securities are the "same" if they produce the same outcomes in terms of combination of returns and risk (Schachermayer 2008).

Similarly, equivalent combinations of assets providing the same outcomes should sell for the same price, provided risk is taken into consideration. Therefore, the equivalence among financial securities involves the comparability of expected returns and risk.

If there is a mispricing for some asset sold on different markets, the LOP is violated, and investors will exploit the price disparity through arbitrage. Thus, the LOP imposes structure on asset prices through the discipline of the profit motive. The LOP is also violated when assets with the same risk profile have different expected returns.

For the LOP to hold, several assumptions must be made, about investors' behavior and preferences, according to given constraints. Specifically, the analysis of LOP assumes the following hold:

- The agents are utility maximizing and prefer more wealth, which considers potential returns, risk, and constraints.

- The agents chose their investment based on dominance, meaning the investment with best outcomes would be chosen.
- An investment generating the same outcome in all possible future states of the world is riskless and should therefore not earn more than the risk-free rate.
- Strategies that produce riskless returns but exceed the risk-free return on a common benchmark, such as risk-free government bonds of some kind, must involve mispriced investments.

In efficient financial markets, deviations from the LOP should not be persistent but very rare and so small as to not be worth the transactions costs involved in exploiting them.

If arbitrage opportunities show up, they will be exploited only by those traders with very little transaction costs for whom the strategy is profitable. The LOP is roughly a synonymous with equilibrium, which balances the forces of supply and demand.

It is possible to give a formal presentation of the LOP by introducing some straightforward math.

Consider an asset traded on two different markets (A and B):

- Transaction and transport costs from one market to another are observable.
- Prices are expressed in the same currency.

The equilibrium determined by the LOP with transaction and transport costs can be formally expressed as

$$P_{\mathrm{A}} = P_{\mathrm{B}} + c_{\mathrm{T}} \quad \Rightarrow \quad \frac{P_{\mathrm{A}}}{P_{\mathrm{B}} + c_{\mathrm{T}}} = 1$$

where:

P_{A} is the price in market A.
P_{B} is the price in market A.
c_{T} is the transaction and transportation costs.

The law of one price adjusted for transport and transaction therefore implies a specific equilibrium, usually named the fundamental law of one price identity (LPI).

The LPI is equal to one in absence of transaction costs, when the prices in the two markets are identical. In case there are transaction and transport costs, the LPI will be always lower than one.

If the difference of prices in the two markets is much lower than transaction and transport costs, then it is not optimal to transfer the good, and the two economies are both self-sufficient in wheat.

The analysis can be extended to the case of multiple markets, other than just two. In this case, markets which do not trade with each other must be analyzed in terms of common markets they trade with.

The price difference between each pair of two markets that do not trade with each other can be determined as the minimum difference in transport and transaction costs between the two markets to a market with which they both trade (Schachermayer 2008).

Long distances make two markets more unlikely to trade with each other and more likely to need a common intermediate market. It is then often argued that the difference between prices of a commodity in two markets increases monotonically with distance.

Sometimes the LPI differs from one, making the market being not free of arbitrage opportunities. As mentioned above, these differentials rapidly drop to zero, setting the market back to equilibrium.

Therefore, the LOP is not a permanent state, but rather an evolving system, where market forces act in order to restore equilibrium, when it has been subject to whatsoever shock.

Perfectly efficient markets will allow for only sporadic and very short violations of the LOP, depending on how strong the local shocks are and how fast the information is diffused to other markets, including distortions (Bjork 2009).

The LOP can be conveniently analyzed from an econometric point of view, with an error correction model, in which model equilibrium is estimated. If markets are not well integrated, there is no immediate LPI but a long-run equilibrium. A violation of the equilibrium between markets is called an innovation or shock and needs to be corrected in order for the equilibrium price difference to be restored.

Assume at time t the prices for market A and B are initially in equilibrium, and the LOP holds. Consider a shock hitting market A, so that the price in the market plus transport and transaction gets higher than in market B.

The demand shifts to the market where the price is lower, and that will cause the price at time $t + 1$ to increase in market B and decrease in market A, due to the shift of demand.

To summarize, the logic behind the error correction model is that prices in both markets will react if the price differential is larger or smaller than transport and transaction costs. In this case, the prices will adjust such that the deviation from equilibrium is decreasing. The error correction model is usually expressed in differences of log prices, and it is given by:

$$\Delta P_{\mathrm{A},t+1} = \alpha_{\mathrm{A}} \ln\left(\frac{P_{\mathrm{A},t}}{P_{\mathrm{B},t} + c_{\mathrm{T},t}}\right) + \varepsilon_{\mathrm{A},t}$$

$$\Delta P_{\mathrm{B},t+1} = \alpha_{\mathrm{B}} \ln\left(\frac{P_{\mathrm{A},t}}{P_{\mathrm{B},t} + c_{\mathrm{T},t}}\right) + \varepsilon_{\mathrm{B},t}$$

where:

$\Delta P_{A,\, t+1}$ is the change of price in market A, from time t to $t + 1$ due to the shock.
α_A indicates the speed at what price in market A is adjusted.
$\varepsilon_{A,\, t}$ is a standardized normally distributed error.

The market efficiency is indicated by the magnitude of the model parameters; the higher they are, the faster will the LPI be restored and the more efficient markets are, if the absolute values of the sum of the parameters should not exceed one.

As long as markets get more efficient, the magnitude of shocks tends to decrease, so the adjustments have to be smaller, and in the last centuries, the speeds of adjustments for markets like commodities decreased dramatically.

1.3.2 Forms of Arbitrage

As mentioned in the previous section, when the LOP is violated, arbitrage opportunities arise, giving investors the chance to achieve positive outcomes without any risk. Arbitrage can be therefore defined as the process of exploiting differences in the price of an asset by simultaneously buying and selling it on different markets and making a risk-free return out of it.

Example 1.8 Assume company WIZ's stock trades on the market at \$25.00 per share on the NYSE and an equivalent of \$25.40 on the LSE. An arbitrageur can purchase the stock in the NYSE and sell it on the LSE profiting of a positive payoff of \$0.40.

In the example of equities, the price should be the same on both exchanges all the time, but if they are not, arbitrage opportunities arise. Arbitrage is a riskless activity because traders are simply buying and selling the same amount of the same asset at the same time, and it is often referred to as "riskless profit."

Another example of arbitrage opportunity is given by mergers and acquisitions. When the market is favorable to that, they buy the shares of companies that are targeted by acquisition, with the objective to cash the money coming from the difference between trading price and the cash payment resulting from the merger.

Technically, such a strategy slightly differs from proper arbitrage in that it carries a small risk that the actual merger will not happen after all, leaving the investor with an amount of shares to be then sold back to the market, without any guarantee on the selling price.

In practice, arbitrage opportunities are very small and quick at disappearing, and only the very large institutional investors and hedge funds can fully exploit them, given their high endowment.

They are capable of taking advantage of arbitrage opportunities because they are able to trade large blocks of shares. Their investment scale is so big that they can gain huge profits even if the spread between two security prices is very small.

At the scale of the individual investor with limited resources, trading fees would neutralize any possible profit, making it impossible to benefit from the very small differences (usually just few cents) between prices that constitute arbitrage.

In the past arbitrage, opportunities were mainly created by the lack of real-time price update in different markets. Nowadays, the introduction of modern technology has decreased the number of arbitrage opportunities from that perspective.

There are several possible arbitrage strategies, depending on the type of activity and the type of financial asset involved in the transaction. Recall that in its purest form, arbitrage contains no element of risk (Schachermayer 2008).

Besides the standard, true form of arbitrage, which is unavailable to most traders, there are other forms of advantage that can be achieved by appropriately trading on assets to exploit riskless profit opportunities.

A popular form of arbitrage widely used in banking is the so-called inward arbitrage. It consists in borrowing from the interbank market, usually characterized by very low rates, and depositing the proceedings on a local currency deposit account at a higher interest rate.

In this case, the profit is given by the spread (difference) between the low interbank borrowing rate, and the much higher deposit rate, that is gained by the bank, following the application of the strategy.

Outward arbitrage, on the other hand, involves depositing local currency in a deposit account of some Eurobank. As oppose as inward arbitrage, the strategy works when the interest rate will be higher in the interbank market, which will enable the bank to earn more on the interest it receives for the use of its cash. It is also an efficient way for the bank to optimize its use of the cash, by investing it abroad, rather than in the local market, when more remunerative.

Example 1.9 Assume an American bank lends at the Eurodollar rate in the interbank market. The money will be then transferred from the parent bank in the United States to a foreign branch. The bank will earn revenues on the spread between the two interest rates. The larger the spread, the more will be made.

Triangular arbitrage involves three currencies and consists in converting one currency to another and then converting the resulting amount to a third currency for final conversion back to the original currency.

If market conditions are favorable, the final amount of original currency obtained could be higher than the initial amount in the same currency, making it possible to benefit from a risk-free and easy profit.

Mismatches in currency exchange rates give the opportunity for such an arbitrage to take place, but this does not happen very often, and opportunities only last for a few seconds on the markets.

Example 1.10 Assume the following exchange rates are currently quoted on the market: EUR/USD $= 0.85$, EUR/GBP $= 1.35$, and USD/GBP $= 1.64$. An arbitrage opportunity would involve:

– Sell \$1,000,000 for EUR: $1,000,000 \times 0.85 = 850,000$ €
– Sell 850,000 € for GBP: $\frac{850,000}{1.35} = £629,629.30$
– Sell £629,629.30 for USD: $629,629.30 \times 1.64 = \$1,032,592.59$

These transactions yield an arbitrage profit of \$32,592.59 (assuming no transaction costs or taxes).

Risk arbitrage is a form of speculation but still considered also a form of arbitrage due to its features. The definition is controversial because there is some degree of risk involved in the strategy, which is then not completely risk-free and true arbitrage.

The most common type of risk arbitrage is the so-called takeover and merger arbitrage, which takes place when there is an undervalued business that has been targeted by another corporation for a takeover bid (Bjork 2009).

Usually the takeover bid raises the value of the share in the market, because it is common for a buying company to offer a buying price, which is higher than current market price, in order to give incentive to shareholders to sell their shares.

Therefore, once the takeover is announced, the market price rises to a level, which is close to the offered takeover price. Arbitrageurs can identify a company targeted for takeover and buy the stock at the pre-takeover price, to sell them after the takeover has been completed.

In case the merger is successful in fact, the investors that bought the stock at the pre-takeover price will benefit from significant profits, dependent on the post-takeover price level, and the number of shares held.

In order for the strategy to be successful, timing is very important, and usually traders entering this type of strategy benefit of access to streaming market news. Within seconds of an announcement, they can act before regular retail traders.

The early trade, immediately after takeover bid announcement, raises the price to an intermediate level, which is the starting level for the risk arbitrage to take place. Trading at this price allows the investor to then wait for the final price to be reached and close the position in order to benefit from the price differential.

Statistical arbitrage is a form of arbitrage that attempts at profiting from price inefficiencies identified through analytical models. The investor attempts to profit from the likelihood that prices will trend toward a historical norm.

The relative-value arbitrage is an uncommon type of arbitrage, which relies on a strong correlation between two different securities. The strategy is to find pairs of asset that are historically strongly correlated and invest on them in times of divergence (temporary lower correlation).

If the strategy is successful, the correlation between the two assets will statistically be likely to turn back to a high level, and the pair will converge again to the same performance, so that the investor can benefit from the change in correlation.

Liquidation arbitrage consists in identifying companies in liquidation process, whose liquidation value is higher than market price. If this is the case, after liquidation the market price of shares will converge toward the higher liquidation (book) value.

Fixed income arbitrage is about identification of violations of the historical patterns for credit spreads, as well as deviations from the term structure relationships, with the belief of a future reestablishment of convergence.

Another case is when on the markets, there is overcompensation for credit risk or liquidity risk, creating another situation of potential profits after adjustments. Central bank intervention in the markets often creates abnormalities that can be exploited.

As other weak forms of arbitrage, fixed income arbitrage is not totally risk-free, and even when duration neutral strategies are adopted, still there is exposure to various other market risks, including term structure shifts and foreign exchange risk.

A special form of arbitrage involves convertible bonds. As will be more clearly explained in Chap. 5 about bond valuation, convertible bonds are issued with the option to be returned by the investor to the issuing company in exchange for a predetermined number of shares in the company.

The structure of a convertible bond is quite complicated in that it embeds a call option on the stock. Therefore, arbitrageurs often rely on sophisticated quantitative models in order to identify bonds that are trading cheap versus their theoretical value.

The three major factors affecting the price of a convertible bond are:

- Interest rate: when rates increase, the bond value of the convertible moves down and the opposite for the embedded option (aggregate movement is lower).
- Stock price: the price of the bond rises when the underlying stock price increases as well.
- Credit spread: when solvability of the issuer deteriorates, credit spread increases leading the bond price to decrease, while the option part moves higher.

As described by Bjork in 2004, the convertible arbitrage consists of buying a convertible bond and hedging (neutralizing) two of the three factors in order to gain exposure to the third factor at a very attractive price (Bjork 2009).

As mentioned above, modern arbitrage is made of very rare and elusive opportunities lasting the time of an eye beat. The price adjustment due to the shift of demand from the more expensive to the cheaper market closes the opportunity almost in real time.

The price convergence will be instant in a perfect market while taking a while in a market with imperfections. The principle behind arbitrage applies to financial instruments, which appear different but result in the same set of cash flows.

In summary, in order for a market to be free of arbitrage, none of the following conditions must be violated:

- The same asset must trade at the same price on all markets.
- Two assets with identical cash flows must trade at the same price.
- The price today of an asset is equal to its future value (if known) discounted at the risk-free rate.

The transactions of buying in the cheapest market to sell in the most expensive must happen simultaneously in order to avoid the market risk due to a sudden change of prices in both markets before the whole strategy is completed.

Given that statement, it is important to stress again how, in practical terms, this is almost impossible or only possible with securities and financial products which can be traded electronically. Such risk-free trading is not available to everyone.

1.4 Summary

A company can take one of the several corporate forms available to start a business. From the simple sole proprietorship to the more complex corporation, organizational forms can accommodate all sizes of business.

The corporate objective is an important element that defines every business and communicates to the outside world what the nature of a firm's business is. Properly identifying the corporate objective as the value maximization is the right start.

The life cycle of a company is an interesting topic involving the displacement of the corporate activities from the early stage to the end of business. It is made of several steps that constitute a cycle of alternate fast growth and steady state.

The financial statement of a corporation is the mirror of the financial outcome of its activities. The balance sheet displays the assets of the firm on one side, opposing it to the liabilities and equity on the other side of the book, with the two sides equating.

While the balance sheet is a picture of the company at the time it is observed, the income statement registers the results of the yearly activity of the firm, separating revenues from expenses.

The cash flow statement is a record of the inflows and outflows involved in the corporate operations. Cash flows must in fact be distinguished from corresponding income statement operations in that they are not always synchronized to them.

Pro forma financial statements allow the company to report forecasts on revenues based on selected figures that would not normally suffice for accounting standards. In this way, the company can illustrate to the reader the relevant financial figures that build up a forecast on future profitability. The percentage of sales approach is normally used by projecting forward the numbers in the financial statements, based on their connection with the sales growth (or decrease).

Arbitrage is a key concept in finance, which defines the conditions under which a market is considered normal and efficient. The law of one price is at the basis of the concept and allows the investors to define the conditions under which arbitrage holds.

Arbitrage can take many forms, depending on the financial instrument involved in the strategy or the type of market involved. Triangular arbitrage and currency arbitrage are among the most common forms.

The understanding of market inefficiencies is crucial in order to define the boundaries of efficiency of the most popular financial theories, including those touching the field of corporate finance.

Problems

- Explain the difference between unlimited liability and limited liability and how it affects the wealth of the firm and its owners.
- Explain the main differences between a sole proprietorship and a general partnership.
- What is a corporation? In what does it differ from a LLC?
- What is the structure of a balance sheet? How are the various entries reported based on the source and employability of capital?
- List and explain the most important current assets in the balance sheet of a firm. Do the same for the long-term assets.
- List and explain the most important current liabilities in the balance sheet of a firm. Do the same for the long-term liabilities.
- Define the process of net income calculation through a multi-step income statement. How does it differ from the single-step case?
- Suppose that a company has 1,300,000 € in sales and the cost of its labor and materials amounts to 750,000 €. What is the gross income margin?
- Assume a company has an EBIT amount of 1,200,000 € and sales were 2,100,000 €. What is the operating profit margin?
- Calculate all ratios explained in the chapter, for company AAA, as from the financial statements presented in the text.
- What is the net profit margin for the company in Exercise 8, if it generates after-tax earnings of 300,000 €.
- Assume the following exchange rates are currently quoted on the market: EUR/USD = 0.87, EUR/GBP = 1.33, and USD/GBP = 1.56. Is there an arbitrage opportunity? How can it be exploited?
- True arbitrage opportunities are rare. When they are discovered, they do not last long. So why is it important to explore arbitrage in detail?

Case Study: Pro Forma Statements

Projectcast Ltd

The Case

Nancy is a finance graduate, with specialization in finance. Last year she applied for a job position in the finance team of Projectcast Ltd, a company active in the production and sale of cooling systems for cars.

After going through several interviews, Nancy recently got hired by the company, with a position in the team taking care of forecast and financial planning. Upon hiring, Nancy is immediately faced with a forecasting task for a client company.

The task focuses on the analysis of the projected balance sheet of the company, over 1 year, in view of some future increase of sales to be forecasted as well.

Basically the case involves the analysis of the company's financials, the possible future state of the world, and actions to be taken to restore potential imbalances in the

Table 1.11 Income statement of Projectcast Ltd.

Income statement	
Sales	5,000,000
COGS	2,500,000
Taxes	40%
Payout ratio	30%

Table 1.12 Balance sheet of Projectcast Ltd.

Balance sheet			
Assets		*Liabilities and equity*	
Cash	1,200,000	Accounts payable	1,000,000
Accounts receivable	3,000,000	Long-term debt	5,500,000
Inventory	2,500,000	Common stocks	6,500,000
Net PPE	8,500,000	Retained earnings	2,200,000

projected balance sheet. Nancy knows that she can apply the percentage of sales approach to the analysis.

She also knows that the future state of the economy and the consequences on the projected increase/decrease of sales can be described as follows:

- Booming with a probability of 10%, with sales increasing by 20%
- Growing slow with a probability of 25%, with sales increasing by 10%
- Steady with a probability of 40%, with sales increasing by 5%
- In moderate recession with a probability of 20%, with sales remaining unchanged
- In a severe crisis with a probability of 5%, with sales decreasing by 10%

After analyzing the sales growth rate, Nancy is ready to forecast the pro forma balance sheet starting from the historical financial statements that look like in Tables 1.11 and 1.12.

The company is currently operating below full capacity, and the payout policy will remain unchanged in the next year.

Questions

- How would you calculate the internal growth rate of the company?
- How would you calculate the sustainable growth rate of the company?
- Determine the amount of external financing needed, according to the POS approach.
- What recommendations would you then give to the client company, based on your analysis?

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Valuation Tools

2

A financial market is an aggregation of possible investors, willing to buy or sell financial assets. Financial valuation is a complex discipline, which involves many tools and methods in order to correctly price the assets and the risks on financial markets.

Corporate valuation is based on the correct valuation and time position of the cash flows deriving from daily corporate activities, as well as the correct choice of tools to evaluate the streams of cash flowing in and flowing out, on the other hand, perhaps inflow and outflow.

The knowledge of the rules of time traveling and mastering the timeline are basic skills that must be acquired in order to proceed to valuation at a later stage, as it is most commonly said in the literature: "a dollar today is not equal to a dollar tomorrow."

Whether it is a revenue, or a cost of some type, or an amortization, or any type of operative or non-operative item, basic tools for valuation allow investors to attach the right value to each of the possible corporate balance sheet items.

On the other hand, beside the corporate side, valuation is also important to acquire the price of openly traded assets on the financial market, which are functional to the survival of the corporation.

After studying this chapter, you will be able to answer the following questions, among others:

- What is the timeline, and how can it be used to position cash flows over time?
- What are the rules of time travel in financial valuation, and how do they affect the valuation of cash flows?
- What types of interest rate can be described, and how can they be compounded at different frequencies?
- What are the main drivers of interest rates?
- What are net present value and internal rate of return? How can the present value of specific cash flows like annuities and perpetuities be calculated?

A. Corelli, *Analytical Corporate Finance*, Springer Texts in Business and Economics, https://doi.org/10.1007/978-3-319-95762-3_2

The first section of the chapter is an introduction to the time value of money, with insights on how the rules of time travel can be applied to valuate cash flow streams. The second section deals with the many types of interest rates that can be calculated, with focus on how the rates change at different compound frequencies and what are the drivers of interest rates. The third section is about present value calculation and the concepts of net present value and internal rate of return, at the basis of the theory of financial valuation.

2.1 The Time Value of Money

Learning Outcomes

- Explain the rules of time travel and learn how to use the timeline.
- Learn how to valuate cash flows streams over time.
- Calculate the present value of annuities and perpetuities.

2.1.1 The Rules of Time Travel

A stream of cash flows can be defined as a series of many cash flows happening at several points in time. The timeline is a linear representation of the timing and can be used to graphically place the cash flows at their correct date.

In order to construct a timeline, consider a loan of amount L received today to be repaid in the following 2 years, with payments of C every year. The timeline in this case will look as in Fig. 2.1.

Example 2.1 Assume investor A borrows 1000 € from investor B today, and ignoring any time value of money, the repayment will happen in two equal yearly installments of 500 € in the next 2 years. The timeline is as in Fig. 2.2.

In the timeline, time 0 represents the present date. Date 1 is 1 year later and represents the end of the first year. At time 1, one will receive the 500 € payment, expressed by the cash flow at time 1.

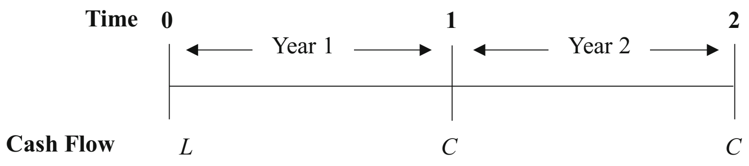

Fig. 2.1 Timeline of a series of cash flows when an amount L is paid at time 0 and an amount C is received at time 1 and 2

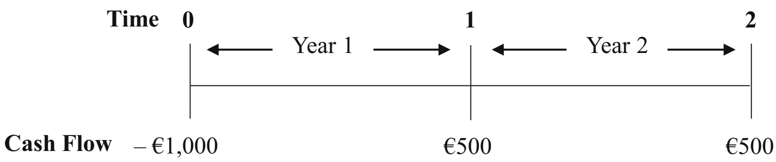

Fig. 2.2 Timeline of a series of cash flows with sample numbers

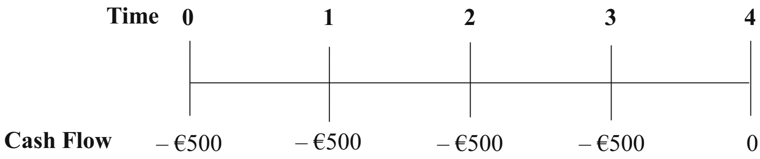

Fig. 2.3 Timeline of a series of a student loan payment

The same will happen at time 2, for the payment covering the period from time 1 to time 2, which generates the cash flow registered at the latter time, adding up another 500 € to the sum of payments over time.

Each point in the timeline can be seen as a date, and the space between two dates is the time between dates. As convention, remember that each point represents the start of the period considered.

To differentiate between the two types of cash flows, we assign a different sign to each. Cash outflows are represented on the timeline with a negative sign, while cash inflows have a positive sign.

For example, if rent is paid each month, one could use a timeline like the one in our first example to represent two rental payments, but the time label would represent months instead of years (Bodie and Merton 2000).

Basic timelines are very simple and just relate to well-structured and easy-to-understand cash flow streams, with a straightforward position on the timeline. However, timelines that are more complicated exist.

However, assigning the right position of cash flows on the timeline can be a difficult task, and it is a crucial step in making right financial decisions. It is then recommended to always draw the timeline when approaching every task.

Example 2.2 A student must pay tuition fees for the next 4 years of his education at some university. In this case, the money must be anticipated at time zero for the first year, and so on, with no payment in the last year. The timeline is as of Fig. 2.3.

In order to make efficient financial decisions, sometimes one must compare cash flows that occur at different points in time. However, the basic principle of comparison is that cash flows at different points in time cannot be compared.

Therefore, in order to make comparisons and sum up or subtract cash flows belonging to a stream, it is necessary to move them backward or forward on the timeline toward a common point. There are three important rules that govern the movement in time of cash flows.

The first rule asserts that only cash flow values at the same point in time can be compared or combined, following the main principle mentioned above. In order to compare or combine cash flows, it is necessary to reduce the analysis to a single time point.

A euro today is not equivalent to a euro in 1 year, and that is why, the first rule is so important. Money today is more valuable, given that it can be invested to earn some interest rate.

It is therefore necessary to discount future cash flows and to account for their equivalent (lower) present value and compound past cash flows, when moving forward in time, in order to get to the higher value the money would get by investing it in time.

The second rule explains how to move a cash flow forward in time and states that in order to do so, the cash flow must be compounded. In order to do so, the interest on the initial amount must be accrued according to the percentage interest rate of reference.

Example 2.3 An investor has 10,000 € to invest for 1 year. In order to calculate the amount he will receive after 1 year if the money is invested at an interest rate of 5%, it is possible to move the cash flow forward in time (compound) by

$$(10,000) \times (1 + 0.05) = 10,500 \text{ €}$$

In general, if the market interest rate for the year is r, then we multiply by the interest rate factor $(1 + r)$ to move the cash flow from the beginning to the end of the year.

The compounding can be applied repeatedly and iteratively to move the amount forward by more than one time step. There is no limit to how far we can get with the compounding as long as we keep the coherence when accruing for the interest. The value will of course grow more and more as we move further ahead in time.

Example 2.4 Assume the above investor wants to invest the 10,000 € for a 2-year term. Assuming the interest rate is fixed at 5% for both years, calculation shows that the amount due after 2 years is

$$(10,000) \times (1 + 0.05) \times (1 + 0.05) = 11,025 \text{ €}$$

It is possible to represent the calculation on a timeline as in Fig. 2.4.

The two examples above show that for an interest rate of 5%, the amount available in 1 year will be obtained by multiplying the initial amount by 1.05, while the amount available in 2 years is obtained by multiplying the initial amount by the same factor twice.

The point is that the interest accrued in 1 year also produces interests. Therefore, in the second period, the amount will be augmented of the additional 5% on the initial amount plus the 5% of the interest amount accrued in the first period.

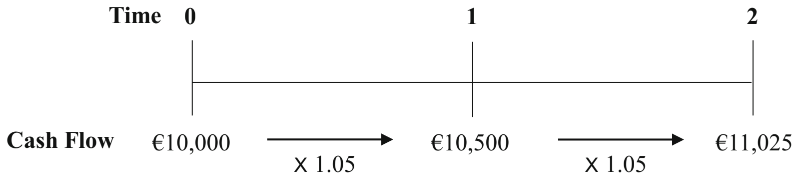

Fig. 2.4 Timeline for an interest-earning investment

The value of a cash flow that is moved forward in time is known as its future value, and depending on whether only the notional amount is compounded or the interest also matured in the previous period, the interest is defined as simple or compounded, respectively.

Consider now a movement forward of more steps than just one or two. Using the same approach, we compound the cash flow a third time, fourth time, and so on. The approach can be extended too many consecutive periods or jumps of many periods.

Example 2.5 Assume the investor wants to invest the 10,000 € for a 3-year term. Assuming the interest rate is fixed at 5% for all years, calculation shows that the amount due after 3 years is

$$(10,000) \times (1 + 0.05) \times (1 + 0.05) \times (1 + 0.05) = 11,576.25 \text{ €}$$

In general, if we have a cash flow now C_0 to compute its future value FV_n for n periods into the future, we must compound it by the n intervening interest rate factors. If the interest rate r is constant, this calculation yields

$$\mathrm{FV}_n = C_0 \times (1 + r) \times (1 + r) \times \ldots \times (1 + r) = C_0(1 + r)^n$$

The third rule of time travel states that in order to move a cash flow backward in time, it must be discounted at the relevant interest rate.

The third rule of time travel shows how to push the cash flows backward in time to get a present value of some amount supposed to be paid or received at some point in the future.

Example 2.6 Suppose one wants to calculate the value today of 10,000 € you anticipate receiving in 1 year. If the current market interest rate is 5%, it is possible to invert the compound relationship shown above to obtain that

$$\frac{10,000}{1.05} = 9523.81$$

To move the cash flow backward in time, one must divide it by the interest rate factor (1 + r) where r is the interest rate—this is the same as multiplying by the discount factor $\frac{1}{(1+r)}$.

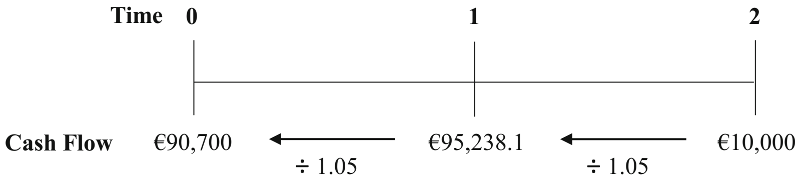

Fig. 2.5 Timeline for the present value of a future amount

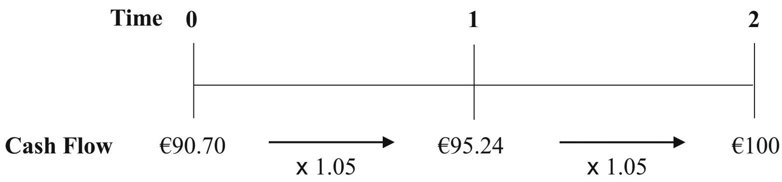

Fig. 2.6 Timeline for the future value of a sum at present value

The process of moving a value or cash flow backward in time—finding the equivalent value today of a future cash flow—is known as discounting.

Example 2.7 An investor knows he will receive an amount of 10,000 € in 2 years from today rather than in 1 year. If the interest rate for both years is 5%, it is possible to draw the timeline as in Fig. 2.5.

An interest rate of 5% makes 10,000 € in 2 years being equivalent to 95,238.1 € in 1 year and 90,700 € today. The amounts represent the same value in different units (different points in time). The arrow points to the left to indicate that the value is being moved backward in time or discounted. Note that the value decreases as we moved the cash flow further back (Kaminsky 2003).

The same example can be again applied to moving forward cash flows and compounding them. One just has to invert the timeline to obtain the one in Fig. 2.6.

An amount of 90,700 € invested today is equivalent to an amount of 95,238.1 € in 1 year and 10,000 € in 2 years. Again, further extension of the timeline is possible, and for a 3-year period, it can be drawn as in Fig. 2.7.

That is, the present value today of a cash flow of 100 € in 3 years discounted at an interest rate of 5% is given by 86,380 €.

In general, to compute the present value today (date 0) of a cash flow C_n that comes n periods from now, we must discount it by the n intervening interest rate factors. If the interest rate r is constant, this yields

$$\mathrm{PV}_0 = C_n \times \frac{1}{(1+r)} \times \frac{1}{(1+r)} \times \ldots \times \frac{1}{(1+r)} = \frac{C_n}{(1+r)^n}$$

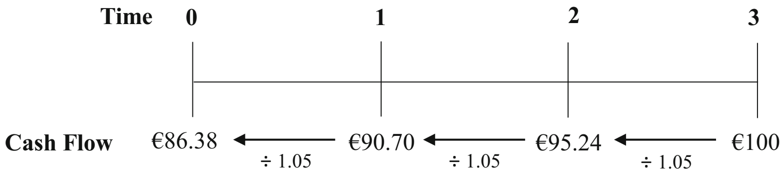

Fig. 2.7 Timeline for the present value of a future amount

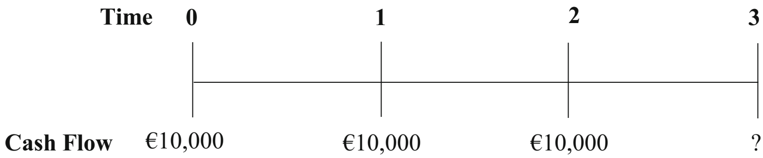

Fig. 2.8 Undefined timeline for a saving plan

2.1.2 Valuation of Cash Flow Streams

Proper application of the rules of time travel allows the investor to compare and combine cash flows occurring at different times. Based on the analysis of the cash flows, it becomes straightforward to give valuations in terms of future value or present value.

Example 2.8 A saving plan consists in saving 10,000 € for three times, including current time and 2 further years. The investor wants to calculate how much money will be available after a further year (therefore 3 years from today), if the interest rate is fixed at 5% for all years.

In this case, the timeline is given in Fig. 2.8.

The rules of time travel again allow to calculate the value of every cash flow depending on the time they belong to. Every single cash flow in this case will have to move for the right amount of time steps, which is one in this case, and be summed up to the correspondent cash flow for the time of arrival of the compounded cash flow.

First, the deposit at date 0 is moved forward to date 1. Because it is then in the same time period as the date 1 deposit, the two amounts can be combined to find out the total in the bank on date 1.

The new aggregate amount can then be moved forward to time 2 in the same way, and compounded first, to be then summed up to the deposit at that time. The last step is to push the overall amount to time 3 by a last compounding. The overall result is shown by the timeline in Fig. 2.9.

There is another possible approach to the problem, consisting in calculating the future value at year 3 of each cash flow separately. Once all the three amounts are in year 3, it is possible to combine them (see Fig. 2.10).

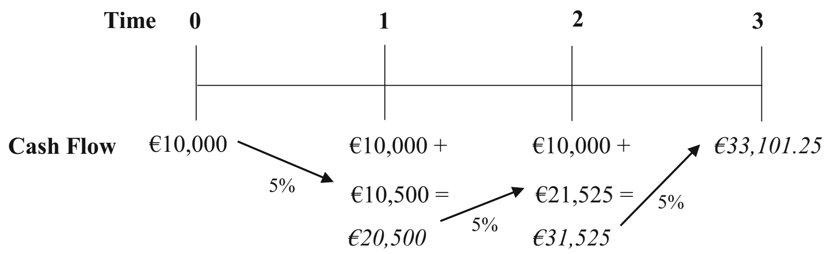

Fig. 2.9 Timeline for a saving plan

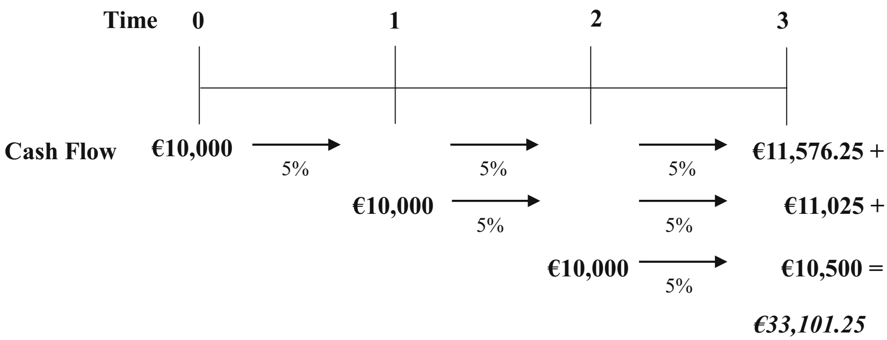

Fig. 2.10 Timeline for a saving plan with separate cash flow calculation

Obviously, both calculations lead to the same result. As long as the computation is consistent with the rules of time travel, the amount obtained is the same in both ways. The order of application of the rules has no role, and the chosen calculation depends on which is more convenient for the problem at hand.

2.1.3 Annuities and Perpetuities

Some types of cash flows have very well-defined properties, so that they get specific names and become easily identifiable. In particular, cash flows that involve equal payments over regular time spans can be defined as annuities and perpetuities.

A regular perpetuity is a stream of equal cash flows that occur at constant time intervals with infinite maturity (never stop). The attribute regular is used to distinguish it from the growing perpetuity, to be discussed later on. The timeline for a perpetuity can be drawn as in Fig. 2.11.

Normally the first cash flow of a perpetuity arrives at the end of the first period (time 1 on the usual timeline). This timing is sometimes referred to as payment in arrears and is a standard convention that is adopted in literature and in this book (Shapiro and Streiff 2001).

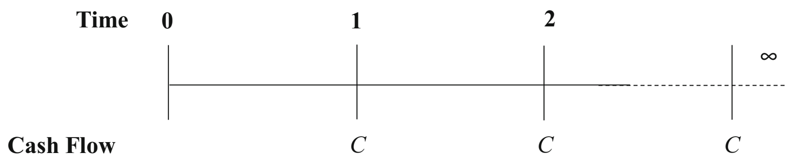

Fig. 2.11 Timeline for a perpetuity

The equation for the present value of a perpetuity is derived from the standard present value calculation, where all cash flows and rates are constant and time steps increase toward infinity, so that it can be simplified as

$$\mathrm{PV}_0 = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots = \sum_{t=1}^{n} \frac{C}{(1+r)^t} \tag{2.1}$$

Notice that $C_t = C$ in the present value equation because the cash flow for a perpetuity is constant. Also, because the first cash flow is in one period $C_0 = 0$.

The trick from the perpetuity present value calculation equation is that even the sum of an infinite number of positive terms becomes finite. This is because the cash flows in the future are discounted for an ever-increasing number of periods, so their contribution to the sum eventually becomes negligible.

Equation (2.1) can therefore be further simplified in order to get a shortcut formula, which simplifies the calculation. The shortcut is derived by calculating the value of a perpetuity by creating our own perpetuity.

By the law of one price, once the calculated and the created perpetuity are taken back at present values, those must be equal given that they come from the same cash flow structure.

Example 2.9 An investor can invest 10,000 € in a bank account paying a 5% interest forever. Every year you can then withdraw the money from the bank, keep the interest part for yourself, and reinvest the initial capital again, at the same conditions.

The situation can be then represented on the timeline in Fig. 2.12.

The strategy creates a perpetuity paying 500 € per year, on the assumption that the bank will remain solvent and the interest rate will not change.

Recall that the law of one price tells us that the same good must have the same price in every market. The bank allows creating the perpetuity for the initial cost of 10,000 €. The present value of the 500 € per year in perpetuity is the self-made perpetuity, which costs 10,000 €.

Generalizing the argument, assume an investment in a bank of an amount P. It is possible to withdraw the interest $C = rP$ every year as per the example above and leave the principal for another round of compounding (Taylor 1986).

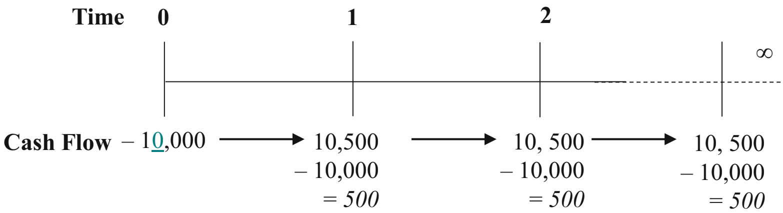

Fig. 2.12 Timeline for a perpetuity with calculations

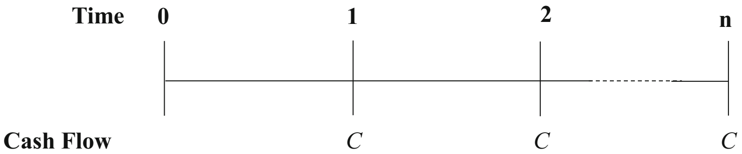

Fig. 2.13 Timeline for an annuity

The condition of present value of a perpetuity with discount rate r and constant cash flows C, starting in one period, can be expressed mathematically as

$$\mathrm{PV_P} = \frac{C}{r}$$

By depositing the amount $\frac{C}{r}$ today, we can withdraw an interest of

$$\frac{C}{r} \times r = C$$

each period in perpetuity.

Similar to perpetuities, a regular annuity is a stream of a finite number of equal cash flows coming at constant time intervals, for a period, which is not infinite. As for the perpetuities, growing annuities are a special case, introduced later on.

The main difference between an annuity and a perpetuity is the absence of an infinite time horizon and infinite cash flows paid. There are n payments that are spanned over n time steps on the timeline. As for the perpetuity, the first cash flow conventionally corresponds to time 1.

The timeline for representing an annuity looks like in Fig. 2.13.

The same convention of the first payment happening at time one, as for the perpetuity, is adopted. The present value of an n-period annuity with payment C and interest rate r is

$$\mathrm{PV_A} = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^n} = \sum_{t=1}^{n} \frac{C}{(1+r)^t}$$

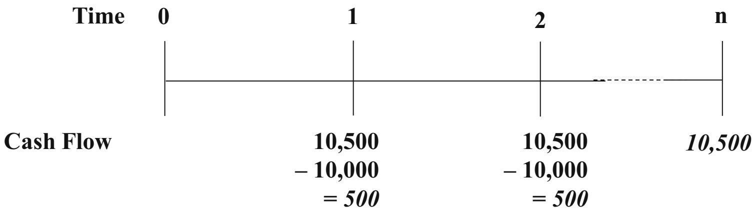

Fig. 2.14 Timeline for an annuity derived from an investment

The derivation of the present value of an annuity is based on the same principles adopted for the perpetuity. Obviously, the change to a finite maturity and finite number of cash flows cuts some of the assumptions made in the previous case.

This is especially true in the second step of the derivation, when deriving the shortcut formula, which in the case of an annuity is more complicated in that it cannot benefit from the assumption of negligible cash flows made for the perpetuity.

The approach to be used to find the shortcut equation is nevertheless the same as before and goes through the creation of an annuity and comparison of present value with an equivalent one.

The process to artificially create an annuity is similar to the one used for building up a perpetuity. Starting with an investment 10,000 € in a bank account paying a 5% interest, after one year there will be 10,500 € in the bank.

By withholding the interest and reinvesting the principal further, one builds up the cash flow system that generates an annuity. The only difference with the case of the perpetuity is that the time is limited to *n* years and does not go to infinite. In that case, the cash flows will look like in Fig. 2.14.

The artificially created annuity generates a positive payoff of 500 € per year for 20 years, and it is possible to get the present value of the annuity by applying the law of one price, as in the case of the perpetuity.

Since it took an initial investment of 10,000 € to create the cash flows on the timeline, the present value of these cash flows is 10,000 €, which is the sum of the present value PV_{CF} of the *n* cash flows of 10,500 € and the present value PV_A of the *n* annuity cash flows of 500 €. It follows that

$$PV_{CF} + PV_A = 10,000 \text{ €}$$

The present value of the annuity can be therefore derived as

$$PV_A = 10,000 \text{ €} - PV_{CF} = 10,000 - \frac{10,000}{(1.05)^{20}} = 10,000 - 3768.89 = 6231.10$$

It follows that the present value of 500 € for 20 years is 6231.10 €. Intuitively, the value of the annuity corresponds to the initial investment in the bank account minus the present value of the principal still in the account after 20 years.

As for the perpetuity, the numerical example can be generalized in order to obtain the general equation for the present value of a regular annuity.

Generalizing the argument, assume an investment in a bank of an amount P. It is possible to withdraw the interest $C = rP$ every year as per the example above and leave the principal for another round of compounding (Watson 1936).

For an initial investment of P, we will receive an n-period annuity of C per period, plus we will get back our original P at the end. P is the total present value of the two sets of cash flows. The present value of receiving C in annuity is therefore given by

$$\mathrm{PV_A} = P - \frac{P}{(1+r)^n} = P\left(1 - \frac{P}{(1+r)^n}\right)$$

By recalling that

$$P = \frac{C}{r}$$

it follows that

$$\mathrm{PV_A} = \frac{C}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

As opposed as for the perpetuity, an annuity also has a future timeFV_n, which starts after the last payment made. It is then possible to calculate the future value, through a simple equation for the future value.

In case one is interested in the future, value of an annuity in n periods forward on the timeline, it is necessary to compound the present value for n periods at the relevant interest rate r.

$$\mathrm{FV}_n = \frac{C}{r}\left(1 - \frac{1}{(1+r)^n}\right)(1+r)^n = \frac{C}{r}\left[(1+r)^n - 1\right]$$

2.2 Interest Rates

Learning Outcomes

- Define the various types of interest rates.
- Learn how to use various compound frequencies.
- Explain what are the drivers of interest rates.

2.2.1 Types of Interest Rates

There are several types of interest rates that can be defined depending on the way they are calculated. The easiest type of rate involves a simple loan, where a specific amount of funds is borrowed, which must be repaid at maturity to the lender, along with some additional amount, represented by the interest.

Commercial loans are usually structured like that. As an example, consider a loan of 10,000 € at 5% rate, implying that the borrower will have to repay 10,500 € in 1 year.

A fixed-payment loan is fully amortized and consists in borrowing an amount of funds that will be repaid with the interest on top, through several fixed regular payments, until maturity is reached (Brealey et al. 2006).

As an example consider the above loan of 10,000 € but given out as a mortgage. This implies it will be repaid in yearly installments of 1260 € for 25 years, implying an interest rate of 12%.

The yield to maturity (YTM) can be defined as the interest rate equating the present value of payments received from a debt instrument to the price or value of that debt instrument today (market value).

The YTM is sometimes also called the internal rate of return (IRR) of an investment, and it is the most accurate and widely applicable measure of interest rates, given that it allows to understand how much interest an investment is yielding to the investor.

The above definitions lead to several types of calculations that can be performed on the loan, according to how it is considered. For a simple loan, for example, the yield to maturity associated to the loan, given the data above, must satisfy the condition that

$$\mathrm{PV} = \frac{\mathrm{FV}}{1+r}$$

so that

$\mathrm{PV}(1+r) = \mathrm{FV} \quad \Rightarrow \quad r = \frac{\mathrm{FV}-\mathrm{PV}}{\mathrm{PV}}$ where:

PV is the present value of the loan.
FV is the future value of the loan at maturity.

Example 2.10 Consider calculating the yield on a payment of 10,500 € in 1 year, whose present value today is 10,000 €. Calculation shows that

$$r = \frac{10,500 - 10,000}{10,000} = 0.05 = 5\%$$

Thus, for simple loans, the yield to maturity equals the simple interest rate.

Example 2.11 In the case of a fixed-payment loan instead, recall the borrower pays 1260 € per year, for 25 years, to repay an initial loan of 10,000 €.

The yield to maturity r must satisfy

$$10,000 = \frac{1260}{1+r} + \frac{1260}{(1+r)^2} + \frac{1260}{(1+r)^3} + \ldots + \frac{1260}{(1+r)^{25}}$$

The calculation for r cannot be performed analytically, but it requires a numerical approach or the use of a financial calculator or spreadsheet. Calculation done via computer or scientific calculator shows that the yield to maturity is equal to 0.1183 or 11.83%. More generally, for any fixed-payment loan, if

$$\mathrm{PV} = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^n}$$

where:

C is the amount of fixed annual payments.
n are the years to maturity.

In finance and corporate finance, the distinction between the nominal interest rates and the actual return on a bond is very important, in that it affects the perception of the profitability of the investment.

The rate of return on a bond in fact is a complex calculation that takes into account not only any interest payments made by the bond but also any changes in the market price of the bond itself, which also adds up.

Example 2.12 A coupon bond has face value of 100 € and a coupon of 5%, and it is bought at par. If the interest rate falls over time, the bond price will increase. In case the investor receives one coupon payment during the time span of the investment and is then able to sell the bond after 1 year for 115 €, the total investment return includes both capital gain and coupon. The total return on the investment will therefore be given by

$$\mathrm{PV} = \frac{115 - 100 + 5}{100} = 20\%$$

Sometimes the strategy can lead to a loss, with the process working the other way round. If the interest rates rise over the next year, so that the bond price falls to 80 €, the gain from the coupon is completely offset by the capital loss. In this case, the total return would be

$$\mathrm{PV} = \frac{80 - 100 + 5}{100} = -15\%$$

2.2.2 Compounding Frequencies

When investing money in a deposit account in a bank or other financial institution, some interest is paid on the sum, which is deposited. The same works when borrowing money, in which case some interest is owed to the institution lending the money.

In fact, the interest paid on deposited money is usually much lower than the interest an individual has to pay to a bank when borrowing money. This is how banks make profits from circulating the money in the financial system and that is how the bank pays the salaries for all their employers, the cost of buying and maintaining their premises, and the dividends paid to shareholders (Brigham 1992).

As mentioned in previous sections, a very important distinction when calculating an interest is between simple and compound interest. Simple interest is paid every term (year), but the interest is not reinvested, so it doesn't attract extra interest if you leave it in the bank.

Compound interest on the other hand implies that the interest payments are reinvested in the same account, at the same interest rate, for the following periods, generating new interest themselves, to add up to the interest generated by the capital amount.

When shifting to compound interest, the issues that determine the right amount to be repaid is how often the interest is calculated and added to the account. The frequency of compounding determines the final amount of the loan plus interest. Common practice for interest rate compounding is to have, for example, weekly, monthly, yearly, etc.

The terminal value of a current amount L, invested for n years, at a rate R per annum, is given by

$$L_n = L(1 + R)^n$$

If the interest rate is compounded m times per year, the terminal value of the investment can be written as

$$L_{n,m} = L\left(1 + \frac{R_m}{m}\right)^{nm}$$

As the compounding frequency tends to infinity, the computation shifts in continuous compounding, which gives the highest value for the terminal value and can be written as

$$L_c = Le^{R_c n}$$

The relation between discrete and continuous compounding is given by

Table 2.1 Example of various compounding frequencies

Compounding frequencies	Interest rate
Annual ($m = 1$)	10.0000%
Semiannual ($m = 2$)	10.2500%
Quarterly ($m = 4$)	10.3813%
Monthly ($m = 12$)	10.4713%
Weekly ($m = 52$)	10.5065%
Daily ($m = 365$)	10.5156%

$$
\begin{aligned}
&L\mathrm{e}^{R_c n} = L\left(1 + \tfrac{R_m}{m}\right)^{nm} \\
&\Rightarrow \mathrm{e}^{R_c n} = \left(1 + \tfrac{R_m}{m}\right)^{nm} \\
&\Rightarrow R_m n = nm\left(\mathrm{e}^{\frac{R_c}{m}} - 1\right) \\
&\Rightarrow R_m = m\left(\mathrm{e}^{\frac{R_c}{m}} - 1\right)
\end{aligned}
$$

To show the effect of compounding frequency on an interest rate of 10% per annum, it is possible to apply the above formulas, and the effective rates at different compound frequencies are shown in Table 2.1.

2.2.3 The Drivers of Interest Rates

Interest rates, like any other financial variable, take their value from the balance of supply and demand of credit. As the forces of supply and demand change, so do interest rates to bring the credit markets back to equilibrium.

It is therefore interesting to analyze what are the drivers of interest rates, for shorter and longer maturities, as well as for the demand and supply side. First, consider the factors that affect long-term interest rate cycles on the demand side.

The risks involved in holding the asset of reference are at the basis of the driving factors from the demand side. The risk perception of the investor may change overtime and so does the valuation of the asset.

The debt holder holds the risk of insufficient funds available to repay for the obligation, to make it difficult to repay the principal and interest when the maturity of the loan approaches.

In case the debt is riskless, like investing in government securities of very advanced countries, there is no risk related to the possibility of repayment, which is guaranteed. For a treasury bond holder, the major risk to the investment is the erosion of value of the coupon payments and the principal through price increases.

Inflation plays a role in valuation of future cash flows, in that an increase in prices of goods and services in the domestic economy results in the decrease of the value of future cash flows.

Given that coupon and nominal principal of a bond are fixed in case of fixed-rate instrument, this translates into a decrease in the value of bond future cash flows, with consequences on the overall valuation of the bond.

The consequence in terms of risk and value is that investors will demand, in the presence of a high inflation, higher interest rates to compensate for the possible loss of value. Oppositely, in case deflation takes hold, the value of the future cash flows actually increases, making the government bond more valuable.

In general, for a fixed income instrument like a bond with fixed rate, the risk is mostly concentrated in the value of future cash flows, given that current coupon payment is a small percentage of the total value (Copeland et al. 2003).

Therefore, it is the expectation of future inflation that is most relevant as a factor, rather than the current rate. It follows that expectations of investors on inflation are a driver of the demand for debt.

The inflation expectations are themselves driven by several factors, which an investor may consider in order to assess the inflation risk in a fixed income investment. There is a strong connection between the economic cycle and the investor perception of inflation.

The actual dynamics behind that relationship are complex, and factors can interact in different ways to form expectations. However, the basic logic entails a few basic steps related to the state of the economy.

More precisely, when the economy starts growing, the companies increase production to meet increasing demand. The increase in production corresponds to a heavier utilization of current capacity.

Labor becomes a scarce resource and salaries increase in order to attract working force. As a consequence, consumer prices increase as well, leading to a vicious circle of salary/prices increase.

The chain reaction makes salaries increase further, and economic agents create expectations of future rise in prices, which leads to building expectations of high inflation in the near future, as well.

The opposite obviously works in times of economic downturn, when both salaries and prices tend to decrease because of economic stagnation, and the expectations of investors and economic agents are then formed based on deflation expectations.

Government expenditure is another important factor in expectation formation for investors. When structural government spending and cyclical spending rise, the economy tends to heat up.

It has the same effect of overproduction of corporation, with a powerful impact on the private sector as well. As government spending rises, the increasing amount of debt the government needs to take on also crowds out private sector debt, leading to interest rate increases.

The aggregation of the overall demand for long-term funds by economic entities can be referred to as the leverage cycle, where leverage is the term usually employed in describing the amount of debt versus assets of either an individual or the economy as an aggregate.

The most direct effect of leverage cycle on the long-term interest rates is direct, with interest rates increasing as long as the demand for debt increases. However, it is not that simple, and the process is mostly discontinuous, so that increasing amounts of leverage do not necessarily lead to increases in interest rates as a steady line.

An important role as a driving factor for the level of domestic interest rates is played by the regulatory framework and the structure of banking system in an economy. In fact, strict regulations preventing inflows and outflows of capital can have a significant impact on domestic interest rates (Webster's Dictionary 1992).

When foreign capitals are blocked to enter the domestic economy, interest rates will tend to be higher than with an open economy, while the opposite occurs in a framework of impossibility of outflows to abroad, with capital held captive domestically.

Central banks worldwide control the cycles of interest rate movements, by employing policy actions that interact with the economic data. Depending on the country or regional area, the central banks take direct control of different types of rates.

For example, in the United States, the Fed directly controls the overnight rate in the United States, but its influence goes far beyond the short rate. This is because the short end of the yield curve is not independent of longer maturities.

Economic data are an important factor driving changes and expectations on future interest rates. Macroeconomic variables are crucial to follow for any financial market but in particular for interest rates.

Every single piece of economic data can be analyzed in order to get important information about the trades in the economy. Mostly, the evaluation of macroeconomic data consists in looking at gross domestic production (GDP), employment data, manufacturing data, and more.

Another important factor to consider when forming expectations on fixed income securities is that such a class of instruments is exposed to interest rate risk. It stems from fixed cash flows being received or paid, with these fixed cash flows made more or less attractive by changes in the broader market interest rates. This change in attractiveness translates into change in the price of the security.

Example 2.13 A bond pays a 5% coupon, and immediately after its purchase by an investor, the average yield on similar investments raises to 7%. The investor can then try to sell the 5% bond, which represents a poor investment, but he will have to do it at a discount. This is because other investors can buy other bonds with higher interest payment. The price of the bond will therefore go down therefore to the increase in interest rates. An opposite argument explains why the 5% bond would gain in value if interest rates on similar instruments fell to 3%.

The reference rate, to what the bond is sensitive to, I changed the too but do you mean the bond is "also" sensitive to the reference rate. If so, then you can ignore my correction. I was not sure about the rate offered in the market on similar assets. Interest rates in the markets are highly correlated, making it crucial to track global flows and developments to effectively manage the interest rate risk.

To summarize, there is a wide range of factors affecting interest rates, and all of them are constantly in a state of flux, with some of them moving up or down in importance according to the state of the economy at some specific time.

More factors could emerge in time, due to the dynamic nature of interest rate markets. In all cases, a logical approach is needed in order to take into account those drivers whose horizons range from decades to days.

When approaching the interest rate markets, it is important to know the major drivers on long and short time scales. At the same time, also being aware of new factors that may arise from time to time, it is an important step in understanding interest rate markets.

2.3 Present Value Calculation

Learning Outcomes

- Explain the concept of net present value.
- Learn how to calculate the internal rate of return.
- Apply the knowledge of time value to growing cash flows.

2.3.1 Net Present Value

The market of interest rates and credit market are characterized by various instruments that give or require payment at different times. From simple loans to coupon bonds, payments can happen only at maturity as well as at multiple times until maturity.

The goal of this section is to describe a unified methodology to approach measurement of interest rates on the various types of instruments available on the market. Given such a framework, valuation of financial assets, projects, and investment opportunities becomes possible and straightforward.

The concept underlying the valuation of credit and interest rate instruments is the present value that captures the idea that a dollar received in the future is less valuable than a dollar received today.

Example 2.14 A bank issues a loan of 100,000 € with a 5000 € interest payment over 1 year. This simple framework makes the interest payment a measure of the interest rate as from

$$r = \frac{I}{L} = \frac{5000}{100,000} = 0.05 = 5\%$$

where:

r is the simple interest rate.
I is the amount of interest paid.
L is the principal amount.

Example 2.15 It is possible to reverse the previous example and calculate the total amount available to the investor at the end of the investing period, given the interest rate of 5% on the initial investment of 100,000 €. Calculation shows that

$$100,000 + (100,000 \times r) = 100,000(1 + r) = 100,000(1.05) = 105,000 \ \text{€}$$

It is then possible to extend the investment for a further year by lending the 105,000 € out again for another year at the same simple interest rate of 5%, and you get

$$105,000 + (105,000 \times r) = 105,000(1.05) = 110,250 \ \text{€}$$

at the end of the second year. Equivalently, we can write

$$100,000 \times (1 + r) \times (1 + r) = 100,000(1 + r)^2 = 100,000(1.05)^2 = 110,250 \ \text{€}$$

and so on for more years, if one wants.

It is possible to generalize the above examples into an equation for the compounded value V of an amount L at an interest rate i for n years (periods).

$$V = L(1 + r)^n$$

By reworking the formula, it is also possible to express the present value L of a future payment V, by the formula

$$L = \frac{V}{(1 + r)^n}$$

2.3.2 Internal Rate of Return

The internal rate of return (IRR) in capital budgeting is a measure of profitability of investments by comparison. According to the context of use, it is also called rate of return (ROR) or effective interest rate.

The term internal refers to the fact that its calculation does not incorporate environmental factors (e.g., the interest rate or inflation), but it only takes into account the cash flows that are generated internally by the investment.

The IRR of an investment measures the annualized effective compounded return rate that makes the NPV of any investment (sum of discounted positive and negative cash flows) equal to zero.

Put another way, the IRR of an investment is the discount rate that makes the net present value of costs (negative cash flows) of the investment equal to the net present value of the benefits (positive cash flows) of the investment.

The budgeting IRR rule states that a higher a project's internal rate of return makes a project or investment to be more desirable. When comparing projects with same up-front investment, the one with highest IRR should be prioritized.

The rule of thumb for a firm should be to undertake all available projects where the IRR exceeds the cost of capital, given the capital constraints and feasibility. Investment in fact may be limited by availability of funds to the firm by the firm's ability to manage multiple projects.

Among other things, the IRR is an indicator of efficiency, quality, and yield of investment, as oppose as NPV, which refers more to the value and magnitude of an investment.

In a scenario where an investment is considered by a firm that has equity holders, this threshold minimum rate corresponds to the cost of capital of the investment (which in turn is determined by the risk-adjusted cost of capital of alternative investments).

The above condition guarantees that equity holders support the investment, given that for an IRR that exceeds the cost of capital, the corresponding investment is profitable and adds value to the company.

The IRR can be calculated from pairs of time and cash flow involved in the project. The NPV is set to zero, and the model is solved for the discount rate, which in turn is the IRR, as an output.

Given pairs of period and cash flow, where n is a positive integer, the total number of periods, and the NPV, the IRR is given by

$$\mathrm{NPV} = \sum_{t=0}^{N} \frac{C_t}{(1+r)^t} \tag{2.2}$$

The common standard is to calculate IRR in yearly term, but the calculation can be made simpler if one considers the actual timing of the cash flows, which is usually monthly, converting the result to a yearly period thereafter.

Example 2.16 Consider an investment represented by the cash flows in the below table

Year	Cash flow
0	−4000
1	1200
2	1410
3	1875
4	1050

The IRR is given by setting the NPV equal to zero, as

$$0 = -4000 + \frac{1200}{(1+\mathrm{IRR})^1} + \frac{1410}{(1+\mathrm{IRR})^2} + \frac{1875}{(1+\mathrm{IRR})^3} + \frac{1050}{(1+\mathrm{IRR})^4}$$

corresponding to

$$\text{IRR} = 14.3\%$$

Instead of the present time, it is also possible to perform the calculation based on any other time, like an end of interval of an annuity, for example. In this case, the value obtained is zero if the NPV is zero.

If cash flows are random, the expected values go in the Eq. (2.2) in place of the actual nonrandom ones. The value of the IRR cannot be found analytically, and numerical methods or graphical methods must be used.

There are numerical methods to solve the equation for the IRR. The secant method, for example, gives the IRR of a period $t + 1$ and can be written as

$$\text{IRR}_{t+1} = \text{IRR}_t - \text{NPV}_t\left(\frac{\text{IRR}_t - \text{IRR}_{t-1}}{\text{NPV}_t - \text{NPV}_{t-1}}\right)$$

where:

IRR_t is considered the nth approximation of the IRR.

The IRR in the above way can be obtained with an arbitrary degree of accuracy.

An interesting case is where the stream of payments consists of a single outflow, followed by multiple inflows occurring at equal periods. It can be written as

$$C_0 < 0, \quad C_0 \geq 0 \ \text{ for } \ n \geq 1$$

In this case, the NPV of the payment stream is a function of interest rate. There is always a single unique solution for IRR given the convexity of the function. A more accurate interpolation equation is given by

$$\begin{aligned}\text{IRR}_{t+1} = \text{IRR}_t - \text{NPV}_t\left(\frac{\text{IRR}_t - \text{IRR}_{t-1}}{\text{NPV}_t - \text{NPV}_{t-1}}\right) \\ \times\left[1 - 1.4 \times \left(\frac{\text{NPV}_{t-1}}{\text{NPV}_t - 3\text{NPV}_t + 2C_0}\right)\right]\end{aligned}$$

This has been shown to be ten times more accurate than the standard secant equation or a wide range of interest rates and initial guesses.

2.3.3 Growing Cash Flows

Regular cash flows are those which are not growing over time but stay constant. As described above, such cash flows are the basis for the calculation of regular annuities and perpetuities.

There is also the case where the cash flows are expected to grow at some (constant) rate in each period. In this case, it is again possible to derive formulas in order to calculate the present value of the associated perpetuity and annuity.

A growing perpetuity is then a stream of cash flows occurring at regular intervals over an infinite period, with an infinite number of payments. Payments grow at a constant rate over time.

The usual convention of the first payment occurring at time 1 is adopted, so that from the first payment to the last (occurring at time t), there are only $t-1$ periods of growth between every t periods, until infinite.

Following the same approach and substituting the so obtained timeline into the general equation for present value of a stream of cash flows yields a summation that simplifies the calculation

$$\mathrm{PV} = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \ldots = \sum_{t=1}^{\infty} \frac{C(1+g)^{t-1}}{(1+r)^t}$$

It is important to note that when $g > r$ cash flows grow faster than they are discounted, the summation diverges, getting larger over time. The sum is then infinite, meaning that it is impossible to reproduce the cash flows associated to the growing perpetuity.

In fact, recall that the approach used to build the formulas for the regular perpetuity is based on replication of cash flows, at a cost equal to the present value of the perpetuity itself.

An infinite present value means that no matter how much money you start with, it is impossible to reproduce those cash flows on your own. This type of perpetuities cannot exist in practice because no one would be willing to offer one at any finite price.

Nobody would ever offer a payment of an amount that grows faster than the interest rate on it. Therefore, the only growing perpetuities that make sense are those with a growing rate lower than the interest rate, so that each successive term in the sum is less than the previous term and the overall sum is finite. It makes sense therefore to assume that $g < r$ for a growing perpetuity.

The equation for a growing perpetuity can be derived by following the same logic applied to a regular perpetuity. Therefore, the amount needed to be deposited today in order to create the perpetuity must be computed.

Recall that for the regular perpetuity, the trick was to create a constant payment forever by withdrawing the interest earned each year and reinvesting the principal. In the case of growing perpetuity, the amount that can be withdrawn each year must be increased, and the principal reinvested each year must grow.

The method to implement such a strategy is to withdraw less than the full amount of interest accrued between each two periods and use the remaining interest to increase the principal amount.

Example 2.17 Consider creating a perpetuity growing at a rate of 3%, by investing 1000 € into a bank account, at an interest rate of 5%. At the end of 1 year, the deposit will be worth 1050 €. By withdrawing only 20 €, it is possible to reinvest 1030 €. This amount will then grow in the following year to

$$1030 \times 1.05 = 1081.5 \text{ €}$$

It is now possible to withdraw an amount

$$20 \times 1.03 = 20.6 \text{ €}$$

leaving a principal of

$$1081.5 - 20.6 = 1060.9 \text{ €}$$

By noting that

$$1030(1.03) = 1060.9 \text{ €}$$

It is clear that both the principal and the withdrawals grow at the same rate. The repeated withdrawals after initial investment is equivalent to creating a perpetuity starting at 20 € and growing at a rate of 3% per year. This growing perpetuity must have a present value equal to the cost of 1000 €.

As in previous cases, the argument can be generalized by recalling that in case of a regular perpetuity, an amount C was deposited in an account and the corresponding accrued interest was withdrawn each year.

In order to increase the withdrawn amount by the growth rate g to implement a growing perpetuity, the principal deposited must grow at the same rate. It means that instead of reinvesting C in the second year, we should reinvest

$$C(1+g) = C + gC$$

In order to increase our principal by gC, we can only withdraw

$$C_{\mathrm{w}} = rC - gC = C(r-g) \tag{2.3}$$

It is possible to withdraw an amount $C(r - g)$ after one period, keeping the account balance and cash flow growing at a rate of g forever. Solving Eq. (2.3) gives the present value PV of a growing perpetuity as

$$\mathrm{PV}_{\mathrm{GP}} = \frac{C}{r-g}$$

The intuition behind the equation of a growing perpetuity starts from the regular one. In a regular perpetuity, enough money had to be put in an account in order to ensure that the interest earned matched the cash flows of the regular perpetuity.

In a growing perpetuity, the amount to be put in the account must be larger in order to finance the growth of cash flows. The additional amount is calculated by difference between the interest rate and growth.

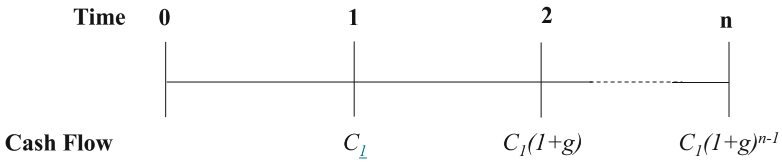

Fig. 2.15 Timeline for a growing annuity

A growing annuity can be represented on a timeline, with initial cash flow, which grows at a rate g in every period considered, until the maturity n of payments, which is finite, as the number n of payments, as shown in Fig. 2.15.

When representing the growing annuity on a timeline, this looks equivalent to a growing perpetuity where the payments from time n+1 onward are removed. The removed cash flows can then be seen themselves as a growing perpetuity starting at $n + one$.

It follows that in order to determine the present value of the growing annuity, the present value of a growing perpetuity starting at time n+1 can be calculated and subtracted to the value of a corresponding perpetuity starting at time 1.

The method translates into the following formulas:

$$\mathrm{PV_{GA}} = \frac{C}{r-g} - \frac{C_{n+1}}{r-g} \times \frac{1}{(1+r)^n}$$

where the first addendum identifies the present value of a growing perpetuity and the second addendum identifies the present value of a growing perpetuity starting at time n + 1. By substituting in

$$C \times (1+g)^n$$

for

$$C_{n+1}$$

we get

$$\mathrm{PV_{GA}} = \frac{C}{r-g} - \frac{C \times (1+g)^n}{r-g} \times \frac{1}{(1+r)^n}$$

By collecting the terms and simplifying the calculation, the present value of a growing annuity, with interest rate r and cash flows growing at a rate g, on a time interval of n periods, is given by

$$\mathrm{PV_{GA}} = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^n\right] \tag{2.4}$$

Recall that an annuity involves a finite number of payments, so that Eq. (2.4) works for $g > r$.

The equation for the present value of a growing annuity is a general solution, and all other formulas for calculation in this section can be directly derived from it. Consider the case of a growing perpetuity, which is a special case for $n = \infty$. If $g < r$, then

$$\frac{1+g}{1+r} < 1$$

and

$$\lim_{n \to \infty} \left(\frac{1+g}{1+r}\right)^n = 0$$

It follows that the equation for a growing perpetuity is

$$\mathrm{PV_{GP}} = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^n\right] = \frac{C}{r-g}$$

which is exactly the equation for a growing perpetuity. By letting the growth rate equal to zero, it is finally possible to derive the present values of a regular annuity and a regular perpetuity.

Starting then from a regular annuity, it is straightforward to derive a simple equation for the future value of a growing annuity. The value n years in the future is given by moving the present value n periods forward on the timeline, thus compounding the present value for n periods at interest rate r as defined as

$$\mathrm{PV_{GA}} = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^n\right]$$

2.4 Summary

The valuation of cash flows implies the ability to move them backward and forward in time in order to make them compatible to each other. This is why the rules of time travel allow to discount and compound cash flows making them homogeneous in time.

Cash flows streams can then be valued according to the amount they show once the time travel is completed and appropriate compounding or discounting has been applied. The overall value of the stream is the sum of the transformed value of each cash flow.

Interest rates can be measured in different forms, and various types of rate are classified, depending on the definition and the framework where the rate is applied. The bottom line is that they all represent the percentage of gain-loss on an investment.

It is possible to compound and discount rates at different frequencies. Depending on the frequency of recalculation of the interest on the interest, the overall amount of interest and the effective rate applied vary according to the frequency.

The dynamics of interest rates is influenced by several drivers that affect the rates and determine their value. Macroeconomic variables are the most important factors that influence the level and shape of the interest rates.

The net present value is an important measure of the profitability of an investment, given by the sum of all its cash flows, appropriately moved in time. The internal rate of return is the rate that makes the NPV of a cash flow stream equal to zero.

Annuities and perpetuities are specific types of cash flow streams with well-defined features that make them unique. An annuity is a stream of non-perpetual cash flows, while the perpetuity has an infinite time horizon.

Both types of cash flows can also be calculated in the form of growing cash flows. By adding a growth term, the formulas for the present value of both annuity and perpetuity change considerably, allowing measuring the present value of growing cash flows.

Problems

1. Arianna deposits money today in an account that pays 6.5% annual interest. How long will it take to double her money?
2. Sonia has 42,180.53 € in a brokerage account and plans to deposit an additional 5000 € at the end of every future year until the account totals 250,000 €. The account is expected to earn 12% annually on the account. How many years will it take to reach the goal?
3. What is the future value of a 7%, 5-year ordinary annuity that pays 300 € each year? If this were an annuity due, what would its future value be?
4. An investment will pay 100 € at the end of each of the next 3 years, 200 € at the end of year 4, 300 € at the end of year 5, and 500 € at the end of year 6. If other investments of equal risk earn 8% annually, what is its present value? Its future value?
5. Cristina is paying 20,000 € of tuition fees to the university every year. Upon successful completion of a certain amount of credits with high grade, she can get a reimbursement of 20% of the fees every 2 years. Assuming Cristina is always having a good performance, how does the timeline look like?
6. A mortgage of 100,000 € is issued to a company, with yearly repayment of 10,000 € including 7000 € capital and 3000 € interest, for 13 years. The company decides to pay all the mortgage in one solution after 4 years of payments. What is the timeline in this case?
7. Consider an investor depositing an amount of 20,000 € in a bank, for 5 years, with an interest to be earned of 3% every year. How much will the investor have at the end of the period?

8. Now assume the interest rate in exercise 7 starts at 3% and grows every year by 0.2% on top of the previous year. Repeat the calculation.
9. Consider an investor who will receive an amount of 50,000 € in 5 years from now. If the interest rate earned on the 5-year time is going to be 2.5% every year, how much is the present value of the sum?
10. Now assume the interest rate in exercise 9 starts at 2.5% and grows every year by 0.15% on top of the previous year. Repeat the calculation.
11. Mahsa can invest 15,000 € in a bank account paying a 4% interest forever. Every year she withdraws an amount equal to the 20% of the deposited money from the bank and reinvests the remaining capital again, at the same conditions. How can the situation be represented on the timeline?
12. Calculate the yield on a payment of 10,000 € in 3 years from now, whose present value today is 9500 €.
13. A fixed-payment loan is paid in yearly instalments of 2450 €, for 15 years, to repay an initial loan of 27,500 €. What is the yield to maturity of the loan?
14. A coupon bond has face value of 100 € and a coupon of 5%, and it is bought at par. In case the investor receives four coupon payments during the time span of the investment and is then able to sell the bond after 2 years for 118 €, what is the total investment return? Can this be solved analytically?
15. Vittorio, manager of Furjan LTD, wants to sell on credit, giving customers 3 months payment deadline. However, the customer Clelia LTD will have to borrow from her bank to carry the accounts payable. The bank will charge a nominal 15% but with monthly compounding. The customer wants to quote a nominal rate to customers that will exactly cover financing costs. What nominal annual rate should Clelia LTD credit customers?
16. Fred wishes to accumulate 1,000,000 € by his retirement date, which is 25 years from now. He will make 25 deposits in your bank, with the first occurring today. The bank pays 8% interest, compounded annually. He expects to get an annual raise of 3%, so he will let the amount he deposit each year also grow by 3%. How much must your first deposit be to meet your goal?
17. Your parents make you the following offer. They will give you 10,000 € at the end of every year for the next 5 years if you agree to pay them back 10,000 € at the end of every year for the following 10 years (i.e., from year 6 on). Should you accept this offer if your discount rate is 12% a year?
18. A rich entrepreneur would like to set up a foundation that will pay a scholarship to one deserving student every year. The first such scholarship will pay 8000 € and is to be awarded in 5 years from now. Then others will follow in perpetuity every year after, and the amount will be indexed at 1.2% per year. How much money should the entrepreneur put in the foundation's account today if that account earns 10% per year?
19. Elisa runs a construction firm. She has just won a contract to build a government office building. Building it will take 1 year and requires an investment of 12,000,000 € today and 6,000,000 € in 1 year. The government will pay her 25,000,000 € upon the building's completion. Suppose the cash flows and their times of payment are certain and the risk-free rate is 5%.

   (a) What is the NPV of this opportunity?
   (b) How can Elisa turn this NPV into cash today?
20. A firm is considering a project that will require an up-front investment of 10,000,000 € today and will produce 11,000,000 € in cash flow for the firm in 1 year without risk. One option is to pay the 9,000,000 € all cash, while another option is to issue a security that will pay investors 5,600,000 € in 1 year. Risk-free rate is 4%.
   (a) Is the project a good investment paying all cash?
   (b) Is the project a good investment issuing the new security?
   (c) What can be concluded?
21. Suppose Guzeliya wishes to retire 40 years from today. She determines that she needs 50,000 € per year once she retires, with the first retirement funds withdrawn 1 year from the day of retirement. She estimates to earn 6% per year on the retirement funds and that she will need funds up to including her 22nd birthday after retirement.
   (a) How much must she deposit in an account today so that there are enough funds for retirement?
   (b) How much must she deposit each year in an account, starting 1 year from today, so that there are enough funds for retirement?

Case Study: Time Value of Money

Lottery in the United States

The Case

In the United States, lotteries are subject to the laws of and operated independently by each jurisdiction. Even though there is no national lottery, organization consortiums of state lotteries organize games spanning nationwide and carrying larger jackpots. That is the case of Powerball, which is in turn one of the major national lotteries.

For those who are not familiar with gambles, the Powerball is one the most popular and richest in prizes lotteries in the country. As any other lottery, it can be played by picking random numbers and a small monetary investment.

The winning of the jackpot is indeed a very unlikely task to accomplish given that the chances of scoring the winning combination are approximately 1 in 300 million, equivalent to looking for a microscopic needle in a gigantic haystack.

Clelia is a student of English Literature who recently won a jackpot of $250,000,000 at Powerball, an event that is now changing her life and turning her life upside down in some sense.

Jackpot winners have the option of receiving their prize in cash (lump sum) or as an annuity paid in 30 yearly instalments, starting from year 1 (no payments at current time). Each annuity payment is 2% higher than in the previous year to adjust for inflation. The lump sum means taking the entire cash value at once, but there is a

Table 2.2 Schedule of pretax payments, over 30 years, of $250,000,000 growing annuity

Year	Annuity payment
1	4,059,095
2	4,140,277
3	4,223,082
4	4,307,544
5	4,393,695
6	4,481,568
7	4,571,200
8	4,662,624
9	4,755,876
10	4,850,994
11	4,948,014
12	5,046,974
13	5,147,913
14	5,250,872
15	5,355,889
16	5,463,007
17	5,572,267
18	5,683,712
19	5,797,387
20	5,913,334
21	6,031,601
22	6,152,233
23	6,275,278
24	6,400,783
25	6,528,799
26	6,659,375
27	6,792,563
28	6,928,414
29	7,066,982
30	7,208,322

Payments increase by 2% per year, and interest rate is constant at 3% per year

catch: the lump sum is less than the value of the total jackpot, namely, 56% of the announced nominal amount.

Assume that federal taxes on lottery winnings are 25% of the nominal amount, followed by a further 14.6% tax deduction during season (topping up to the maximum US federal tax rate of 39.6%) which applies to both a lump sum and a yearly annuity payment.

Clelia is currently challenged with making the decision of whether to cash the lump-sum payment or to accept the annuity payments every year for the next 30 years, without loss in the nominal amount, besides taxes.

She is not much expert on financial matters, so she decides to ask her friend Allison (a financial analyst in an important consulting firm) to help her with the decision. Winners of the Powerball have 60 days to decide which form of payment they prefer.

Allison accepts to help her friend and decides to start a thorough analysis of both the payment options, given some reasonable assumptions about the evolution of the financial variables involved.

In particular, she wants to compare the future value of the sum obtained by accepting the lump-sum payment to that obtained by accepting the annual annuity payments to be reinvested at the reference interest rate (yield).

Assume that in case the annuity is chosen, the yearly payments can be reinvested at the available interest rate in the economy, which in turn depends on the state of the economy and is supposed to be constant at 3% for the next 30 years. Also, assume returns to be taxed at the same 39.6% as for the lump-sum payment.

Questions

1. If you were Allison, which type of payment would you recommend to Clelia, in case the before-tax return is 3%? Use Table 2.2 to start your analysis.
2. How does your answer to question 1 change in case the reinvestment rate is 6%?
3. Build up a scenario analysis and a graph to describe the relationship between the future value of the lump-sum payment and the future value of the annuity, for possible interest rates ranging from 1% to 5%.
4. Under which conditions is it, in general, convenient to accept the lump sum rather than the annuity payments?
5. Beside the mere numbers and rates, which are other factor(s) to consider when choosing a lump-sum payment instead of an annuity over the years?
6. What considerations would you put forward to the fact that most categories of winners do prefer a lower lump-sum payment?

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3 The Relationship Between Risk and Return

The relationship between risk and return on the financial market is an issue of primary importance in finance, and it spans all the fields of specialization, including corporate finance.

In fact, one of the most important principles in financial markets states that securities with higher risk are supposed to give a higher expected return in order to be appealing for an investor.

On the other hand, securities with lower risk are those chosen by the investors with lower risk appetite. They therefore offer a lower return and are more indicated for conservative investment strategies.

The risk-return trade-off is at the foundation of modern finance, and finding the right balance between return demand and risk exposure is at the basis of good management of a business.

All the theory of financial risk and return has its roots in the work done in the 1950s about the Modern Portfolio Theory. Nowadays it is then possible to give a mathematical interpretation to the relationship.

After that, the Capital Asset Pricing Model constitutes a further step in identifying the elements of risk and return that optimize an investment in some specific asset and describes the optimal portfolios.

After studying this chapter, you will be able to answer the following questions, among others:

- How can return and risk in financial markets be represented?
- What are the basics of the relationship between risk and return?
- How can Portfolio Theory be used to select the efficient portfolios in the space of return and volatility?
- What are the assumptions of the Capital Asset Pricing Model and how can it be derived?
- How is it possible to determine the right expected return for an investment?

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The first part of the chapter is dedicated to the definition of expected return and volatility as a measure of risk universally used in finance. The second section is about the Modern Portfolio Theory and its implications for investment decisions and asset pricing. The last section deals with the Capital Asset Pricing Model and its application for determining the rate of return demanded by investors for a particular level of risk.

3.1 Expected Return and Volatility

Learning Outcomes

- Understand the concept of portfolio return.
- Understand the concepts of portfolio volatility and correlation.
- Learn how to use Monte Carlo simulation methods.

3.1.1 The Portfolio Return

Financial returns are profits on an investment, including changes in value of the assets held, or dividends coming from owning it, as well as other cash flows, which the investor receives from the investment.

More appropriately, in general, financial returns are referred to as profits on an investment as percentage of the amount invested, for investments made over a specific period.

The period is typically a year, in which case the rate of return is referred to as an annual return, but as mentioned for the interest rates compounding frequencies in Chap. 2, other time periods can be assumed for calculation.

It is a measure of the profitability of an investment, and, by just inverting the calculation, it allows to understand the time it will take to partially or fully recover an amount invested in some project.

Example 3.1 Consider an investor holding 10,000 € and receiving 1200 € in the first year of the investment. The rate of return is then 12%, and the investor will recover the initial 10,000 € in almost 7 years. Different investors have different required rates of return at different levels of risk.

There are several ways financial returns can be measured, which creates a problem of transparency when financial institutions try to market their investment opportunities or try to communicate with clients.

In an ideal world, clients would be updated annually with their portfolio's rate of return, enabling them to compare their performance to the appropriate benchmarks. However, in reality, some advisors may not be providing these results to their clients.

Therefore, there is a question mark on what returns measurement should be used and how the possible measurement methods differ from each other. Moreover, there

Table 3.1 Market values of the portfolio

Date	Market value (€)	Cash flow (€)	Net (€)
December 2014	350,000		
January 2015	357,000		
February 2015	365,000	+30,000	395,000
March 2015	401,000		
April 2015	407,000		
May 2015	412,000	−15,000	397,000
June 2015	403,000		
July 2015	410,000		

are issues related to the contributions and withdrawals from the portfolio, to complicate the picture.

There are four common methodologies for return calculation, and they can sometimes differ substantially from each other, depending mostly on the combination of the timing of cash flows, their size relative to the portfolio value, and the volatility of the portfolio's market value.

Example 3.2 An investor holds a portfolio with market value of 350,000 € in December 2014. The following table shows the market values for the first quarter of 2015, with contributions and withdrawals, as in Table 3.1.

The first methodology for return calculation is called money-weighted rate of return, most commonly known as IRR, already described in Chap. 2. As from the definition, the average return makes the net present value of the sum of all cash flows (during the measurement period) equal to zero.

Assuming that all contributions to the portfolio are positive cash flows, while all the withdrawals from the portfolio are negative cash flows, it is possible to use the method for calculating the rate of return in the presence of relatively small cash flows, compared to the portfolio value (Satchell 2007).

Assuming that all the cash flows earn the same rate of return when they are invested, its return can differ substantially from the true time-weighted rate of return when large cash flows occur during periods of significantly fluctuating portfolio values.

The linked internal rate of return (LIR) is an approximate time-weighted rate of return that can be calculated by performing a geometrical link of monthly money-weighted rates of return.

When the time horizon is 1 year or less, geometric linking involves converting the monthly money-weighted returns to relative form ($1 + r_{\mathrm{MW}i}$), multiplying them together, and subtracting 1 from the result, as from the formula

$$r_{\mathrm{LIR}} = [(1 + r_{\mathrm{MW}1}) \times (1 + r_{\mathrm{MW}2}) \times \ldots \times (1 + r_{\mathrm{MW}n})] - 1$$

where:

r_{MW1} is the monthly money-weighted rate of return at time 1.

In case the external cash flows hitting the portfolio have not been large in combination with volatile portfolio fluctuations during the monthly measurement period, the measured LIR should be very close to the true time-weighted rate of return.

The modified Dietz rate of return (MDR) is a method used by the Canadian company PWL Capital, in order to calculate returns for their customers. The approximation to a true time-weighted rate of return is achieved by weighting.

Each cash flow in fact is weighted by the proportion of the measurement period it is present or absent from the portfolio. Monthly Modified Dietz rates of return are usually calculated and geometrically linked together, similar to the LIR.

The MDR is based on monthly valuation of the portfolio; therefore, the rate it yields can be widely different from the true time-weighted rate of return in the presence of large external cash flows occurring during monthly measurement periods characterized by high volatility.

$$r_{\mathrm{MD}} = \frac{V_{\mathrm{M1}} - V_{\mathrm{M0}} - CF}{V_{\mathrm{M0}} + \sum \mathrm{CF}_i w_i}$$

with

$$CF = \sum CF_i$$

where:

V_{M0} is the full market value of the portfolio at the beginning of the period.
V_{M1} is the full market value of the portfolio at the end of the period.

and

$$w_i = \frac{n_{\mathrm{CD}} - n_{\mathrm{D}}}{n_{\mathrm{CD}}}$$

where:

n_{CD} is the total number of days in the period.
n_{D} is the number of days from the beginning of the period that the CF_i occurs.

All cash flows in the above formulas are assumed to occur at the end of the day.

The final and most relevant portfolio performance measurement tool is the time-weighted rate of return (TRR). It is the most accurate return measure in most cases but requires daily valuations of the portfolio, as soon as a cash flow occurs (Lettau and Van 2008).

Periods of return calculation are divided into subperiods, and for each of them, the total return calculation is performed. These subperiod returns are then geometrically linked together to obtain the TRR over the measurement period.

$$r_{\mathrm{TW}} = [(1 + r_{t,1}) \times (1 + r_{t,1}) \times \ldots \times (1 + r_{t,n})] - 1$$

where

$$r_{t,1} = \frac{V_{\mathrm{M1}} - V_{\mathrm{M0}}}{V_{\mathrm{M0}}}$$

3.1.2 Volatility and Correlation

Many applications in finance require volatility as an input, including corporate finance, for valuation. Knowing how to measure volatility and correlation is therefore crucial, as well as knowing what the economic drivers of volatility in financial markets are.

A financial manager has the aim to understand what the determinants of volatility on financial markets are. Moreover, it is important to highlight the connections between volatility and the risk factors driving it.

Volatility can be measured in several ways, and many approaches are possible. Historical volatility is the most commonly used method, while other methods are implied volatility and econometric modeling.

Historical volatility can be measured as the standard deviation (square root of variance) of the log changes in price, measured at regular time intervals. Estimating volatility from historical data implies starting from the observation of market prices at fixed time intervals (Connor et al. 2010).

The frequency of observation can range from daily to yearly, depending on the nature and purpose of the analysis. There are several methods to scale the volatility conveniently from one frequency to another.

The observation is set on n prices at some point in time, at the end of any time interval i in the time range. First, the returns for each time interval must be calculated, in order to compute the variance afterward.

Each return r_i is calculated from a couple of prices, and therefore $n + 1$ prices are needed in order to calculate n returns. Define

$$\begin{aligned} r_i &= \ln(S_i) - \ln(S_{i-1}) \\ &= \ln\left(\frac{S_i}{S_{i-1}}\right), \quad i = 1, 2, \ldots, n \end{aligned}$$

where:

S_i is the stock price at the end of i-th interval (time i).
S_{i-1} is the stock price at the end of $(i-1)$-th interval (time $i - 1$).

Volatility is measured according to the basic statistical properties of variance and standard deviation. As a convention, it is convenient to define the sum of square deviation from the mean d^2 as

$$d^2 = \sum_{i=1}^{n} \left(r_i - \bar{r}\right)^2$$

where $\bar{r}$ is the mean of the returns over time, calculated as

$$\bar{r} = \frac{1}{n} \sum_{i=1}^{n} r_i$$

Once d^2 is defined, the standard deviation estimator can be derived as the square root of the unbiased sample variance.

$$s_\sigma = \sqrt{\frac{\sum_{i=1}^{n} \left(r_i - \bar{r}\right)^2}{n - 1}}$$

Basic knowledge of statistics is sufficient to recall that the estimator s is actually biased, as opposed to s^2 which is an unbiased estimator for the population variance σ^2.

One of the tricky points in the historical volatility estimation is the right choice of n, which is not easy. In fact, choosing a high number of observation increases the accuracy of the estimation but at the cost of including data that go too far back in time which can bias the analysis since the far past may be irrelevant.

Recall that the frequency of observation can be of any kind, and it is possible to scale the volatility to get the annualized value of it from the volatility measured at other frequencies. The yearly estimate of standard deviation therefore is

$$\widehat{\sigma} = s_\sigma \sqrt{m}$$

where m is the number of reference periods in a year, so that $m = 252$ for daily observations, $m = 52$ for weekly observations, $m = 12$ for monthly observations, and so on. The standard error of the estimation is approximately equal to $\widehat{\sigma}/\sqrt{2n}$.

If the volatility measures are high, it means there have been many fluctuations in market price. A low volatility shows the price has been stable over time. It is also possible to compare the volatility of one instrument with other instruments or indices, in order to get a measure of the relative volatility.

In the short run, volatility changes over time, while in the long run, it is possible to identify trends to almost fixed values, well contained in between narrow boundaries.

In order to identify those trends, a technique is to measure volatility over different time windows and average it (Esch et al. 2005).

Comparison between short-term volatility and long-term trend is very useful to determine the trend of volatility over time. For example, a high short-period low volatility combined with a long-period high volatility means the stock has recently calmed down in price fluctuations.

Implied volatility is another method for standard deviation measurement, widely used in finance. It is the volatility implied by an analytical model for pricing financial assets. In particular, it is the volatility implied by the Black-Scholes-Merton (BSM) model for pricing financial derivatives.

The description and analysis of the analytical model are beyond the scope of the chapter and are dealt with in Chap. 9. However, the concept of implied volatility is so important in finance, that it is worthwhile to present it by using appropriate math.

The purpose of this part is to introduce the concept of implied volatility and analyze the use of it in financial analysis. According to BSM model, the price of a European call option written on a stock is given by

$$c = SN(d_1) - K\mathrm{e}^{-rT}N(d_2)$$

with

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\Delta t}{\sigma\sqrt{\Delta t}} \\ d_2 &= d_1 - \sigma\sqrt{\Delta t} \end{aligned}$$

where:

S is the price of the stock, as observed in the market at the time of calculation.
K is the strike price, as specified in the option contract.
σ is the volatility of the underlying stock.
r is the risk-free rate.
Δt is the time left until maturity of the option.
$N(.)$ is the cumulative normal distribution.

The volatility parameter σ is a constant in the model. By inverting the BSM model, one can achieve a measure of the volatility that is implied by the actual market price of the option.

The problem is that the model is complicated so that it is not possible to be inverted analytically. Therefore, numerical procedures are needed in order to invert the model and get the value for σ.

In the last year, financial mathematicians have worked on advanced financial approximation formulas for the calculation of implied volatility. One of the most popular approximation formulas has been developed by M. Brenner and G. Subrahmanyam, who define implied volatility as

$$\sigma = \sqrt{\frac{2\pi c}{S\Delta t}}$$

The above approximation works for the money options, namely, the options for what the condition $S = Ke^{-rT}$ holds.

Another popular method of approximation was presented by Curtis and Carriker in 1988, and it is called the Direct Implied Volatility Estimate (DIVE), whose formula is given by

$$d_1 = \frac{\sigma\sqrt{\Delta t}}{2}$$
$$d_2 = -\frac{\sigma\sqrt{\Delta t}}{2}$$

The BSM formula then becomes

$$\begin{aligned} c &= S\left[N\left(\frac{\sigma\sqrt{\Delta t}}{2}\right) - N\left(-\frac{\sigma\sqrt{\Delta t}}{2}\right)\right] \\ &= S\left[N\left(\frac{\sigma\sqrt{\Delta t}}{2}\right) - \left(1 - N\left(\frac{\sigma\sqrt{\Delta t}}{2}\right)\right)\right] \\ &= S\left[2N\left(\frac{\sigma\sqrt{\Delta t}}{2}\right) - 1\right] \end{aligned} \tag{3.1}$$

Formula (3.1) can be inverted in order to get the implied volatility (Brenner and Subrahmanyam 1988) as

$$\sigma = \frac{2}{\sqrt{\Delta t}} N^{-1}\left(\frac{c+S}{2S}\right)$$

where:

$N^{-1}(.)$ is the inverse of the cumulative normal distribution.

Implied volatility is a good proxy of the market sentiment about the volatility of a particular asset. Some investors calculate implied volatility on actively traded options and then manipulate it in order to price less actively traded options on the same underlying stock.

Another type of volatility is the one that is estimated through econometric models, in particular the so-called Autoregressive Conditional Heteroscedasticity (ARCH) and General ARCH (GARCH) models (Green 2011).

Econometric models like those allow to model variance in a framework of autoregressive returns, dependent on historical values, and heteroscedasticity, which is the condition of nonconstant conditional variance.

A convenient way to describe the relation between two variables is through covariance, which describes the interdependence among the variables telling if it is direct or inverse, and also to what degree the variables move in the same or opposite direction.

Recall the variance of a random variable x with mean μ is given by

$$
\begin{aligned}
\sigma^2 &= E\left\{[x - E(x)]^2\right\} \\
&= E\left[(x-\mu)^2\right] \\
&= \int (x-\mu)^2 f(x)dx \\
&= \int x^2 f(x)dx - \left[\int xf(x)dx\right]^2 \\
&= E(x^2) - E^2(x)
\end{aligned}
$$

Using the integration method, it is then possible to describe the covariance between two variables X and Y, which is given by

$$
\begin{aligned}
\sigma_{x,y} &= E\left[(x-\mu_x)(y-\mu_y)\right] \\
&= E(xy) - E(x)E(y) \\
&= \int\int xyf(x,y)dxdy - \left[\int\int xf(x,y)dxdy \int\int yf(x,y)dxdy\right] \\
&= \int\int xyf(x,y)dxdy - \left[\int xf_x(x,y)dx \int yf_y(x,y)dy\right]
\end{aligned}
$$

where:

$f(x, y)$ is the joint density.
$f_x(x, y)$ and $f_y(x, y)$ are the marginal densities.

Correlation is very similar to covariance in that it measures the relation between two variables, but, as opposed to covariance, it also gives information about the degree to which the two variables are related.

The correlation coefficient ranges from -1 to +1, making it a standardized measure of the interdependence between two variables. A coefficient of +1 corresponds to positive perfect correlation. When the variables are not related at all, the value of correlation is zero, and the perfect negative correlation corresponds to a value of -1.

The correlation coefficient, denoted by $\rho_{x,\ y}$, of random variables x and y is defined to be

$$\rho_{x,y} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

where:

σ_{xy} is the covariance between x and y.
σ_x is the standard deviation of x.
σ_y is the standard deviation of y.

The coefficient is independent of the measurement unit and has the same sign of covariance.

Example The joint density of two random variables x and y is

$$f(x, y) = \begin{cases} y^{-2x} & x \geq 0, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

By integrating, the expected values are given as

$$E(x) = \int_0^\infty x f_x(x, y) dx = \ldots = \frac{1}{2}$$
$$E(y) = \int_0^1 y f_y(x, y) dy = \ldots = \frac{1}{3}$$
$$E(xy) = \int_0^\infty \int_0^1 xy f(x, y) dx dy = \ldots = \frac{1}{12}$$

The covariance is given by the expectation of the product minus the product of the expectations, as defined by the formula

$$\begin{aligned} \sigma_{x,y} &= E(xy) - E(x)E(y) \\ &= \frac{1}{6} - \frac{1}{12} \\ &= \frac{1}{12} \end{aligned}$$

The correlation between x and y is simply the expected product of the corresponding standard scores as can be shown by

$$\begin{aligned} \rho_{x,y} &= \frac{\sigma_{x,y}}{\sigma_x \sigma_y} \\ &= \frac{E\{[x - E(x)][y - E(y)]\}}{\sigma_x \sigma_y} \\ &= E\left\{ \left[\frac{x - E(x)}{\sigma_x}\right] \left[\frac{y - E(y)}{\sigma_y}\right] \right\} \end{aligned}$$

In finance, the measure of correlation between two financial assets is related to the issue of portfolio diversification. Diversification is the practice of investing in many different assets in order to reduce the overall portfolio volatility through correlation effect.

The correlation coefficient can be expanded to give a measure of the correlation between two assets, starting from the historical realized returns based on n observations, as described by

$$\widehat{\rho}_{x,y} = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\left[n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]}}$$

which provides the sample correlation coefficient of the two assets rather than the population coefficient.

Example 3.3 Consider the following table with five observations of the returns of two assets x and y.

Stock	$n = 1$	2	3	4	5
x	13.2%	−2.35%	4.53%	10.42%	−3.44%
y	10.3%	3.45%	−1.23%	8.33%	1.21%

Calculation yields

	x_i	y_i	$x_i y_i$	x_i^2	y_i^2
1	13.2%	10.3%	1.36%	1.74%	1.06%
2	−2.35%	3.45%	−0.08%	0.06%	0.12%
3	4.53%	−1.23%	−0.06%	0.21%	0.02%
4	10.42%	8.33%	0.87%	1.09%	0.69%
5	−3.44%	1.21%	−0.04%	0.12%	0.01%

so that

$$\sum_{i=1}^{n} x_i = 22.36\%,\ \sum_{i=1}^{n} y_i = 22.06\%,\ \sum_{i=1}^{n} x_i y_i = 2.05\%,$$
$$\sum_{i=1}^{n} x_i^2 = 3.21\%,\ \left(\sum_{i=1}^{n} x_i\right)^2 = 5.00\%,\ \sum_{i=1}^{n} y_i^2 = 1.90\%,\ \left(\sum_{i=1}^{n} y_i\right)^2 = 4.87\%$$

and the sample correlation is given by

$$\widehat{\rho}_{x,y} = \frac{(5 \times 2.05\%) - (22.36\% \times 22.06\%)}{\sqrt{[(5 \times 3.21\%) - 5.00\%][(5 \times 1.90\%) - 4.87\%]}} = 0.7416$$

3.1.3 Maximum Likelihood Methods

In case of volatility from econometric models, those must be estimated in order to get the parameters that explain the variance. Maximum likelihood estimation (MLE) is a general method for estimating the parameters of an econometric model (McLeish 2005).

As the name suggests the method is based on the fit of the modeled calculated variance to the real one, observable on the market. It involves a random variable with some probability density whose form is known, but not the parameter vector.

Consider a random variable $x_i = x_1, x_2, \ldots, x_n$ with probability density function $f(x_1, x_2, \ldots, x_n; \theta)$. As mentioned above MLE involves choosing the values of the parameters that give the highest probability to match the observed data.

The trick behind MLE is to view the time series under analysis as a series of draws from a probability distribution. Assuming there are n random variables conditional on n parameters, the joint probability density function is given by

$$f(x_1, x_2, \ldots, x_n; \theta_1, \theta_2, \ldots, \theta_n) \tag{3.2}$$

The usual interpretation of formula (3.2), as the function of the variable x_t for given parameters θ, in the case of MLE, is reversed. The aim now is to look at f as a function of the parameter given the value of the variable.

The MLE can be used for both time series models and replicated experiments, being its versatility in statistics.

For a variable x with n observations $x_1, x_2 \ldots, x_n$ and a joint probability density given by

$$(x_1, \ldots, x_n) \mapsto p_{n,\theta}(x_1, \ldots, x_n)$$

which depends on a parameter θ, the likelihood function is the stochastic process

$$\theta \mapsto p_{n,\theta}(x_1, \ldots, x_n)$$

The MLE for the parameter θ is the value maximizing the likelihood function. It is now possible to condition repeatedly in order to represent the likelihood appropriately. In fact, the likelihood corresponding to the observations $x_1, \ldots, x_n$ as $\theta \mapsto p_\theta(x_1, \ldots, x_n)$ can be decomposed as

$$\theta \mapsto p_\theta(x_1, \ldots, x_n) = p_\theta(x_1) \times p_\theta(x_2|x_1) \times \ldots \times p_\theta(x_n|x_{n-1}, \ldots, x_1)$$

In case the variable under analysis is continuous, the analysis changes slightly. The probability density function in fact can be written as

$$f(x; \theta_1, \theta_2, \ldots, \theta_k)$$

In addition, it depends on k unknown parameters $\theta_1, \theta_2, \ldots, \theta_k$.

Again, a draw of n independent observations $x_1, x_2, \ldots, x_n$ from an experiment has an associated likelihood function that can be written as

$$L(x_1, x_2, \ldots, x_n|\theta_1, \theta_2, \ldots, \theta_k) = \prod_{i=1}^{n} f(x_i; \theta_1, \theta_2, \ldots, \theta_k) \tag{3.3}$$

The right-hand side of Eq. (3.3) is clearly very difficult to calculate, being an iterated product of densities. A convenient way to solve the issue is to work with the log of the likelihood, given that it is a monotone transformation that preserves the concavity of the target function, leaving the maximization problem unchanged.

The log likelihood function is given by:

$$\begin{aligned}\ell(x|\theta) &= \ln L(x_1, x_2, \ldots, x_n|\theta_1, \theta_2, \ldots, \theta_k) \\ &= \sum_{i=1}^{n} \ln f(x_i; \theta_1, \theta_2, \ldots, \theta_k)\end{aligned}$$

So, maximizing the log likelihood is much easier than the likelihood L, and the MLEs of θ_1, θ_2, …, θ_k are given by the simultaneous solutions of k equations such that

$$\frac{\partial \ell(x|\theta)}{\partial \theta_j} = 0, \quad j = 1, 2, \ldots, k$$

MLE is a very good estimator for big samples, tending to infinite. For finite samples, there are better tools.

In general, the MLE method is used to estimate parameters of well-known distributions, as, for example, the normal distribution. In this case, the MLE distribution for the parameters mean μ and standard deviation σ for the normal distribution can be derived by first introducing its probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Assume the observations x_1, x_2, …, x_n are realizations of a normally distributed variable x. The likelihood function is given by

$$\begin{aligned}L(x_1, x_2, \ldots, x_n|\mu,\sigma) &= \prod_{i=1}^{n}\left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X_i-\mu}{\sigma}\right)^2}\right] \\ &= \frac{1}{\left(\sigma\sqrt{2\pi}\right)^N} e^{-\frac{1}{2}\sum_{i=1}^{N}\left(\frac{X_i-\mu}{\sigma}\right)^2}\end{aligned}$$

The log likelihood is

$$\ell(x|\theta) = -\frac{n}{2}\ln(2\pi) - n\ln\sigma - \frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2$$

The first-order conditions are defined by setting the partial derivatives with respect to mean and standard deviation equal to zero.

$$\begin{aligned}\frac{\partial(\ell)}{\partial\mu} &= \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i - \mu) = 0 \\ \frac{\partial(\ell)}{\partial\sigma} &= -\frac{n}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(x_i - \mu)^2 = 0\end{aligned}$$

Solving the two equations simultaneously gives the solution

$$\widehat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\widehat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (x_i - \mu)^2}$$

The estimators are correct in the limit, meaning their accuracy increases with the size of the sample. When the sample equals the whole population, the estimators are true values. As for the general MLE, the above estimators have in-the-limit properties, being unbiased, sufficient, consistent, and efficient.

Snapshot 3.1

Risk and Return in Excel

The calculation of risk and return of some particular assets starts from the time series of the historical market prices of the asset, at any frequency (daily, weekly, monthly, etc.). Let us consider the case of daily prices.

Date	Price
Day_n	P_n
Day_{n-1}	P_{n-1}
...	...
Day_1	P_1

Note that here Day_n is the most recent day in the dataset, while Day_1 is the oldest.

One can then calculate the daily returns by using any the following formulas:

Date	Price	Return	or	Return	or	Return
Day_n	P_n	$r_n = \frac{P_n - P_{n-1}}{P_{n-1}}$		$r_n = \frac{P_n}{P_{n-1}} - 1$		$r_n = \ln\left(\frac{P_n}{P_{n-1}}\right)$
Day_{n-1}	P_{n-1}	$r_{n-1} = \frac{P_{n-1} - P_{n-2}}{P_{n-2}}$		$r_{n-1} = \frac{P_{n-1}}{P_{n-2}} - 1$		$r_n = \ln\left(\frac{P_n}{P_{n-1}}\right)$
...	...	...		...		...
Day_2	P_2	$r_2 = \frac{P_2 - P_1}{P_1}$		$r_2 = \frac{P_2}{P_1} - 1$		$r_n = \ln\left(\frac{P_n}{P_{n-1}}\right)$
Day_1	P_1	...		...		...

It is clear that for n prices in the dataset, a total of $n - 1$ returns can be calculated.

The return of the asset can be calculated as the average of daily returns multiplied by the scaling factor 252 (number of days in a financial year) as

(continued)

Snapshot 3.1 (continued)

$$\boxed{= \text{AVERAGE}\left(\text{`Column of returns'}\right) * 252}$$

The scaling factor depends on the frequency of the dataset and can be adjusted accordingly. For weekly data, the factor is 52, for monthly data is 12, and so on.

The volatility of the returns can be calculated by using the command in Excel for the calculation of the standard deviation of the population of data as

$$\boxed{= \text{STDEV.P}\left(\text{`Column of returns'}\right) * \text{SQRT252}}$$

In this case, the scaling factor is square rooted because the standard deviation is already the square root of the variance, which is scaled as the returns.

3.2 Modern Portfolio Theory

Learning Outcomes

- Understand the trade-off between risk and return on financial markets.
- Learn about market efficiency.
- Understand the main concepts of Modern Portfolio Theory.

3.2.1 The Risk/Return Trade-Off

Financial economist Harry Markowitz developed the Modern Portfolio Theory (MPT) in the 1950s, claiming that the information given on the risk and return of the single assets in a portfolio is not enough to conclude about the overall risk and return.

In practice, MPT tries to give a mathematical description of how diversification affects the portfolio risk and return by modeling the mathematical relationship between risk and return of the assets in a normal market.

Standard deviation is the measure of risk in the MPT framework, and the model aims at defining all the possible combination of risk and return available in the market and then identifying the optimal portfolio.

MPT relies on assumptions on distribution of prices and returns. In particular, they are assumed to follow a normal distribution. So in order to understand the relationship between risk and return, it is important to recall the basic properties of the normal distribution.

- The distribution is symmetric and bell shaped.
- It is continuous for all values of X between $-\infty$ and ∞ so that every interval of real numbers has a non-null probability.
- The distribution is totally defined by two parameters, μ and σ, that determine the shape of the distribution.
- The probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where:
μ is the mean of the distribution.
σ is the standard deviation of the distribution.

- About $\frac{2}{3}$ of all cases fall within one standard deviation from the mean, that is

$$\Pr(\mu - \sigma \leq x \leq \mu + \sigma) \approx 0.68$$

- About 95% of cases lie within two standard deviations from the mean, that is

$$\Pr(\mu - 2\sigma \leq x \leq \mu + 2\sigma) \approx 0.95$$

Recall from basic statistics that the normal distribution (Fig. 3.1) is completely defined by the two parameters mean and standard deviation. In financial terms, if one assumes that returns are normally distributed, the mean corresponds to the expected return, and the standard deviation describes the volatility of returns (Figs. 3.2 and 3.3).

The intuition is that the relationship between the risk and the return of some investment is given by the same corresponding relationship between the mean and standard deviation of the distribution of the returns.

If prices or returns are supposed to be normally distributed, it is possible to infer the properties of the normal distribution to them.

Example 3.4 If we have a time series of normally distributed financial returns, with mean 0.04 and standard deviation 0.22, we can say that the asset associated with that particular series of data has an expected return of 4% (over the time horizon considered) and a volatility (risk) of 22%.

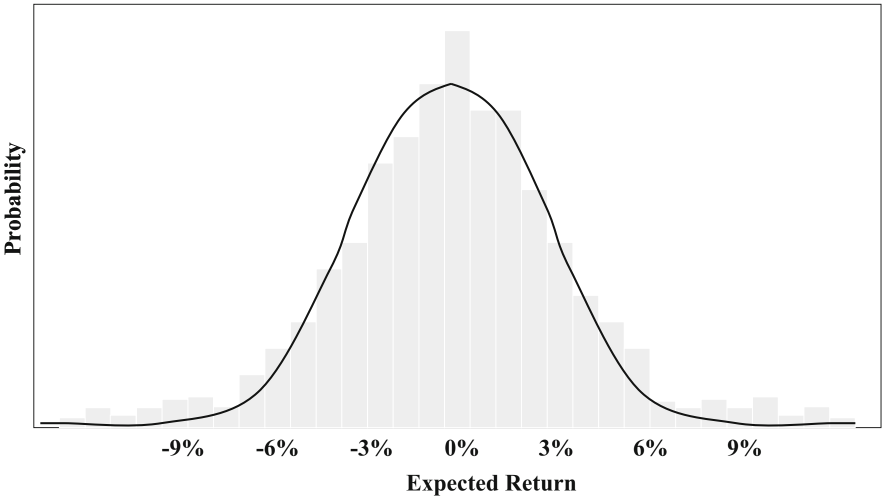

Fig. 3.1 Financial returns of most securities are normally distributed. In this case, the graph shows an average return of almost 3% for that particular security

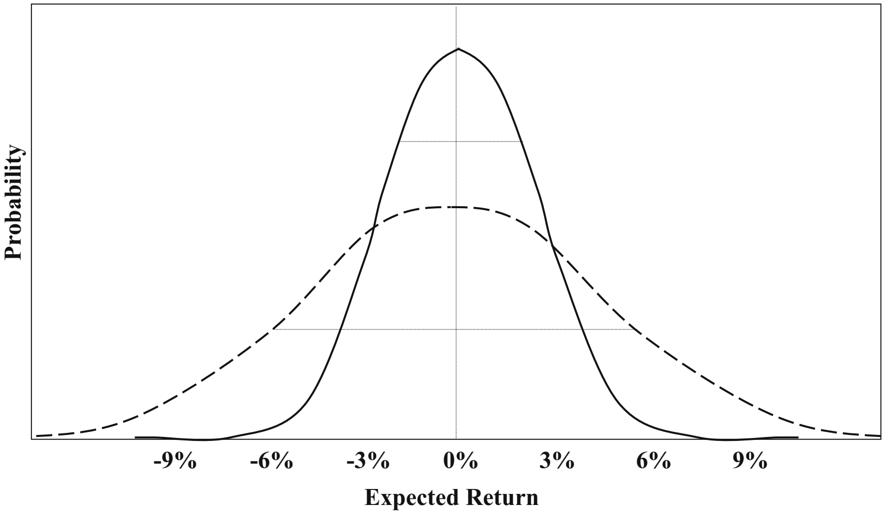

Fig. 3.2 Distribution of possible returns of two different investments. They both have the same expected return, but one of them (dashed line) offers a wider spread of possible outcomes, making other investments safer and more attractive for most of the investors

Given the distributional properties of returns that are normally distributed, it is clear that the relationship between risk and return is direct, in order to get a.

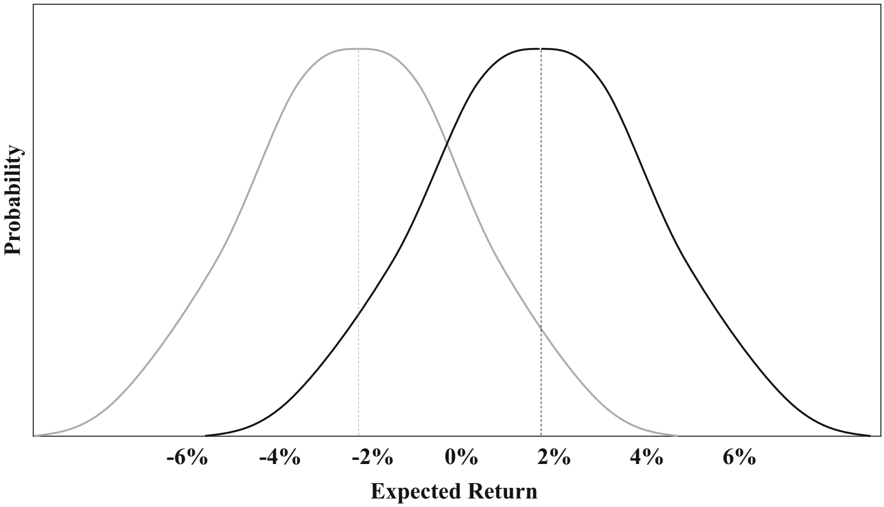

Fig. 3.3 Two Normal distributions, with same standard deviation but different mean (expected return)

Investment returns reflect the degree of risk carried by the investment, and investors should be able to determine what level of return is appropriate for a given level of risk.

Define the portfolio value as

$$V_t = \sum_{i=1}^{n} \frac{\alpha_i}{S_{i,0}} S_{i,t}$$

where:

α_i is the amount invested in asset i.
$S_{i,\ 0}$ is the value of asset i at time 0.
$S_{i,\ t}$ is the value of asset i at time t.

The expected return of a portfolio is straightforward to calculate, being the weighted average of the expected returns of the assets composing it. Weights assigned to the calculation reflect the proportion of each asset on the total portfolio value.

$$
\begin{aligned}
r_{\mathrm{p}} &= \frac{V_t - V_{t-1}}{V_{t-1}} \\
&= \dots \\
&= \sum_{i=1}^{n} w_i r_i
\end{aligned}
\tag{3.4}
$$

where:

r_i is the expected return on stock i.
n is the number of stocks in the portfolio.
w_i is the weight (proportion) of asset i in the portfolio.

Example 3.5 Suppose stocks A and B have expected returns $E(r_{\mathrm{A}}) = 12.5\%$ and $E(r_{\mathrm{B}}) = 20\%$. The expected return of the portfolio composed of 75% of stock A and 25% of stock B is given by

$$E(r_{\mathrm{p}}) = 0.75 \times 12.5\% + 0.25 \times 20\% = 14.38\%$$

The calculation of the standard deviation of a portfolio is trickier, in that it does not only involve the volatility of the single assets in the portfolio but also their covariance. The co-movements of the asset returns must be taken into account through covariance and correlation measures (Merton 1972).

The combined analysis of the single volatilities and covariance among couples of assets leads to the formula for the variance of the portfolio, which is given by

$$
\begin{aligned}
\sigma_{\mathrm{p}}^2 &= E\left[\left(r_{\mathrm{p}} - E(r_{\mathrm{p}})\right)\right]^2 \\
&= \dots \\
&= \sum_{i=1}^{n} \sum_{j \neq i=1}^{n} w_i w_j \sigma_{ij}
\end{aligned}
\tag{3.5}
$$

where:

σ_i^2 is the variance of the i-th asset.
w_i is the weight of an asset i in the portfolio.
w_j is the weight of an asset j in the portfolio.
σ_{ij} is the covariance between asset i and j.

Equation (3.5) can be written in matrix notation (Best 2010) as

$$\sigma_{\mathrm{p}}^2 = \mathbf{w}^{\mathrm{T}} \mathbf{\Sigma} \mathbf{w}$$

where:

$\mathbf{w} = (w_1, w_2, \ldots, w_n)$ is the vector of weights of assets in the portfolio.

$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma_{1,n} \\ \vdots & \iddots & \vdots \\ \sigma_{n,1} & \cdots & \sigma_n^2 \end{pmatrix}$ is the covariance matrix of the portfolio.

The covariance between any couple i,j of asset returns is given by

$$\begin{aligned}\sigma_{ij} &= E\{[r_i - E(r_i)][r_j - E(r_j)]\} \\ &= \ldots \\ &= E(r_i r_j) - E(r_i)E(r_j)\end{aligned}$$

The calculation of the correlation coefficient between the returns on two stocks is given by

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$

where:

σ_i is the standard deviation of asset i.
σ_j is the standard deviation of asset j.

Example 3.6 Suppose stock A and stock B have expected returns $E(r_{\mathrm{A}}) = 12.5\%$ and $E(r_{\mathrm{B}}) = 20\%$ respectively, and standard deviations $\sigma_{\mathrm{A}} = 5.12\%$ and $\sigma_{\mathrm{B}} = 20.49\%$, correlation coefficient is $\rho_{\mathrm{AB}} = -1$. First, we compute the covariance to be

$$\sigma_{\mathrm{AB}} = \frac{-1}{0.0512 \times 0.2049} = -0.0105$$

The variance of a portfolio of 75% stock A and 25% stock B is

$$\begin{aligned}\sigma_{\mathrm{p}}^2 &= (0.75)^2(0.0512)^2 + (0.25)^2(0.2049)^2 + 2 \times 0.75 \times 0.25 \times -0.0105 \\ &= 0.00016\end{aligned}$$

3.2.2 Optimal Portfolios

The aim of a financial investor is to maximize the expected return on their investment, given an accepted level of risk. The MPT defines the analytics of the relationship between risk and return in a complete market.

MPT says that the investors minimize the risk of the portfolio, hence its volatility, for a given level of return. This is done by choosing the right amount of each security to include in the portfolio, so that the total portfolio variance is minimized.

One must take into account not only how the single assets price changes, according to the volatility, but also how the changes for all asset compare, so to use covariance to quantify the diversification effect.

The approach of the theory is mathematical, and it is based on the construction of the ideal portfolio that fits the needs of an investor minimizing the risk given some fixed return. The relationship between risk and return is taken under consideration for the analysis.

Each security carries its own risk, and mixing many securities in the same portfolio should reduce the risk through diversification effect. The emphasis is in fact on the power of covariance to reduce the overall risk.

The MPT resembles a series of mathematical procedures to identify the optimal portfolio in the set of all possible portfolios and choose it in the context of wealth-maximizing, risk-adverse investors.

The model can be represented on a graph in the risk-return space. Different investors have different risk tolerance and return appetite. Taking into consideration the utility function of the investors, the model identifies a set of feasible risk/return combinations.

Securities can be combined in portfolios (and portfolios of portfolios) in a way to minimize the risk for some level of return. The set of portfolios with minimum variance for a given return is called efficient frontier.

Linear algebra is not sufficient to handle calculation when portfolios are very large. It is much more convenient to use matrix algebra, in order to simplify the problem. Matrix algebra formulas are faster to process and much easier to implement on the computer (Fig. 3.4).

An optimization problem, for minimizing the variance, leads to the analytical derivation of the efficient frontier. As mentioned above, it is better to show the calculation in matrix algebra, for the sake of simplicity.

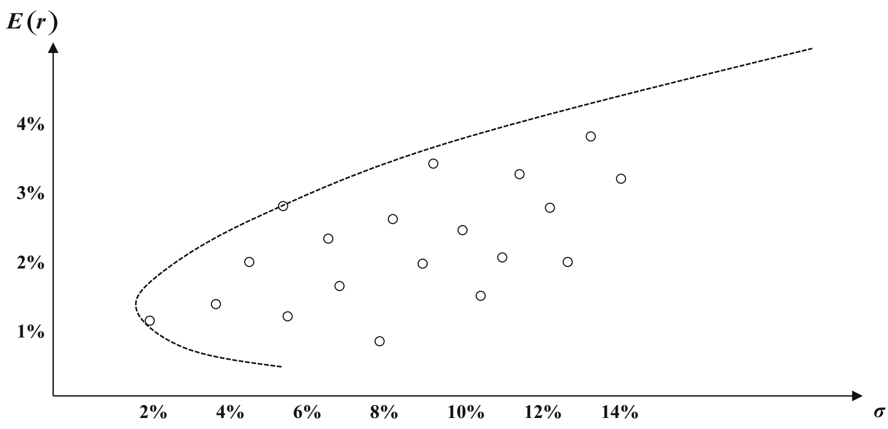

Fig. 3.4 Efficient frontier for portfolio or risky assets. Various combinations of available assets generate different portfolios, with different combinations of risk and return (dots). The portfolios lying on the upper side of the curve are efficient

The minimum variance portfolio with expected return μ is the solution of the following minimization program

$$\min\left(\frac{1}{2}\mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w}\right)$$

subject to

$$\begin{aligned}\mathbf{w}^{\mathrm{T}}\mathbf{1} &= 1\\ \mathbf{w}^{\mathrm{T}}\mathbf{r} &= \mu_{\mathrm{P}}\end{aligned}$$

where:

$\mathbf{w}$ is the vector of weights assigned to each asset.
$\boldsymbol{\Sigma}$ is the variance-covariance matrix of the asset returns.
$\mathbf{r}$ is the vector of the expected returns of the assets.
$\mathbf{1}$ is a vector of ones.
μ_{P} is the expected return of the portfolio.

It is possible to simplify the problem, for example, by reducing the amount of constraints. Allowing for short selling in fact, weights can take any value, even negative, removing one of the constraints.

The Lagrangian function of the problem is

$$L \equiv \frac{\mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w}}{2} + \lambda_1\left(\mu_{\mathrm{P}} - \mathbf{w}^{\mathrm{T}}\mathbf{r}\right) + \lambda_2\left(1 - \mathbf{w}^{\mathrm{T}}\mathbf{1}\right)$$

where:

λ_1 and λ_2 are the Lagrange multipliers.

with first-order conditions given by the partial derivatives of the Lagrangian function with respect to the weights vector and the Lagrangian multipliers, which can be written as

$$\begin{aligned}\frac{\partial L}{\partial \mathbf{w}} &= \boldsymbol{\Sigma}\mathbf{w} - \lambda_1\mathbf{r} - \lambda_2\mathbf{1} = 0\\ \frac{\partial L}{\partial \lambda_1} &= \mu_{\mathrm{P}} - \mathbf{w}^{\mathrm{T}}\mathbf{r} = 0\\ \frac{\partial L}{\partial \lambda_2} &= 1 - \mathbf{w}^{\mathrm{T}}\mathbf{1} = 0\end{aligned}$$

Some straightforward calculation leads to the solution for the optimal weights, which minimize the variance of the portfolio, as defined by

$$\mathbf{w}^* = \lambda_1\left(\boldsymbol{\Sigma}^{-1}\mathbf{r}\right) + \lambda_2\left(\boldsymbol{\Sigma}^{-1}\mathbf{1}\right) \tag{3.6}$$

It is very common in the literature to now simplify the notation by indicating the following:

$$\begin{aligned} A &= \mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{r} > 0 \\ B &= \mathbf{r}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{r} > 0 \\ C &= \mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{1} > 0 \\ \Delta &= BC - A^2 > 0 \end{aligned} \tag{3.7}$$

The popular Cauchy-Schwarz inequality shows that $\Delta > 0$, due to the condition that the covariance matrix is non-singular and not all assets have the same mean. It follows that

$$\mathbf{r} \neq k\mathbf{1}$$

where:

k is some constant.

Following the result in Eq. (3.7) and given the constraint of the optimization problem, the Lagrange multipliers are given by

$$\lambda_1 = \frac{C\mu_{\mathrm{P}} - A}{\Delta}$$

$$\lambda_2 = \frac{B - A\mu_{\mathrm{P}}}{\Delta}$$

The financial economist Merton (1972) shows that the equation of the minimum variance portfolio, as a consequence of the optimization algorithm, is given by

$$\sigma^2 = \frac{C}{\Delta}\left(\mu_{\mathrm{P}} - \frac{A}{C}\right)^2 + \frac{1}{C}$$

This is clearly a quadratic equation describing a parabola in the space of return and volatility. The program can be implemented as an interesting application with software and real data, in order to draw the efficient frontier of the market analyzed and conclude about efficiency of single portfolios.

The last step is to substitute the explicit values of the Lagrange multipliers into Eq. (3.6) in order to get the explicit extended formulation of the vector of portfolio weights as solution of the optimization problem, defined as

$$\begin{aligned} \mathbf{w}^* &= \left(\frac{C\mu_{\mathrm{P}} - A}{\Delta}\right)\left(\boldsymbol{\Sigma}^{-1}\mathbf{r}\right) + \left(\frac{B - A\mu_{\mathrm{P}}}{\Delta}\right)\left(\boldsymbol{\Sigma}^{-1}\mathbf{1}\right) \\ &= \frac{1}{\Delta}\left[\mu_{\mathrm{P}}\left(C\boldsymbol{\Sigma}^{-1}\mathbf{r} - A\boldsymbol{\Sigma}^{-1}\mathbf{1}\right) + \left(B\boldsymbol{\Sigma}^{-1}\mathbf{1} - A\boldsymbol{\Sigma}^{-1}\mathbf{r}\right)\right] \end{aligned}$$

The representation shows a portfolio whose variance is minimized for some specific value of expected return. It is possible to show that the minimum variance portfolio is obtained for a value μ_{P} corresponding to

$$\mu_{\mathrm{P}} = \frac{A}{C}$$

so that the optimal portfolio is given by

$$\mathbf{w}^{*}_{\mathrm{OPT}} = \frac{1}{\Delta}\left[\frac{A}{C}\left(C\boldsymbol{\Sigma}^{-1}\mathbf{r} - A\boldsymbol{\Sigma}^{-1}\mathbf{1}\right) + \left(B\boldsymbol{\Sigma}^{-1}\mathbf{1} - A\boldsymbol{\Sigma}^{-1}\mathbf{r}\right)\right]$$

The derivation of the efficient frontier for a market with only risky assets can be extended to the case when a risk-free asset is also present. Such an asset is like a constant in the model, so that the following condition holds:

$$\sigma^2_{\mathrm{f}} = \sigma_{i\mathrm{f}} = 0$$

where:

σ^2_{f} is the variance of the risk-free asset.
$\sigma_{i\mathrm{f}}$ is the covariance between the risk-free and the risky asset.

First, consider a convex combination of two portfolios Π_1 and Π_2 that lie on the efficient frontier, which is given by

$$\alpha\Pi_1 + (1-\alpha)\Pi_2, \quad \forall\alpha, \quad -\infty < \alpha < \infty$$

Now consider the minimum variance portfolio (MVP) from the above derivation in the space of return and risk, in a market with only risky assets, having expected return r_{p} and variance σ^2_{p}.

In case a proportion α of the investor's wealth is invested in the MVP and the remaining $(1-\alpha)$ is invested in the risk-free rate, the new combined portfolio Π has new expected return and variance.

The expected return of the portfolio is defined as

$$E(r_{\Pi}) = \alpha E(r_i) + (1-\alpha)r_{\mathrm{f}}$$

where:

$E(r_i)$ is the expected return on the risky asset.

In addition, the variance can be written as

$$\begin{aligned}\sigma^2_{\Pi} &= \alpha^2\sigma^2_{\mathrm{A}} + (1-\alpha)^2\sigma^2_{\mathrm{f}} + 2\alpha(1-\alpha)\sigma_{\mathrm{A,f}} \\ &= \alpha^2\sigma^2_{\mathrm{A}}\end{aligned}$$

The new equation leads to another frontier, for different values of α. The tangent portfolio r_m covers all the portfolios with expected return in the range between zero and $E(r_m)$, for $0 < \alpha < 1$.

When $\alpha > 1$, the frontier includes portfolio with an expected return higher than $E(r_m)$. It corresponds to leveraged portfolios, which are created by borrowing money at r_f and investing it into the tangent portfolio.

Now let us change notation and define a portfolio composed of a proportion w_f of the risk-free asset and a proportion $w_i = (1 - w_f)$ of the risky asset. Since the two assets are uncorrelated, the properties of variance hold, as defined by

$$\sigma_p^2 = (1 - w_f)^2 \sigma_i^2$$

where σ_p^2 is the portfolio variance. Straightforward mathematics shows that the portfolio weights can be expressed as

$$w_f = 1 - \frac{\sigma_p}{\sigma_i}$$
$$w_i = \frac{\sigma_p}{\sigma_i}$$

Therefore, we can calculate the portfolio return as

$$E(r_p) = \left(1 - \frac{\sigma_p}{\sigma_i}\right) r_f + \left(\frac{\sigma_p}{\sigma_i}\right) E(r_i)$$

The Capital Market Line takes the form

$$E(r_p) = r_f + \left(\frac{\sigma_p}{\sigma_i}\right) [E(r_i) - r_f]$$

The risk-free asset r_f adds up in the previous optimization as a zero-risk element in the market. Therefore, the efficient frontier expands, increasing the range of investment opportunities (Fig. 3.5).

The model then turns from a curve (the efficient frontier) to a line, which is called Capital Market Line (CML). It represents the capital allocation between the risk-free security and the risky asset. The tangent point between the new linear frontier and previous curve is the tangency portfolio, also known as the market portfolio.

The CML is the line that can be drawn between the risk-free rate, on the vertical axis, and the tangency point. The new line is considered dominant to the efficient frontier because the inclusion of a risk-free asset in the economy allows better portfolios to be formed.

The line is the new frontier for efficient portfolios so that the investor can move on it to choose the desired mix of return and risk. Introducing the risk-free rate allows for borrowing and lending funds in order to leverage or deleverage the investment portfolio.

The presence of the risk-free rate allows the investor to borrow funds at that rate and invest them in the risky asset, in order to increase the leverage. On the other

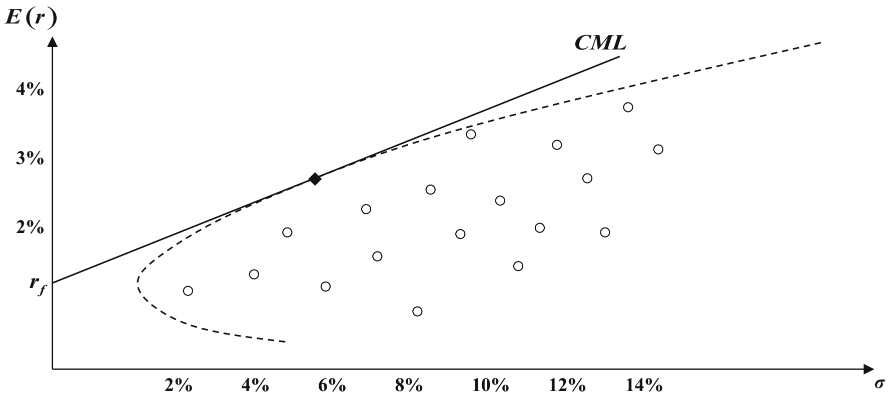

Fig. 3.5 Introducing a risk-free asset in the market allows choosing efficient portfolios on the straight line. Combinations of risk-free assets and risky assets lead to different points on the line

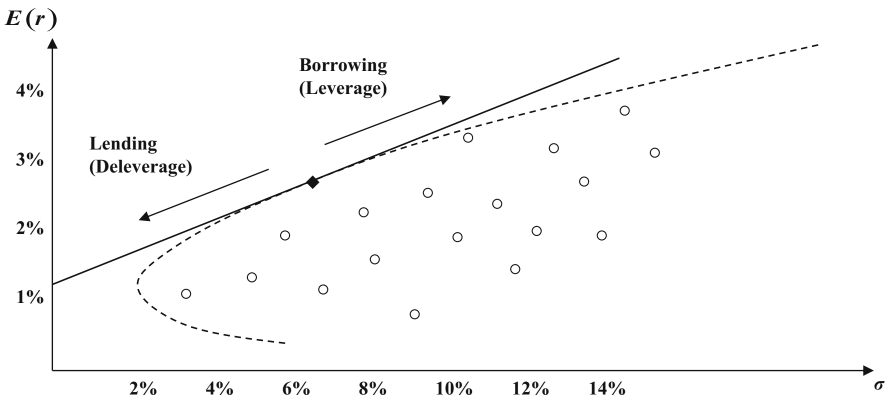

Fig. 3.6 Lending and borrowing money are possible to leverage or deleverage the position over the optimal portfolio

hand, it is also possible to save the proceedings from sale of risky asset at the risk-free rate, so to reduce the riskiness.

The theory has important implications. For example, risk-averse investors want to hold portfolios that are combinations of the risk-free asset and the tangency risky portfolio, at appropriate balance.

The fact that all investors hold the same risky portfolio must be unique, and it identifies like the market portfolio. The only choice left to risk-averse investors is the proportion to put in the risk-free asset (Fig. 3.6).

3.2.3 The Market Price of Risk

The market price of risk can be defined as the extra return on the risk-free rate, required by an investor to compensate the risk bore on the market by investing in some specific asset.

In derivatives, theory quantities are modeled as stochastic, and randomness leads to risk. The purpose of financial analysis is to determine how much extra return an investor should expect for taking risk (Renneborg 2006).

The abovementioned BSM derivative pricing model does not express the market price of risk directly, since the risk in an option position, for example, can be hedged away by taking a position on the underlying asset. This concept will be clearer after Chap. 9, but this information is important for the continuation of this section.

The derivation of the market price of risk goes through considering two different claims written on the same underlying variables. Consider the properties of derivatives dependent on the value of a single variable θ, following the stochastic process

$$d\theta = \mu_\theta \theta dt + \sigma_\theta \theta dW$$

where μ_θ and σ_θ are the expected growth rate and volatility of θ, respectively. It is assumed that the parameters depend only on the variable itself and the time.

According to the Ito's lemma, it is possible to define the process of any derivative v_i written on θ, as

$$\begin{aligned} dv_i &= \left(\frac{\partial v_i}{\partial \theta}\mu_\theta \theta + \frac{\partial v_i}{\partial t} + \frac{1}{2}\frac{\partial^2 v_i}{\partial \theta^2}\sigma_\theta^2 \theta^2\right)dt + \left(\frac{\partial v_i}{\partial \theta}\sigma_\theta \theta\right)dW \\ &= m_i dt + s_i dW \end{aligned}$$

Consider then the process for the variable transformation $\varsigma_i = \ln v_i$. The application again of the Ito's lemma leads to

$$\begin{aligned} d\varsigma_i &= \left(\frac{1}{v_i}m_i - \frac{1}{2v_i^2}s_i^2\right)dt + \left(\frac{1}{v_i}m_i\right)dW \\ &= \mu_i dt + \sigma_i dW \end{aligned}$$

The next step is to consider the above process for two different derivatives

$$\begin{aligned} d\varsigma_1 &= \mu_1 dt + \sigma_1 dW \\ d\varsigma_2 &= \mu_2 dt + \sigma_2 dW \end{aligned}$$

and then form a portfolio V of the two, by including them with weights w_1 and w_2 (summing up to unity), respectively. Letting Π denote the natural log of the portfolio value, the process followed by $d\Pi$

$$d\Pi = (w_1\mu_1 + w_2\mu_2)dt + (w_1\sigma_1 + w_2\sigma_2)dW$$

In order to investigate the risk premium, it is crucial to know under what circumstances the above portfolio is riskless. Choosing values

$$w_1 = -\frac{\sigma_2}{\sigma_1 - \sigma_2}$$
$$w_2 = \frac{\sigma_1}{\sigma_1 - \sigma_2}$$

The portfolio process becomes

$$d\Pi = \left(\frac{\sigma_1}{\sigma_1 - \sigma_2}\mu_2 - \frac{\sigma_2}{\sigma_1 - \sigma_2}\mu_1\right)dt + \left(\frac{\sigma_1}{\sigma_1 - \sigma_2}\sigma_2 - \frac{\sigma_2}{\sigma_1 - \sigma_2}\sigma_1\right)dW$$

It follows that the term in dW vanishes, meaning that the volatility of the process is zero. Only the drift of the process is left, and in order for the portfolio to be riskless, the drift must be equal to the risk-free rate, to comply with no arbitrage conditions. It follows that

$$\frac{\sigma_1}{\sigma_1 - \sigma_2}\mu_2 - \frac{\sigma_2}{\sigma_1 - \sigma_2}\mu_1 = r$$

which implies

$$\frac{\mu_1 - r}{\sigma_1} = \frac{\mu_2 - r}{\sigma_2} = \lambda \tag{3.8}$$

The parameter λ can be defined as the market price of risk of the underlying variable θ, and it depends on the single derivatives. Moreover, it must be the same for all the claims written on the same underlying variable.

The market price of risk is a measure of the trade-off between risk and return associated to some variable. The product of the quantity of risk, σ, and its price, λ, gives the amount of corresponding return.

In investment finance, it is called Sharpe ratio, and it is mostly used to assess the level of compensation obtained by the investor for the risk taken. It is especially useful to compare two assets with different returns and volatility.

The higher the Sharpe ratio, the higher the expected return from an asset in relation to the risk it carries. Therefore, good investment strategies consist in picking the investment with the highest Sharpe ratio.

Based on the formula (3.8), it is now possible to explicitly define the relationship between the risk premium demanded by investors, in terms of excess return, for holding derivative i and the price of risk, as

$$\mu_i - r = \lambda\sigma_i$$

The risk premium is therefore the market price of risk λ, multiplied by the amount of risk derivative i holds, σ_i.

From basic stochastic calculus, the process followed by any derivative v can be written as

$$dv = \mu v dt + \sigma v dW$$

In the traditional risk-neutral world, the leading assumption is that investors do not demand any compensation for the risk taken, leaving the market price of risk equal to zero. As a consequence $\mu = r$, and the process followed by r can be written as

$$dv = rvdt + \sigma vdW$$

Other worlds can now be defined, based on the specific level of market risk characterizing them. Generalizing the framework, the growth rate of the process drift can be written as

$$\mu = r + \lambda\sigma$$

so that

$$dv = (r + \lambda\sigma)vdt + \sigma vdW$$

The market price of risk of a variable determines the growth rates of all the securities dependent on that variable.

Snapshot 3.2

Portfolio Optimization in Excel

In order to apply portfolio optimization in Excel, first it is necessary to create a matrix of the n observed returns for the chosen m assets in the market and the covariance matrix, as in the following example:

E(R)	*0.178*	*0.216*	...	*0.049*	**Var(R)**	*0.178*	*0.216*	...	*0.049*

Using the formulas from Markowitz Portfolio Theory, the following step is to calculate, from the expected return and variances of the m single assets, the portfolio expected return and portfolio variance

R(P)	*0.178*	**Var(P)**	*0.178*

In order to set for the optimization, a line of weights of the assets in the portfolio must be filled.

	1	2	...	m
Weights	0.111	0.223	...	0.015

It is then possible to start the program, using the Solver Tool in Excel and implementing the following steps:

1. Set your desired portfolio return.
2. Start the Excel Solver.

(continued)

Snapshot 3.2 (continued)

3. Minimize the portfolio variance.
4. Change "By changing variable cells" to the range of cells containing the portfolio allocation.
5. Set the constraints (the total portfolio allocation must add up to one, and the portfolio return should be you desired value).
6. Click Solve.

Once the set of optimal weights are obtained, they can be used to build efficient portfolios for different combinations of expected return and variance. In order to get the efficient frontier, it is sufficient to plot the line crossing these portfolios, using the graph function in Excel.

3.3 The Capital Asset Pricing Model

Learning Outcomes

- Understand the Capital Asset Pricing Model.
- Draw and describe the Capital Market Line and Security Market Line.
- Acquire introductory knowledge of advanced factor models for asset pricing.

3.3.1 Model Assumptions

The previous sections have been dedicated to developing the CML. Once the line is generated, a further step consists in finding a way to measure systematic risk, given that diversification can eliminate only a part of it.

Modern Portfolio Theory describes systematic risk and how it should be compensated. After diversification takes out all asset-specific risk in fact, the systematic risk is the only one to be compensated.

A solution to the issue of risk measurement and compensation is given by the Capital Asset Pricing Model (CAPM), a model describing and solving the issue through mathematical modeling of systematic risk.

The CAPM framework starts with the risk-free rate as a benchmark for measuring a risk premium that determines the expected return on some specific asset. The premium is a compensation for the risk borne by the investor.

After the premium is calculated, the following step is to multiply it by a beta coefficient, which is a measure of the riskiness of the asset in proportion to the market risk. This is the link between expected return on the asset and the risk premium in the market.

Modern Portfolio Theory is then meant at determining the right expected return on a portfolio of assets. One of the main defining points of the method is diversification, toward elimination of all non-systematic risk.

Moreover, the investors hold combinations of just the risk-free asset and the tangency portfolio. Finally, systematic risk of the asset is a proportion of the market risk, and the risk premium of an asset is proportional to its systematic risk.

Recall that the CML describes the relationship between the excess return of an asset and the excess return of the market, as a line in the risk-return space.

$$r_i - r_{\mathrm{f}} = \beta(r_{\mathrm{m}} - r_{\mathrm{f}})$$

where:

r_{m} is the return of the market.
β is the dependence factor between market and asset.

The CAPM identifies the tangency portfolio, lying on both the efficient frontier and CML, as the market portfolio, and relating the price of any asset on the market to the market itself.

Consider a model where there are n assets in the market. Each asset $i = 1, \ldots, n$ has a market capitalization

$$\mathrm{Mcp}_i = V_i n_i$$

where:

V_i is the price per share of asset i.
n_i is the number of shares outstanding of asset i.

It follows that the total capitalization of the assets in the market is

$$\mathrm{Mcp}_{\mathrm{m}} = \sum_{i=1}^{n} \mathrm{Mcp}_i$$

The weights for each asset are given by

$$\begin{aligned} w_i &= \frac{\mathrm{Mcp}_i}{\sum_{i=1}^{n} \mathrm{Mcp}_i} \\ &= \frac{\mathrm{Mcp}_i}{\mathrm{Mcp}_{\mathrm{m}}} \end{aligned}$$

The CAPM result is based on the assumption that investors invest their money between the risk-free and the tangency portfolio, which is held in same proportions by all the investors.

The main assumptions underlying the derivation of CAPM are:

- Markets are complete with price-taking investors.
- Markets are frictionless and transactions are free of costs.
- The holding period of the assets is the same for all investors.
- Investors optimize the investment in the risk-return space, as by MPT.
- Funds are lent and borrowed at the same risk-free rate.
- The market is characterized by homogeneous information for all investors.

The CAPM states that if an investor holds some amount of the market portfolio, the risk compensation for single assets should be a consequence of how the single asset behaves compared to the market (Sharpe 1964).

If the volatility of an asset is the same as another asset (or the market), then also the expected return must be equal. By indicating the variance of the asset by σ^2, the risk premium per unit of risk λ can be written as

$$\lambda_{\mathrm{P}} = \frac{[E(r_{\mathrm{m}}) - r_{\mathrm{f}}]}{\sigma_{\mathrm{m}}^2}$$

There is a direct relation between the risk premium and the excess return on the market. The relation is instead inverse with the market risk, indicated as variance. The right compensation for each asset is calculated according to the covariance between the asset and the market.

The expected premium on asset i should be equal to the risk premium per each unit of risk multiplied by the relationship of the asset with the market expressed in the form of covariance.

$$E(r_i) - r_{\mathrm{f}} = \sigma_{i\mathrm{m}} \frac{[E(r_{\mathrm{m}}) - r_{\mathrm{f}}]}{\sigma_{\mathrm{m}}^2}$$

In case of a null covariance with the market, the asset has no risk attached and can only earn the risk-free rate, in normal market conditions. For a value of the covariance equal to 1, the asset is perfectly related to the market, earning the market return.

The formula of the CAPM can be simplified as

$$E(r_i) = r_{\mathrm{f}} + \frac{\sigma_{i\mathrm{m}}}{\sigma_{\mathrm{m}}^2}[E(r_{\mathrm{m}}) - r_{\mathrm{f}}]$$

The factor $\frac{\sigma_{i\mathrm{m}}}{\sigma_{\mathrm{m}}^2}$ called the beta β (beta) of the asset shows the proportionality of the asset risk premium to the market risk premium. The resulting model is the CAPM and is typically written as:

$$E(r_i) = r_{\mathrm{f}} + \beta_i[E(r_{\mathrm{m}}) - r_{\mathrm{f}}]$$

Example 3.7 Suppose the covariance of some stock A with the market is 15% and the variance of A is 12%. The market expected return is 11% and the risk-free rate is 5%. We can calculate then the expected return of A applying the CAPM

$$E(r_{\mathrm{A}}) = 0.05 + \frac{0.15}{0.12}[0.11 - 0.05] = 0.125$$

The expected return for the stock is 12.5%.

The implementation of CAPM involves some practical issues related to the input factors and their estimation. For example, the market portfolio is hard to proxy because a portfolio of all assets in the market would be needed.

It is therefore necessary to find a good proxy for the market return, which is usually represented by index funds. It is a sort of smaller portfolio representative of the market, including all the most dominant assets, to capture the essence of it.

One of the most famous stock indices in the world is the Standard & Poor's (S&P) 500 which resembles 500 representative stocks as a proxy of the market. In this specific case, CAPM implementation goes through an elaborated model that applies to reality. Consider estimating the beta coefficient for asset $i = 1, 2, \ldots, m$ over a discrete time horizon of n time points. For every $k = 1, 2, \ldots, n$, the variable average (expected) return for the asset i is defined by

$$\hat{\bar{r}}_i = \frac{1}{n}\sum_{k=1}^{m} r_{i,k}$$

where:

$\hat{\bar{r}}_i$ is the average return of asset i.
r_{ik} is the k-th sampled value of asset return.
$r_{\mathrm{SP},\,k}$ is the k-th sampled value of the S&P 500 index return.

For the index is

$$\hat{\bar{r}}_{\mathrm{SP}} = \frac{1}{n}\sum_{k=1}^{m} r_{\mathrm{SP},k}$$

The estimation of the index variance σ^2_{SP} is

$$\widehat{\sigma}^2_{\mathrm{SP}} = \frac{1}{n-1}\sum_{k=1}^{m}\left(r_{\mathrm{SP},k} - \hat{\bar{r}}_{\mathrm{SP}}\right)^2$$

And the covariance $\sigma_{\mathrm{SP}i}$ between asset i and the index is estimated by

$$\widehat{\sigma}_{\mathrm{SP}i} = \frac{1}{n-1}\sum_{k=1}^{m}\left(r_{\mathrm{SP},k} - \hat{\bar{r}}_{\mathrm{SP}}\right)\left(r_{ik} - \hat{\bar{r}}_{i}\right)$$

The beta value estimation is finally given by

$$\widehat{\beta} = \frac{\widehat{\sigma}_{\mathrm{SP}}^{2}}{\widehat{\sigma}_{\mathrm{SP}}}$$

3.3.2 The Security Market Line

The CAPM model applied to an asset gives the right discount rate for the future cash flows generated by it. Such a rate depends on the riskiness of the asset compared to the market, as defined by the beta.

The beta is in fact proportional to the riskiness of the asset. The higher the beta, the higher the discount rate, and the lower the present value of the future cash flows, with a resulting lower present value of the asset.

The opposite works for low-beta stocks, which are less sensitive to market changes and less risky, yielding lower discount rate and higher present value for the stock. Recall that one of the main findings of CAPM is the identification of the market portfolio as the tangent portfolio (Schneeweis et al. 2010).

Moreover, the risk associated to an asset is dependent on its covariance with the market portfolio. The market affects the asset in that the systematic risk is not diversifiable and must be compensated, according to the relationship between excess return of the asset and excess return of the market.

It is possible to derive the CAPM from decomposing a portfolio between an asset and the market. Consider forming a portfolio p by investing an amount w_i in a risky asset I and an amount $w_{\mathrm{m}} = (1 - w_i)$ in the market portfolio m. The expected return of the portfolio is then given by

$$E\left(r_{\mathrm{p}}\right) = w_{\mathrm{m}}E(r_{\mathrm{m}}) + w_iE(r_i)$$

where:

$E(r_{\mathrm{m}})$ is the expected return of the market.
w_{m} is the weight of the market in the portfolio.

Recall that the risk can be represented through the volatility (standard deviation), which can be defined as

$$\sigma_{\mathrm{p}} = \sqrt{{w_{\mathrm{m}}}^2\sigma_{\mathrm{m}}^2 + {w_i}^2\sigma_i^2 + 2w_iw_{\mathrm{m}}\sigma_{i\mathrm{m}}}$$

where:

σ_{m}^2 is the variance of the market.
$\sigma_{i\mathrm{m}}$ is the covariance between the market and the asset.

The analysis then deals with the slope of the efficient frontier and the slope of the CML, with a focus on their tangency point. At that point, the slope of the CML is given by

$$\mathrm{Slope}_{\mathrm{CML}} = \frac{[E(r_{\mathrm{m}}) - r_{\mathrm{f}}]}{\sigma_{\mathrm{m}}}$$

First of all, the slope of the efficient frontier (curve) at the tangency point can be obtained by differentiating the expected portfolio return $E(r_{\mathrm{P}})$ with respect to portfolio volatility σ_{P}. The chain rule applies as

$$\mathrm{Slope}_{\mathrm{EFR}} = \frac{\partial E\left(r_{\mathrm{p}}\right)}{\partial \sigma_{\mathrm{p}}} = \frac{\dfrac{\partial E\left(r_{\mathrm{p}}\right)}{\partial w_i}}{\dfrac{\partial \sigma_{\mathrm{p}}}{\partial w_i}}$$

The chain is solved piecewise, starting from the numerator of the expression at the extreme right-hand side that can be written as

$$\frac{\partial E\left(r_{\mathrm{p}}\right)}{\partial w_i} = E(r_i) - E(r_{\mathrm{m}})$$

The denominator is the sensitivity of the portfolio volatility with respect to the weight of asset i

$$\begin{aligned}\frac{\partial \sigma_{\mathrm{p}}}{\partial w_i} &= \frac{1}{2}\left[w_{\mathrm{m}}{}^2\sigma_{\mathrm{m}}^2 + w_i^2\sigma_i^2 + 2w_i w_{\mathrm{m}}\sigma_{i\mathrm{m}}\right]^{-\frac{1}{2}} \times \left[-2w_{\mathrm{m}}\sigma_{\mathrm{m}}^2 + 2w_i\sigma_i^2 + 2(1-2w_i)\sigma_{i\mathrm{m}}\right]\\ &= \frac{\left[-w_{\mathrm{m}}\sigma_{\mathrm{m}}^2 + w_i\sigma_i^2 + (1-2w_i)\sigma_{i\mathrm{m}}\right]}{\sigma_{\mathrm{p}}}\end{aligned}$$

The calculation in theory is very difficult. But the properties of the model at the tangency point can be used to simplify it. At tangency point all investors choose the market portfolio.

Therefore the proportion of asset i is zero, and the variance of the portfolio turns into the variance of the market, the only variable left. Therefore, the derivative of the variance wr to the weight w_i becomes

$$\left.\frac{\partial \sigma_{\mathrm{p}}}{\partial w_i}\right|_{\substack{w_i = 0 \\ \sigma_{\mathrm{p}} = \sigma_{\mathrm{m}}}} = \frac{\sigma_{i\mathrm{m}} - \sigma_{\mathrm{m}}^2}{\sigma_{\mathrm{m}}}$$

The chain rule leads to the expression for the slope of the efficient frontier as

$$\text{Slope}_{\text{EFR}} = \frac{\dfrac{\partial E(r_{\text{p}})}{\partial w_i}}{\dfrac{\partial \sigma_{\text{p}}}{\partial w_i}} = \frac{E(r_i) - E(r_{\text{m}})}{\dfrac{\sigma_{i\text{m}} - \sigma_{\text{m}}^2}{\sigma_{\text{m}}}} = \frac{[E(r_i) - E(r_{\text{m}})]\sigma_{\text{m}}}{\sigma_{i\text{m}} - \sigma_{\text{m}}^2}$$

The equilibrium is defined by the equality of the two slopes at tangency point. The market portfolio is the most efficient portfolio, and the equality is such that

$$\frac{[E(r_i) - E(r_{\text{m}})]\sigma_{\text{m}}}{\sigma_{i\text{m}} - \sigma_{\text{m}}^2} = \frac{[E(r_{\text{m}}) - r_{\text{f}}]}{\sigma_{\text{m}}}$$

so that

$$E(r_i) = r_{\text{f}} + \beta_i[E(r_{\text{m}}) - r_{\text{f}}]$$

where

$$\beta_i = \frac{\sigma_{i\text{m}}}{\sigma_{\text{m}}^2}$$

The graph of the beta against the volatility is called Security Market Line (SML). It is the line where all efficient portfolios lie.

If the utility function of the investors is concave, the findings of CAPM theory are consistent with the direct relationship between the risk and the expected return of an investment (Fig. 3.7).

A simple regression allows estimating the beta of an asset starting from the time series of risk-free rate and market returns. The regression takes the form

$$\begin{aligned} r_{i,t} &= \alpha_{i,t} + \beta_{i,t} r_{\text{m}} + \varepsilon_{i,t} \\ \varepsilon_i &\sim N(0, \sigma^2) \end{aligned}$$

where:

$\beta_{i,\ t}$ is the beta of asset i at time t.

The volatility of the return can then be expressed as

$$\sigma_i = \beta_i^2 \sigma_{\text{m}} + \sigma_\varepsilon \tag{3.9}$$

where:

σ_ε is the volatility of the error term.

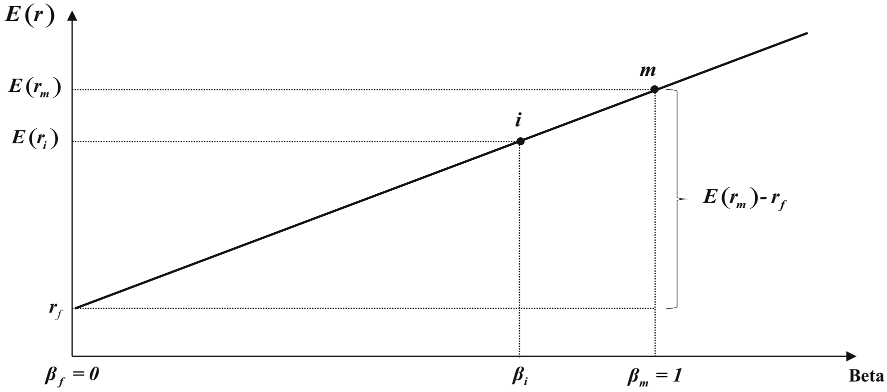

Fig. 3.7 The CAPM relation identifies the Security Market Line (SML) on which all the efficient combinations of expected return and asset betas lie. The slope of the curve identifies the relation between asset and market excess return

The layout of formula (3.9) separates the variance of the market return from the variance of the error term, and it is known as the variance decomposition of returns. It highlights the systematic risk, related to the market, from the diversifiable risk, related to the error term.

When the error term is close to zero, the portfolio is well diversified, and no more action is needed in order to eliminate risk. Only systematic risk is left, and it is rewarded by the market, according to the beta of the asset.

Equation (3.8) can be extended to an equally weighted portfolio of n assets as defined by

$$\sigma_{\mathrm{p}} = \beta_{\mathrm{p}}^2 \sigma_{\mathrm{m}} + \frac{1}{n} \sigma_{\mathrm{p}}^2$$

where:

β_{p} is the beta of the portfolio.

The term on the extreme right-hand side shows the diversification effect; with portfolio, variance reduced when n is particularly high. The covariance between any couple of assets i and j can be expressed as

$$\begin{aligned} \sigma_{i,j} &= \operatorname{cov}\left[\alpha_i + \beta_i r_{\mathrm{m}} + \varepsilon_i, \alpha_j + \beta_j r_{\mathrm{m}} + \varepsilon_j\right] \\ &= \ldots \\ &= \beta_i \beta_j \sigma_{\mathrm{m}}^2 \end{aligned}$$

3.3.3 Beyond CAPM

After the development of CAPM as an asset-pricing model, its potential has been increased by the development of other models that take inspiration from it and extend the features of the original framework.

One example is the consumption-based CAPM (CCAPM), a model that expands the analysis introducing a consumption factor for calculating the expected return on an investment.

The CCAPM structure implies that the (expected) risk premium on the asset in this case is proportional to the covariance between the asset return and the consumption calculated for the analyzed period.

Consider a multi-period model with an infinitely lived representative household and an expected lifetime utility function defined as

$$E(u_t) = u(c_t) + E\left[\sum_{i=1}^{\infty} d^i u(c_{t+i})\right], \quad 0 < d < 1$$

where:

c_t is the consumption at time t.
$u(c_t)$ is the utility associated to consuming at time t.
d is a discount function.
$u(c_{t+i})$ is the utility associated to consuming at time $t + i$.

The level of consumption c_t, subject to a budget constraint, is chosen at time t. If the price for a financial security is p_t, the household can buy the security and redeem later at $t + 1$, to finance consumption at that time.

The first-order condition on the expected utility shows the optimal quantity of financial asset demanded by the investor as

$$dE[c_{t+1}u'(c_{t+1})] - u'(c_t)p_t = 0 \tag{3.10}$$

The result is the utility from consuming at $t + 1$ when financing through settlement of a marginal unit of the asset, net of the marginal utility lost from not consuming when purchasing the security.

Given that the one-period return on the asset can be written as

$$r_{t+1} = \frac{c_{t+1}}{p_t} - 1$$

Equation (3.10) changes to

$$\begin{aligned}
& dE[c_{t+1}u'(c_{t+1})] - u'(c_t)p_t = 0 \\
& \Rightarrow dE\left[\frac{c_{t+1}}{p_t}u'(c_{t+1})\right] - u'(c_t) = 0 \\
& \Rightarrow dE[(1 + r_{t+1})u'(c_{t+1})] - u'(c_t) = 0 \\
& \Rightarrow dE\left[(1 + r_{t+1})\frac{u'(c_{t+1})}{u'(c_t)}\right] = 1
\end{aligned}$$

This is the CCAPM standard equation, showing that in equilibrium, the product of 1 plus the expected value of asset return and the marginal rate of substitution of the consuming utilities is equal to 1.

The model for all the securities, including the risk-free asset, in what case the formula becomes

$$d(1 + r_{\mathrm{f}})E\left[\frac{u'(c_{t+1})}{u'(c_t)}\right] = 1$$

It is then possible to express CCAPM in terms of excess returns, by subtraction of the risk-free rate as described by

$$\begin{aligned}
& dE\left[(1 + r_{t+1})\frac{u'(c_{t+1})}{u'(c_t)}\right] - d(1 + r_{\mathrm{f}})E\left[\frac{u'(c_{t+1})}{u'(c_t)}\right] = 0 \\
& \Rightarrow \ldots \\
& \Rightarrow E\left[(r_{t+1} - r_{\mathrm{f}})\frac{u'(c_{t+1})}{u'(c_t)}\right] = 0
\end{aligned}$$

The product of the expected excess rate and the marginal rate of substitution of consumption is equal to zero.

Another popular variation of the CAPM was proposed by Eugene Fama and Kenneth French in 1992. After years of empirical analysis of financial returns, they found out that two classes of stock have the tendency of performing better than average.

Small capitalization stocks and high book-to-value ratio stocks showed returns higher than average of all classes of stocks. They therefore modified the standard CAPM formula in order to capture the effect of three factors.

The Fama-French formula is exposed to factors related to the returns of small minus big (SMB) capitalization stocks and high minus low (HML) book-to-value stocks. The resulting model is a three-factor formula in the form

$$E(r_i) = r_{\mathrm{f}} + \beta_i[E(r_{\mathrm{m}}) - r_{\mathrm{f}}] + \beta_{i,\mathrm{SMB}}\mathrm{SMB} + \beta_{i,\mathrm{HML}}\mathrm{HML} \tag{3.11}$$

where:

SMB is the expected excess return on SMB factor.
HML is the expected excess return on HML factor.

$\beta_{i,\ \mathrm{SMB}}$ is the beta of small capitalization stock factor, for asset i.
$\beta_{i,\ \mathrm{HML}}$ is the beta of high-minus-low book-to-value factor, for asset i.

The model improves the CAPM by adding new information, and the dynamics of the regression is pretty interesting, given that when returns increase with book-to-value, stocks with high ratio must be more risky than average.

The strong points of the model are the great diversity of the factors included in formula (3.11), based thus on heterogeneous variables. The indices weigh stocks according to capitalization, meaning that factor actually defines the market portfolio of reference.

Finally, it is interesting to have a look at another extension of the CAPM developed by Stephen Ross in 1976, called the Arbitrage Pricing Theory (APT). It is based on the idea that asset returns can be predicted by relating it to many common risk factors.

ATP does not require homogeneous behavior among investors, nor does it claim that the tangency portfolio is the only risky asset that will be held by the investors. It assumes that expected returns on a security should be related to the security's covariance with the common factors.

Suppose that returns are driven by a set of factors F_1, F_2, …, F_m. The model in case of m factors can be expressed in the form

$$E(r_i) = r_{\mathrm{f}} + \beta_{i,F_1}[E(r_{F_1}) - r_{\mathrm{f}}] + \beta_{i,F_2}[E(r_{F_2}) - r_{\mathrm{f}}] + \ldots + \beta_{i,F_m}[E(r_{F_m}) - r_{\mathrm{f}}]$$

where:

$E\left(r_{F_j}\right)$ is the expected return on factor j.
$\beta_i^{F_1}$ is the beta of factor j, for asset i.

The parameters can be estimated by regressing the model, as for the CAPM, in the form

$$r_i = \alpha_i + \beta_{i,F_1} r_{F_1 1} + \beta_{i,F_1} r_{F_2} + \ldots + \beta_{i,F_1} r_{F_m} + \varepsilon_i$$

ATP estimation is based on multivariate regression analysis.

3.4 Summary

The relationship between risk and return is of primary importance in financial analysis and corporate finance. Through the interaction of profitability and risk of all assets on the market, it is possible to determine the overall risk of portfolios.

Good understanding of the risk-return paradigm goes through the analysis of variables like portfolio return, volatility, and correlation and the understanding of techniques like maximum likelihood estimation.

Modern Portfolio Theory is the result of the formalization of the risk-return relationship in a framework of efficient markets and rational investors. The risk-return trade-off is formalized through a parabola in the risk-return space, with the upper part being the curve of the efficient portfolios in the market.

Optimal portfolios are those who lie on the upper side of the curve and represent all the portfolios giving the maximum return for a given level of risk (volatility). They therefore represent the optimal choices for each level of risk.

The formalization of the relationship also allows giving a representation of the market price of risk, which can be defined as the unit price for each unit of risk an investor is willing to bear.

The Capital Asset Pricing Model is the result of establishing a relationship between the risk of some asset and the risk of the market. By introducing a risk-free asset in the space of possible investment opportunities, it is possible to derive the line of the optimal portfolio to invest in, tangent to the curve previously derived.

The beta measures the relationship between excess return of an asset and the excess return of the market portfolio, giving the idea of the specific riskiness of each asset and a price to the market risk taken by an investor.

Models that are more sophisticated expand the idea underlying the CAPM by identifying specific factors that affect the relationship between the asset and the market, therefore making the relationship more factor-specific.

Problems

1. Briefly explain the risk-return relationship, as from modern portfolio theory.
2. Why do some investors put a large portion of their portfolios into risky assets, while others invest largely in the risk-free asset?
3. Explain why the following assumptions in particular are required for the Capital Asset Pricing Model to hold:
   (a) A single investor cannot impact the share price.
   (b) Time horizon is the same for all investors.
   (c) All assets are marketable and infinitely divisible.
4. Is it entirely correct, as an analyst recently commented, that market crashes are never advertised in advance?
5. Recall that criticisms of EMH claim that the rate of return on small-cap stocks tend to be higher than for large-cap.
   (a) Is this compatible with CAPM?
   (b) How could such a claim be tested?
6. Assume beta coefficients are estimated for a large number of assets, finding that average rates of return of assets and their beta coefficients are not significantly correlated.
   (a) What does the result imply in terms of validity of the CAPM?
   (b) Would it make sense to completely reject the CAPM on the basis of this evidence?

7. What happens to the riskiness of a portfolio if assets with very low or negative correlations are combined?
8. Comparing diversifiable and non-diversifiable risk, which one do you think is more important to financial managers in corporations?
9. Discuss financial risk from the perspective of the CAPM.
10. What features define the points on the efficient frontier? Do portfolios exist above the frontier?
11. Explain the concept of beta, its calculation, and meaning.
12. What does the SML indicate? What are the main differences with the CML?
13. Derive the intermediate passages to solve Eq. (2.1) in the chapter.
14. Consider the following investments

Investment	Expected return	Standard deviation
A	6.2%	10.5%
B	8.1%	11.2%
C	7.2%	12.3%
D	6.5%	10.8%

Which investment would you prefer between the following pairs?
(a) A and D?
(b) B and C?
(c) C and D?

15. Derive the intermediate passages to solve Eq. (3.4) in the chapter.
16. Assume two assets A and B for which the following estimates have been derived:

$$E(r_{\mathrm{D}}) = 12\%, \sigma_{\mathrm{D}} = 19\%$$

$$E(r_{\mathrm{E}}) = 15\%, \sigma_{\mathrm{E}} = 22\%$$

Consider the portfolios that can be formed investing 50% of total wealth in A and 50% in B. What is the portfolio's standard deviation if the assets are perfectly positively correlated?

17. A portfolio that consists of the following assets:

Stock	Investment (€)	Beta
A	250,000	1.15
B	330,000	−0.65
C	570,000	1.18
D	1,150,000	0.78

The market return is 13% and the risk-free rate is 2%.
(a) Calculate the beta of the portfolio.
(b) Calculate the required rate of return of this portfolio.

18. Assume the market portfolio has an expected return of 12% and a volatility of 28%. There is a risky asset i on which limited information is available. It is known that the expected return of the asset is 9%, the volatility is bounded between 18% and 32%, and the covariance between the asset and the market is bounded between 0.014 and 0.026.
    (a) Find the interval of possible values of $\rho_{i,\,\mathrm{M}} = \mathrm{Corr}(r_i, r_\mathrm{M})$ and of β_i.
    (b) What range of values for the risk-free rate do you find realistic? Give an interval in which the risk-free rate should be.
    (c) Find the interval of risk-free rates implied by the CAPM in the above system.
    (d) How would you price an asset with the same risk characteristics as asset i?
19. Denote by X a stock and by M the market. The correlation coefficient $\rho_{\mathrm{P},\,\mathrm{M}}$ between the stock and the market is 0.80. The volatility of stock X is 25% and the volatility of the market is 12%.
    (a) Calculate the systematic variance, the unsystematic variance, and β_P.
    (b) Show that the beta of a portfolio equals the weighted average of the assets' betas.
20. When estimating CAPM from data, a regression error must be taken into account and the formula modified accordingly. The error is normally distributed. Suppose there are three risky assets with the following betas and values of $\sigma^2_{\varepsilon_j}$ in the CAPM.

j	β_j	$\sigma^2_{\epsilon_j}$
A	1.3	0.008
F	1.1	0.012
E	0.6	0.010

Suppose also that the market excess returns have a variance of 0.016.
(a) What is the beta of an equally weighted portfolio of these three assets?
(b) What is the variance of the excess return on the equally weighted portfolio?
(c) What proportion of the total risk of asset A is due to market risk?

21. Consider the following data for a one-factor economy. All portfolios are well diversified.

Portfolio	$E(R_i)$	Beta
A	10%	1.0
B	4%	0
C	9%	2/3

(a) Is there any arbitrage opportunity in the market?
(b) If so, what would the arbitrage strategy be?

22. The weight measurement of individuals picked in two different areas of the world X and Y is given in the following table:

i	X_i	Y_i
1	73	67
2	58	59
3	56	62
4	69	53
5	73	75
6	75	52
7	63	61
8	67	58
9	72	64
10	69	59
11	66	62

(a) Calculate the sample correlation of the weight in the two areas of the world.

Appendix: Liquidity CAPM

A version of CAPM based on the liquidity effect was developed in 2000 by Jacoby et al. The model can be described by the formula

$$E(r_i - c_i) = r_\mathrm{f} + \beta_i^* [E(r_\mathrm{M} - c_\mathrm{M}) - r_\mathrm{f}]$$

where

$$\beta_i^* = \frac{\mathrm{cov}(r_i - c_i, r_\mathrm{M} - c_\mathrm{M})}{\mathrm{var}(r_\mathrm{M} - c_\mathrm{M})}$$

The advantage of the revised model is to capture the impact of liquidity costs on the systematic risk, quantified by the liquidity-adjusted beta β_i^*.

By manipulating the beta equation, it is possible to isolate and decompose the covariance as shown by

$$\begin{aligned}\beta_i^* &= \frac{\mathrm{cov}(r_i - c_i, r_\mathrm{M} - c_\mathrm{M})}{\mathrm{var}(r_\mathrm{M} - c_\mathrm{M})}\\ &= \frac{\mathrm{cov}(r_i, r_\mathrm{M})}{\mathrm{var}(r_\mathrm{M} - c_\mathrm{M})} + \frac{\mathrm{cov}(c_i, c_\mathrm{M})}{\mathrm{var}(r_\mathrm{M} - c_\mathrm{M})} - \frac{\mathrm{cov}(r_i, c_\mathrm{M})}{\mathrm{var}(r_\mathrm{M} - c_\mathrm{M})} - \frac{\mathrm{cov}(c_i, r_\mathrm{M})}{\mathrm{var}(r_\mathrm{M} - c_\mathrm{M})}\end{aligned}$$

The liquidity-CAPM equation can be then reformulated as

$$E(r_i) = r_\text{f} + E(c_i) + \left[E(r_\text{M} - c_\text{M} - r_\text{f})\left(\beta_i + \beta_i^{L1} - \beta_i^{L2} - \beta_i^{L3}\right)\right]$$

where

$$\beta_i = \frac{\text{cov}(r_i, r_\text{M})}{\text{var}(r_\text{M} - c_\text{M})}$$

$$\beta_i^{L1} = \frac{\text{cov}(c_i, c_\text{M})}{\text{var}(r_\text{M} - c_\text{M})}$$

$$\beta_i^{L2} = \frac{\text{cov}(r_i, c_\text{M})}{\text{var}(r_\text{M} - c_\text{M})}$$

$$\beta_i^{L3} = \frac{\text{cov}(c_i, r_\text{M})}{\text{var}(r_\text{M} - c_\text{M})}$$

The four beta coefficients resulting from the decomposition can be commented as follows:

– β_i is the classic beta of the CAPM formula, when liquidity issues are not considered.
– β_i^{L1}, or $\text{cov}(c_i, c_\text{M})$, represents the *commonality in liquidity*. Expected return increases with the covariance between the asset's illiquidity and the market illiquidity, because investors demand a premium for holding a security that becomes illiquid when their portfolio (market) becomes illiquid.
– β_i^{L2}, or $\text{cov}(r_i, c_\text{M})$, measures the sensitivity of asset return to market liquidity. This beta loads *negatively* with expected returns, because investors are willing to give up return on an asset with a high return in times of market illiquidity.
– β_i^{L3}, or $\text{cov}(c_i, r_\text{M})$, measures the sensitivity of asset liquidity to market return. This beta also loads *negatively* with expected returns, because investors are willing to give up return on a security that is liquid in a down market. When the market declines, investors are poor, and the ability to easily sell becomes valuable.

Case Study: Risk and Return

Financo Ltd

The Case

Financo Ltd is a primary financial institution, managing the wealth of few selected high-profile investors, with big amounts to invest. It is therefore crucial for the company to manage the clients' portfolios with the highest possible attention to return and risk.

Led by Mr. Edward, CEO and certified financial analyst, the company is currently managing a total of $12 billion of assets, with a team of 50 skilled portfolio managers.

Financo Ltd has a very good reputation among investors in the region, and it is able to acquire new customers every semester. Most of their portfolios have survived the financial crisis years with minimum losses while recovering immediately afterward.

Mr. William is a young but wealthy investor who is seeking for new opportunities to make his money grow and achieve good diversification. He is willing to invest an initial amount of $25 million.

He therefore reaches the Financo headquarters and Mr. Edward welcomes him. After a brief chat with the client and some introduction of the company, the CEO decides to take care of this particular customer himself.

In fact, it is now few weeks that Mr. Edward is analyzing the markets, trying to understand how the company can improve his services to the customers by extending the range of assets to invest in.

In particular, Mr. William has a low-medium risk profile, with preference for income-paying and noncomplex assets. He has been very clear in telling Mr. Edward that he is not willing to consider investments in credit derivatives, as an example.

Even though the range of possibilities allowed by the customer is not extremely wide, Mr. Edward knows he can offer to his customer a wide choice of financial products to be included in the ideal portfolio. Given his recent time spent on learning from the market, there are few types of assets that are interesting.

One possibility is to invest in the equity. Even though the equity market has been subject to drawbacks in the last few months, this newly discovered company seems to be appealing, in that it has been recently made public through an interesting IPO.

Since not much public financial information is available for such a company, the analysis of the investment opportunity must be carried out by comparing the company to the comparable firms in the high-tech sector.

Actually, the average correlation of the industry with the market return is 21.50%. The volatility of the stock is 23.50%, and the volatility of the market is 21.50%. The market return calculated on a 52-week basis is 10.50%, and the risk-free rate in the economy is set at 2.21%, which corresponds to the yield to maturity of government long-term bonds.

Another stock is available from the same market, representing equity of a major well-established corporation. In this case, it is known that the correlation of the stock return with the market return is 19.50%, while the volatility of the stock is 20.50%.

Mr. Edward recommends to focus on the two stocks and combine them in different ways. He then asks Financo to start the analysis of such an investment opportunity, in order to identify the best combination of the available stocks.

Questions

1. If you were to follow the Portfolio Theory like Mr. Edward, what would you do first in order to measure the profitability of the idea?

2. Use CAPM to calculate the expected return on each single asset.
3. What profitability measure can be used in order to assess the relative validity of each single asset and combinations of them?
4. Calculate the return and risk associated with an equally weighted portfolio of the two stocks.
5. Now repeat point 4 for a portfolio made of 80% of the first stock and 20% of the second stock and another portfolio made of 20% of the first stock and 80% of the second stock.
6. Concluding, what is the best combination of the two stocks in terms of relationship between return and risk?

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4 Business Analysis

Every business decision must start from an in-depth analysis of the potential investment opportunities. In order to understand the right choice for project undertaking, all possible choices must be analyzed.

Capital budgeting goes through the application of well-known rules that allow the analyst to understand the potential of every investment opportunity, by setting criteria to be used in order to make the best possible choice in terms of profitability.

Project valuation is then an important part of the corporate finance literature, in that financial managers can use probabilistic approaches in order to value a project, given possible future states of the world.

The definition of scenarios and simulations in fact allows to implement efficient valuation practices and to obtain a fair valuation of the profitability of each investment opportunity through careful identification of the most likely outcomes.

Once good projects are identified, then it is very important to properly calculate the cost of capital associated with financing a profitable investment opportunity, so as to minimize the funding cost.

Cost of capital valuation can be done for both equity capital and debt capital and involves applying a fairly simple mathematical tool to the valuation of financial assets, so to realize what is the cost involved in fund raising.

After studying this chapter, you will be able to answer the following questions, among others:

- What are the rules of capital budgeting, and how can they be implemented for project selection?
- How is it possible to decide which rule is best to apply for any specific case?
- How can probabilistic approaches be used for project valuation?
- What are the models and procedures for debt and equity capital valuation?
- What is the difference between levered and unlevered capital?

A. Corelli, *Analytical Corporate Finance*, Springer Texts in Business and Economics, https://doi.org/10.1007/978-3-319-95762-3_4

The first section of the chapter is about the rules of capital budgeting, how they can be defined, and the differences between them in terms of application and final output. The second section deals with the probabilistic approaches to project valuations. The third section focuses on the cost of capital and how it can be calculated for both debt capital and equity capital, with insight on the concept of levered and unlevered capital, and how it impacts on valuation.

4.1 Capital Budgeting Decision Rules

Learning Outcomes

- Learn how to apply the net present value rule for capital budgeting.
- Learn how to apply the internal rate of return rule for capital budgeting.
- Learn how to apply other rules for capital budgeting.

4.1.1 The Net Present Value Rule

Capital budgeting is the process, also named investment appraisal, for determining the feasibility (in terms of economic and financial convenience) of some long-term investment and their worthiness to fund them through the firm's capitalization structure.

Capital can be input in the form of equity, such as retained earnings, or debt, and the capital budgeting definition extends to the process of allocating resources for major capital, or investment, expenditures.

As per theory of corporate finance, the capital budgeting of investments has the primary goal to increase the value of the firm to the shareholders, and maximize it, given the financial constraints (McNeil et al. 2005).

Long-term projects are expected to generate cash flows over several years, making the choice of the right projects a crucial task. The decision to accept or reject a capital budgeting project depends on an analysis of the cash flows generated by the project and its cost.

In the financial theory of capital budgeting, two main rules apply for the decision of acceptance or rejection of a project, namely:

- Net present value (NPV)
- Internal rate of return (IRR)

A capital budgeting decision rule should satisfy the following criteria:

- Consider all of the project's cash flows.
- Consider the time value of money.
- Always lead to the correct decision when choosing among mutually exclusive projects.

In theory a firm should pursue all investment opportunities that enhance shareholders' value. The problem is that resources are not infinite; therefore, there are constraints related to the amount of capital available for new projects, at any point in time.

Before examining the various capital budgeting rules available to financial analysts, it is important to have a look at the assumptions governing capital budgeting in general, which are as follows:

- Decisions are based on cash flows, not income.
- Timing of cash flows is important.
- Cash flows are based on opportunity cost: cash flows that occur with an investment compared to what they would have been without the investment.
- Cash flows are analyzed on an after-tax basis.
- Financing costs are ignored because they are incorporated in the cost of capital.

There are some concepts in capital budgeting that are very common among managers, who find them useful. Sunk costs, for example, are costs already incurred by the firm, so that they do not affect the actual costs.

Opportunity costs are those related to the value of a resource that is used in some way but could have been used for another purpose. Those costs should be considered at the current market value of the asset in use.

The concept of incremental cash flow is also very important, and it focuses on the cash flow that is generating by making some decision about the project and net of the cash flow that could be realized without making the decision (Scott et al. 1999).

Externality is the effect of an investment on other things apart from the investment. Those effects hit the society or other external agents that are somehow connected to the firm, and they can be positive or negative. Both should be considered.

An example of externality is cannibalization, defined as the situation where an investment takes customers and sales from another part of the company. They should be considered in the analysis because they are incremental cash flows.

Decision criteria can be evaluated according to three main issues. First of all, one must assess whether the decision rule adjusts for the time value of money. Secondarily, it is important to understand whether the decision rule adjusts for risk. The final issue is the informative power of the decision rule about whether value is being created for the firm.

Recall that the NPV of a project indicates whether the impact of investing in it on the value of the firm will be positive or negative. Specifically, projects with a positive NPV are expected to increase the value of the firm.

Based on the above concepts, the NPV decision rule states that the company should undertake the independent projects having a positive NPV. When projects are mutually exclusive, the project with the largest NPV should be chosen.

Consider two different projects, A and B, available to the company. They can be any investment opportunity involving some capital expense and producing an outcome of some type and value.

Financial constraints do not allow the company to invest in both projects, so an analysis is needed to understand which project to undertake, if any. Two investments that are alternative to each other but cannot be both undertaken are called mutually exclusive.

Project A cash flows can be indicated as $\{C_{A,\,0}, C_{A,\,1}, \ldots, C_{A,\,N}\}$, while for project B the cash flows are shown by the series $\{C_{B,\,0}, C_{B,\,1}, \ldots, C_{B,\,N}\}$.

Formally, the NPV rule states that project A is preferred to project B if

$$\mathrm{NPV_A} > \mathrm{NPV_B}$$

where:

$\mathrm{NPV_A}$ is the net present value of project A.
$\mathrm{NPV_B}$ is the net present value of project B.

The present value of projects' cash flows corresponds to the economic actual value of the project, as

$$\mathrm{NPV_A} = C_{A,0} + \frac{C_{A,1}}{(1+r)} + \frac{C_{A,1}}{(1+r)^2} + \ldots + \frac{C_{A,N}}{(1+r)^N} = \sum_{t=1}^{N} \frac{C_{A,t}}{(1+r)^t}$$

Given the right choice for r, the value is the price at what the project could be sold at present in the market. The associated NPV represents the wealth increment brought by the project, and then if NPV > 0 the project is augmenting the wealth.

The relationship can be then formalized as

$$C_{A,0} + \sum_{t=1}^{N} \frac{C_{A,t}}{(1+r)^t} > C_{B,0} + \sum_{t=1}^{N} \frac{C_{B,t}}{(1+r)^t}$$

According to the NPV decision rule, a project should be accepted only if it has a positive or null NPV. Projects with negative NPV should be rejected. In case resources allow for only one project to be undertaken, when comparing two or more exclusive projects having positive NPVs, accept the one with the highest NPV (Khan 1993).

When cash flows are uneven, so not constant over time, the calculation is different and can be summarized in the following example:

Example 4.1 Assume an initial investment on fixed assets of 832,000 €. It is expected to generate cash inflows of 341,100 €, 407,000 €, 582,400 €, and 206,500 € at the end of the first, second, third, and fourth year, respectively. At the end of the fourth year, the residual value of assets is 90,000 €. To calculate the present value of the investment for a discount rate of 18%, it is necessary to first calculate the PV factors associated with the 4 years. The rest of the problem can be solved more efficiently in table format as shown below:

Year	0	1	2	3	4
Net cash inflow		341,100	400,700	582,400	206,500
Salvage value					90,000
Total cash inflow		341,100	400,700	582,400	296,500
PV of cash flows		289,068	287,776	354,467	152,931
Total PV of cash inflows	1,084,242				
Initial investment	(−)832,000				
Net present value	252,242				

4.1.2 The Internal Rate of Return Rule

Another popular capital budgeting rule is based on the internal rate of return of the project, therefore named the IRR rule. Recall that the IRR is defined as the discount rate for which the NPV equals zero. It is the compound rate of return that you get from a series of cash flows.

Suppose there is a project that generates cash flows $\{C_0, C_1, \ldots, C_N\}$. Consider discounting the series of cash flows by the IRR of the project, so to get a null NPV, as described by

$$C_0 + \frac{C_1}{(1+\mathrm{IRR})} + \frac{C_2}{(1+\mathrm{IRR})^2} + \ldots + \frac{C_N}{(1+\mathrm{IRR})^N} = C_0 + \sum_{t=1}^{N} \frac{C_t}{(1+\mathrm{IRR})^t} = 0$$

Assume now discounting the same series of cash flows by the appropriate discount rate r for the project, given by the nature of the project itself and the market conditions. The IRR rule states that the project should be accepted if its IRR $> r$ and rejected it if its IRR $< r$.

The logic behind that is that when r is lower than IRR, the project cash flows give a positive NPV, therefore making the investment worthwhile. The IRR is the compound return you get from the project. Since r is the project's required rate of return, it follows that if the IRR $> r$, the project gives more than required.

As for the NPV rule, the IRR rule also can be extended to set the criteria for the choice between two or more projects. Consider trying to decide between two mutual exclusive projects.

The IRR rule states that project A is preferred to project B if $\mathrm{IRR_A} > \mathrm{IRR_B}$. The logic is clear if one thinks that the IRR is the project compound rate of return. When choosing between two projects using the IRR rule, it is logic to prefer the higher compound rate of return.

Both budgeting rules are logical and sound reasonable when applied to projects. In most cases both NPV and IRR rule will give the same answer on whether it is worth it to invest in a specific project or not.

There are some cases, however, where NPV and IRR give different answers. In such cases, one should always use the NPV to decide between projects. The logic is that if individuals are interested in maximizing their wealth, they should use NPV, which measures the incremental wealth from undertaking a project.

4.1.3 The Payback Rule

The length of time required to recover the initial amount invested in a project is called payback period. The payback period (P_{P}) capital budgeting rule takes its name from that concept.

In order to apply the rule, first of all the financial manager must establish the maximum acceptable payback period length, indicated with $\tilde{P}_{\mathrm{P}}$. The practice suggests that common average payback period is around 3 years.

In determining whether to accept or reject a particular project, the payback period decision rule is

Accept if $P_{\mathrm{P}} < \tilde{P}_{\mathrm{P}}$.
Reject if $P_{\mathrm{P}} > \tilde{P}_{\mathrm{P}}$.
Indifferent where $P_{\mathrm{P}} = \tilde{P}_{\mathrm{P}}$.

For mutually exclusive alternatives, accept the project with the lowest P_{P} if $P_{\mathrm{P}} < \tilde{P}_{\mathrm{P}}$.

Example 4.2 Consider a firm that is offered two mutually exclusive projects, A and B, where the firm's required rate of return is 10% and the project cash flows are given in the below table:

Time	Project A cash flows (€)	Project B cash flows (€)
Year 0	−10,000	10,000
Year 1	2000	6000
Year 2	8000	3000
Year 3	250	10,000
P_{P}	2.0	2.1
NPV	−1380	5480

The payback rule suggests that project A should be accepted; however, the NPV indicates that if A is accepted, the share price will fall. It appears that the payback method is not consistent with the goal of shareholder wealth maximization.

Compared to other rules, the PP rule shows some major drawbacks. For example, it ignores the time value of money and also does not account for the cash flows occurring after the payback period. Moreover, it ignores the scale of the investment.

The rule is appealing in that it provides a measure of the actual money at risk for the project. However, uncertainty increases for cash flows coming further in the future, making the rule not the best tool to account for risk (Levy and Sarnat 1994).

The advantage of implementing a measure like the payback rule is that it gives some measure of the money at risk in the investment. At the beginning of the investment, in fact there is a lot of uncertainty about future cash flows.

The economic environment and the cash flows from the project may be less favorable than initially forecasted. The uncertainty would increase for those cash flows in the more distant future.

But analyzing risk can be done more efficiently by using different methods than the payback criterion. If the measure of risk is what one is interested in, more than other pieces of information, there are two tools for analyzing the risk associated with more distant cash flows.

Recall that the discount rate associated to a series of cash flows can be decomposed in the risk-free part plus a risk premium on top, calculated as the reward demanded by investors for taking the risk of the investment. The discount rate can be therefore defined as

$$r_{\mathrm{DSC}} = r_{\mathrm{f}} + \lambda$$

where:

r_{f} is the risk-free rate.
λ is the risk premium.

It is interesting to analyze what the impact is of a higher discount factor on the present value factor (discounted unit value). The discount rates relate to the time horizon accordingly as can be indicated in the following table, containing a numerical example:

Discount factor	Year 1	Year 2	Year 3	Year 4
10%	0.91	0.83	0.75	0.68
15%	0.87	0.76	0.66	0.57
Difference	3.95%	7.03%	9.38%	11.13%

The risk premium clearly reduces the value of one dollar in the future, to a lower present value at current time. The reduction gets more substantial for longer periods, and it increases for periods 2–4.

4.1.4 The Profitability Index Rule

Another capital budgeting technique that can be employed when the company has limited supply of capital with which to invest in positive NPV projects is the profitability ratio or profitability index (PI).

Capital rationing problem is the lack of funds that forces a company to target the limited capital on just few projects, the ones that result in being the very best among the range of possible investment choices.

Given that the objective is to maximize shareholder wealth, the objective in the capital rationing problem is to identify that subset of projects that collectively have the highest aggregate net present value. In order to compute each project's PI, the formula to be implemented is

$$\mathrm{PI} = \frac{\mathrm{NPV}}{I}$$

where:

I is the amount invested in the project.

Once all the PIs for the projects in analysis are calculated, they must be ranked from the highest to the lowest, in order to select the appropriate investments from the top to the last until the capital budget is exhausted.

The method is based on the underlying idea that the index can provide the subset of projects that maximize the aggregate NPV. This is not always the case and in some cases this information can be inaccurate.

In case the company has limited resources to spread over several projects, then the profitability index cannot give complete information about the profitability of combinations of project, since it focuses on the single projects and their comparison.

If it is already known that there is a limited available budget, then it is possible to analyze the project(s) from the point of view of independent projects or combinations (portfolios) of them, against the available budget. This is done by calculating the weighted average profitability index (WAPI).

The formula for calculating the WAPI of a group of n projects is

$$\mathrm{WAPI} = \sum_{i=1}^{n} \frac{\mathrm{PI}_i A_i}{\mathrm{L}}$$

where:

PI_i is the profitability index of project i.
A_i is the amount invested in project i.
L is the total budget to invest, available to the company.

Example 4.3 Consider a company that has the following projects on the table, and decide in what project(s) to invest in, given a maximum total budget of 600,000 €.

Project	NPV (€)	Investment (€)	PI
A	460,000	400,000	1.15
B	282,500	250,000	1.13
C	388,500	350,000	1.11
D	324,000	300,000	1.08

It is clear that with the available budget, the company can only invest in either project B and D together, project B and C together, or project A only. So to choose which one is the right portfolio of projects, among the above three, one must calculate the WAPI associated to each of them:

$$\mathrm{WAPI}_{\mathrm{B,D}} = \frac{1.13 \times 250}{600} + \frac{1.08 \times 300}{600} + \frac{0.00 \times 50}{600} = 1.01$$

$$\mathrm{WAPI}_{\mathrm{B,C}} = \frac{1.13 \times 250}{600} + \frac{1.11 \times 350}{600} = 1.12$$

$$\mathrm{WAPI}_{\mathrm{A}} = \frac{1.15 \times 400}{600} + \frac{0.00 \times 200}{600} = 0.77$$

It is clear that given the assigned budget, the best way to invest the money is to invest in the combination of projects B and C, which has the highest WAPI.

The choice of the right capital budgeting rule to apply for investment decisions primarily relies on the nature and sequence of the cash flows generated by a project. These can be in fact conventional or nonconventional.

Conventional cash flows are such that there are one of many outflows followed by a series of inflows, and the change in sign occurs therefore only once. In general, every series of cash flows can be considered conventional as long as it involves only one change in sign in the whole series, either from plus to minus or from minus to plus.

Nonconventional cash flows involve initial outflow(s) followed by a series of both inflows and outflows, with more than one change in sign during the length of the timeline.

Another important distinction for choosing the right rule to apply is between independent versus mutually exclusive projects. Two projects are defined as independent when cash flows from one project are independent from those of another project.

It implies that there is no connection or overlap between projects, and all of them are being evaluated and could potentially all be selected as long as their projected cash flows will produce a positive NPV or generate an IRR greater than the firm's hurdle rate.

On the other hand, two projects are defined as mutually exclusive when they directly compete with each other. This happens, for example, in the case when some machinery or equipment in a factory must be changed.

Usually it happens that more than one supplier can make an offer on a brand new piece of equipment to replace the old one, and only one of the many offers will be accepted so that they are all in competition.

With regard to capital available, a distinction can be made between unlimited funds and capital rationing. Recall that capital rationing exists when a company has fixed amounts of funds to invest.

The existence of capital constraints will lead to the existence of the capital rationing issue, therefore not allowing the company to invest in more than just a few selected projects.

Another important concept is project sequencing, a concept related to the situation where many projects are evaluated through time. It happens when investing in one project gives the option to invest for other projects in the future. For example, an investor can decide to open a restaurant this year and if the financial results are positive after some years, a hotel could then be built next to the restaurant.

The conclusion is that the NPV criterion, with appropriately set discount rates, already accounts for the fact that risk increases with the time horizon.

Moreover, there is much less information about the more distant future than the immediate future, and if circumstances change in the future, the design of the project would also change accordingly (Emery et al. 1998).

4.2 Project Valuation

Learning Outcomes

- Learn how to use scenario analysis for project valuation.
- Learn how to design decision trees for project valuation.
- Learn how to run simulations for project valuation.

4.2.1 Scenario Analysis

Standard rules for capital budgeting are based on assumptions about the knowledge of the right amount of cash flows associated to a project and their valuation according to some criteria.

In real life, most cases show that there is uncertainty in the future, and it is not easy to predict future cash flows without using some specific tools. This section is devoted to the definition and explanation of some of the probabilistic tools that can be used for project valuation.

The first valuation tool that helps take into account the uncertainty under different possible states of the world is the scenario analysis, evaluating the impact of simultaneous changes in a number of sensitive variables (Dutta and Babbel 2013).

The simplest implementation of scenario analysis is called back testing, which involves developing a specific asset in case history should repeat itself. It consists in creating just one scenario of how the situation could evolve.

The actual real scenario analysis moves further in the analysis, by considering more factors than the back testing, with a deeper view of the historical scenarios to replicate. It takes a more in-depth look at the future while taking into account historical data and economic knowledge.

There are four basic steps involved in general preparation of a scenario analysis. They can be listed as follows:

- Identification of the relevant factors to build the scenario around
- Choice of the number of scenarios to be analyzed for each selected factor
- Estimation of the outcomes for each possible scenario
- Determination of the probabilities to assign to each scenario

The usual representation of results of the analysis is through outcomes generated, but if probabilities are assigned to each scenario, it is also possible to express the outcomes as weighted averages (expected value) assuming probabilities for each scenario can be estimated, in order to weigh the average accordingly.

Each factor gets assigned a value in order to depict a specific scenario. It is common practice, for example, to simulate a best-case and worst-case scenario in order to bound the range of possible outcomes from some possible investment decision.

The analysis usually starts from historical analysis, by taking into account past returns as a basis for the modeling of the scenario. In fact, on returns many variables can be calculated as volatility and correlation.

After the scenario is completed and all the above points are fulfilled, the last step is the use of the scenario result to plan strategies.

Example 4.4 Consider the following table summarizing three possible states of the world:

Economy	Probability	Stock A return	Stock B return
Boom	20%	15%	10%
Normal	60%	5%	−5%
Recession	20%	−10%	20%

It is possible to calculate expected returns as

$$\mu = \sum_{i=1}^{n} p_i x_i \rightarrow \mu_{\mathrm{A}} = 4.0\%, \mu_{\mathrm{B}} = 3.0\%$$

and the standard deviations

$$\sigma = \sqrt{E(x^2) - (E(x))^2} \rightarrow \sigma_{\mathrm{A}} = 8.0\%, \sigma_{\mathrm{B}} = 10.3\%$$

Once the first scenario is created, it is then possible to change the values assigned to the probabilities in different states of the world, in order to generate new scenarios. It is also possible to change the returns assigned to each state. The ultimate goal is to generate extreme scenarios.

In terms of regulation, scenario analysis is included in the framework of the Basel banking supervision accords issued by the Basel Committee on Banking Supervision (BCBS).

According to the regulatory framework, the financial institutions subject to regulation should use internal and external loss data in order to create scenarios, together with some additional microeconomic factors.

Criteria are loose in terms of the process of scenario formation, given the information processed. Regulated inputs have to be used for regulatory capital calculation in the Basel framework.

Once results are obtained, they can be used for internal purposes, for guiding the business planning, as well as indicators of market functioning, chasing what might happen to financial market returns, such as bonds, stocks, or cash, in each of those economic scenarios.

4.2.2 Decision Trees

Decision trees are a popular tool for financial applications, and their use extends to many activities in social science. The general main use of trees is to help with making good choices, especially the ones involving high levels of risk.

Decision trees allow to compare many different alternatives in a visually efficient graphical format, and compared to other methodologies, decision trees offer some remarkable advantages:

- The graphical approach allows to visualize at a glance the various alternatives and possible outcomes.
- The method allows for a high flexibility, depicting complex alternatives and being easy to modify according to new information.
- A tree can be always split into sub-trees, allowing for almost infinite levels of complexity in the analysis.
- They are not in contrast with other forms of decision methods but arise as a useful complementary tool.

The decomposition of trees into multiple sub-trees allows for a deeper analysis of some specific actions and outcomes. However, it is never too good to put too much information on it, to not lose the intuition effect (Werner 2010).

The starting point for building a tree is the bottom of it, with backward analysis over the branches, through stepping back. This is because sometimes the further development of the tree discloses the need for changing the information in the previous steps.

The tree is developed in nodes from which new branches depart, at any step. There are two types of nodes. Chance nodes are indicated by a circle and are followed by an event that is not under the control of the agent. Decision nodes (squares) are the nodes followed by some decision (Fig. 4.1).

The purpose of decision tree analysis is to forecast future outcomes, assigning probabilities to possible events. Bayesian probability models help to analyze the problem when the analysis involves complex decisions.

Example 4.5 The expansion of a factory has a cost of 1,500,000 €. If the management decides to not do anything, and the state of the economy is good, the expected revenues are 3,000,000 €. If the economy is bad, revenues will only be 1,000,000 €.

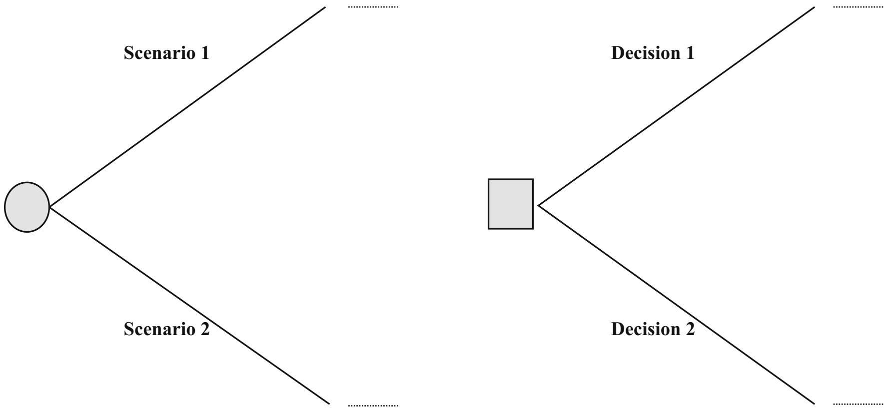

Fig. 4.1 Chance nodes (left) are defined as the nodes where some new information is expected afterward. Decision nodes (right) are the nodes when it is requested to make a decision between the following alternatives

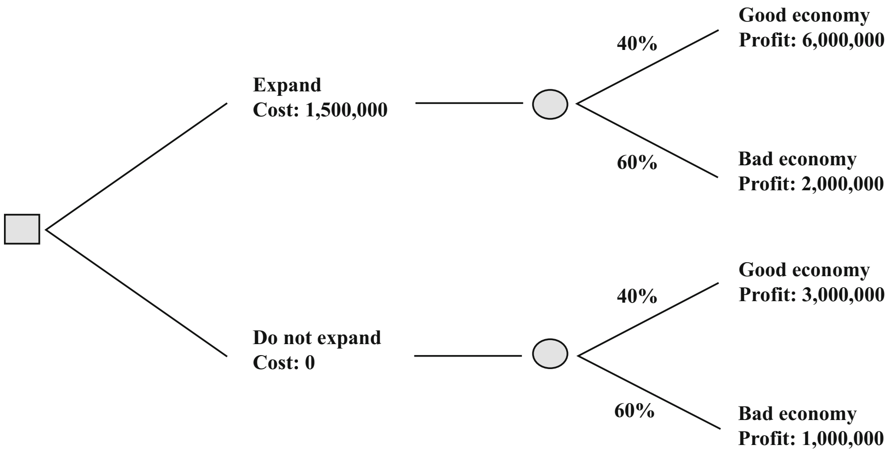

Fig. 4.2 An example of a decision tree where decision nodes are alternate with information nodes, whose output is subject to branching probabilities

If the expansion is approved, the firm will register revenues of 6,000,000 € and 2,000,000 € if the economy is bad. Assume there is a 40% probability of a good economy and a 60% chance of a bad economy.

It is possible to represent the problem on a decision tree as in Fig. 4.2.

The net present values from expansion and no expansion are

$$\mathrm{NPV}_{\mathrm{EX}} = 0.4 \times 6,000,000 + 0.6 \times 2,000,000 - 1,500,000 = 2,100,000 \ \text{€}$$

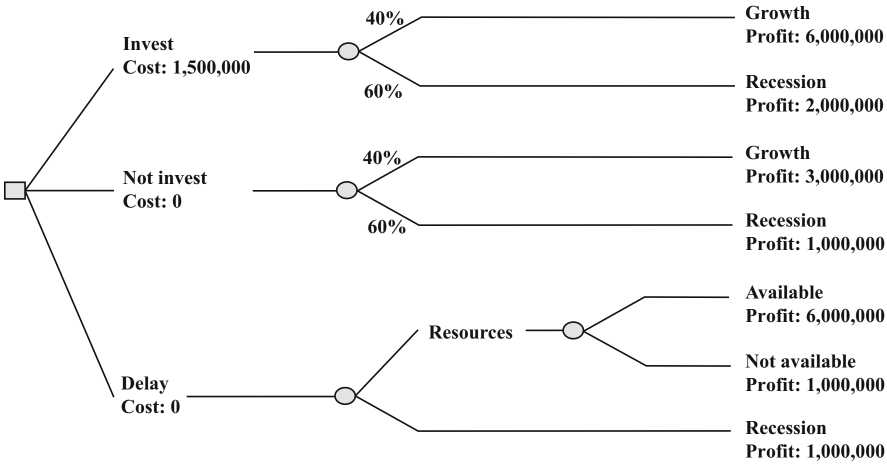

Fig. 4.3 Decision trees can be as complicated as the problem they represent. Also the availability of resources can be an input to the tree

$$\mathrm{NPV}_{\mathrm{NE}} = 0.4 \times 3{,}000{,}000 + 0.6 \times 1{,}000{,}000 = 1{,}800{,}000 \ \text{€}$$

The NPV from expanding is higher; therefore, the factory should expand.

The final payoffs are determined by the expected outcome, given some forecasted probabilities determined according to each possible future state of the world. Sometimes the probabilities are linked to dependent uncertainties among steps, which increases the degree of complication in the analysis.

Example 4.6 Now consider an additional alternative to wait and see how the economy goes, bearing the risk that there will not be enough resources available if waiting. The tree becomes as in Fig. 4.3.

Backward induction on the tree gives a new NPV for the choice of waiting, as

$$\mathrm{NPV}_{\mathrm{Wait}} = 0.4(0.5 \times 6 + 0.5 \times 1) + 0.6 \times 1 = 2{,}000{,}000 \ \text{€}$$

The new NPV from waiting is higher than not expanding but still lower than in the case of immediate expansion. This is an example of dependent uncertainties, where the dependency is between the uncertainty about the state of the economy and the one about the availability of resources.

4.2.3 Simulations

Simulations are a very useful tool in case distributional assumptions are available. The method involves drawing one outcome from a distribution to generate a set of cash flows and values. The process is then repeated several times in order to derive a distribution for the value.

The degree of accuracy of simulations is very high because, as opposed to other probabilistic approaches, the number of variables to be changed and the potential outcomes are not constrained to some limited number.

When past data are not available for a historical analysis, it is necessary to pick some appropriate statistical distribution to capture the shape of the input variables and estimate the parameters (Lea and Cohen 2004).

Correlation plays a major role in simulations, given the fact that most statistical distributions carry statistical features that are hardly approximated by the actual variables in the real world.

When simulation inputs are strongly correlated, the analyst must choose between letting only one input vary, or embed the correlation in the simulation algorithm, which makes the computation more cumbersome.

The number of inputs and iterations determines how many simulations are required in every case. Other criteria include the type of distribution involved in the analysis and the range of possible outcomes.

The simulation can be run using several methods. One of the most popular is the Monte Carlo method, which can be used when there is limited knowledge about data population and sampling is complicated.

In the case of an exploratory approach, exogenous variables are changed in order to generate specific courses of action, defined by models that are usually built from historical data (Jackel 2002).

Sometimes the best solution is a hybrid approach with human and artificial intelligence mixed in a real-time simulation.

Example 4.7 Assume the estimation of some quantity $\theta = E[h(\mathbf{x})]$, where $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$ is a random vector in $\mathbb{R}^n$, $h(.)$ is a function from $\mathbb{R}^n$ to$\mathbb{R}$, and $E[|h(\mathbf{x})|] < \infty$ must be carried on.

The following Monte Carlo algorithm can be used:

1. For $i = 2$ to n.
2. Generate $\mathbf{x}_i$.
3. Set $h_i = h(\mathbf{x}_i)$.
4. Set $\widehat{\theta}_n = \dfrac{h_1 + h_2 + \ldots + h_n}{n}$.

That gives a good estimator for the parameter.

When choosing a random number generator, the choice is between speed and reliability. It is therefore recommended to keep this trade-off in mind and go for a balanced solution.

Uniform number generators are characterized by a generally good overall performance in terms of both speed and reliability, with an average score in both aspects without excelling in any of the two.

A faster generator is not necessarily a better one, but a good random generator is usually fast. Also, a good equi-distribution is necessary but not sufficient condition for the quality of a generator.

In finance a very useful class of simulations, the explanatory simulation, is widely used. Based on historical data for modeling real world realistically, it spares from the computational burden of standard simulation.

Computing power is used to carry on the simulation, including the risk-adjusted NPV, which is simulated by using inputs that are not always fixed, besides being well defined by the user.

The aim of the simulation is to replicate the performance of the target project, with the outcome of a distribution of possible NPVs over a range of discount rates and other variables.

Monte Carlo integration is a useful computational math tool which is able to solve computationally common problems in financial mathematics. It achieves good results by approximating complicated integrals.

Consider the problem of estimating an analytically tricky integral of a function f over some domain D, given by

$$F = \int_D f(\mathbf{x}) d\mu(\mathbf{x})$$

where

$\mu(\mathbf{x})$ is the mean of the variable $\mathbf{x}$.

The function f can have many dimensions, and the solution to the integral is such that it is not possible to get it analytically. Assuming there is a pdf p defined over the same domain, the equation can then be written as

$$F = \int_D \frac{f(\mathbf{x})}{p(\mathbf{x})} p(\mathbf{x}) d\mu(\mathbf{x})$$

That is equal to

$$E\left(\frac{f(\mathbf{x})}{p(\mathbf{x})}\right)$$

which is the expected value of $\frac{f(\mathbf{x})}{p(\mathbf{x})}$ with respect to a random variable distributed according to $p(\mathbf{x})$. Whenever $f(\mathbf{x}) \neq 0$, this is true for all $p(\mathbf{x}) \neq 0$.

The expectation $E\left(\frac{f(\mathbf{x})}{p(\mathbf{x})}\right)$ can be also calculated as an average. First of all random samples must be generated according to p. Then the ratio $\frac{f}{p}$ is calculated for each sample, and the average is proxy for the value for the expectation.

The convergence to the expected value is higher for the sample size increasing. The expected value is the actual solution to the integral. That is the process of Monte Carlo integration.

4.3 Sub-disciplines of Business Analysis

Learning Outcomes

- Understand the use of enterprise analysis.
- Learn how to perform business requirements analysis.
- Explain other common analysis techniques.

4.3.1 Enterprise Analysis

Understanding the needs of a business, the strategy, and identifying actions to put in place for the business to meet its goals is called enterprise analysis, or strategic enterprise analysis.

The span of the analysis goes from the identification of the business problem or need, the proposed solution to it (when existing), and also the assessment of whether the proposed solution is the best available. The analysis also involves developing detailed analysis of what the solution entails, its risks, and its feasibility in the existing organizational climate (Grinblatt and Titman 2002).

Enterprise analysis involves a consistent amount of research and examination. This is the reason why the overall process is usually run at project's inception, or when the project is small, throughout it.

A thorough enterprise analysis endeavor will include points like:

- Proper evaluation of the viability and effectiveness of the proposed business initiatives.
- Identifying the needs of the business, despite the proposals on the table.
- The ideal solution to address the problem or need must be described.
- Evaluating the risk and return associated to the proposed strategy is also important.
- Some important questions are: What is the scope of the proposed solution? What tools and processes are involved in getting the solution?
- The environment for the problem-solving must be created with aids like visuals and business cases

In many companies the senior analysts take care of the enterprise analysis process, given the high level of specialization of such a task. It is however useful for many other employees to get knowledgeable about the analysis.

There are a number of roles in the business world, with an interest in pre-project research and solution justification. These roles include but are not limited to business analysts, project managers, stakeholders, business owners, and software engineers.

The enterprise analysis introduces the analyst to the overall requirement process. It identifies the needs of the business, and it confirms the appropriate solutions. It is the foundational research that undergirds any successful set of requirements.

Many steps of business analysis in a company are usually performed in coordination with the IT department. This is not the case of the enterprise analysis, which is usually carried on irrespectively (Pinches 1996).

The focus of the analysis is on the business, and the contribution of the IT sector is welcome, but it is not the focus. Solutions are focused on business, including changes in business processes, models, and strategies.

A *Guide to the Business Analysis Body of Knowledge®* (*BABOK® Guide*) is the globally recognized standard for the practice of business analysis. It is a collection of the most widely accepted analysis standards, as established by the business analysis community worldwide.

The guide was published in 2005, as a consulting tool for analysis and standardization of the standardized and generally accepted at that time. The first formal release was version 1.6 in June 2006.

The guide defines the knowledge underlying the performance of correct business analysis, taking into account the evolution of the approach, and determining the skills that a practitioner should demonstrate to work in the field.

BABOK standards relate to the framework of business analysis and indicate the various steps to be performed to deliver a solution that will provide value to the sponsoring organization.

The overall analysis goal is influenced by how each single task is performed, with either a direct or an indirect effect. The elements of the tasks like the performing order and the form of execution can vary, as well as their relative importance.

The *BABOK® Guide* identifies five main steps involved in enterprise analysis. The process combines the following elements:

The first step entails the definition of the business needs. This is a task that can be done even before project inception, but the risk is that the analysis will be inaccurate, and the identified need may not be the true one that the business needs to address in order to achieve its goals.

The identification of the need is a crucial step, in that an incorrect identification of the need will hinder the achievement of a viable solution and all efforts toward that end will be wasted.

Many types of event, like market opportunity, decrease in revenues, or other factors, may generate a business need and lead to analysis as a consequence. It is common in this case for most firms to try to resolve the issue without investigating the underlying business need.

In order to improve such a behavior, the business analyst has the duty of investigating the underlying causes of the need. This helps identify the possible solutions and collect them in order to be presented to the stakeholders.

In order for the task to produce the best outcome, it is very important that group thinking is avoided, as well as preconceived ideas and wrong assumptions. The view on the problem should be as objective as possible and effective as well.

As outlined in the BABOK standards, the accurate definition of the business need requires the analyst to identify:

- The signals of problem in the organization that can be quantified, like losses in profits, etc.
- The outcome expected after applying the selected solution, like higher revenues, cost reduction, etc.
- The potential speed of implementation of the chosen solution to the problem and the consequences of doing nothing
- The underlying real source of the problem to be addressed

The second step is the capability analysis, which defines whether the organization can meet the identified need. As from BABOK, if an organization does not have sufficient capabilities to meet a business need, the analyst must indicate the missing capabilities that must be added.

Once the business need is identified, and the capabilities have been sorted out, the third step involves choosing the right approach to the problem, as the most viable solution to the business need.

At this stage a careful description of what is needed to implement in the business must be given. Changes in items like software, website, and business processes must be clearly indicated, if that is the case, as well as the combinations of those.

More than one solution, or multiple parts to a solution, may be proposed. If capability gaps prohibit a smooth implementation of the ideal solution, solution alternatives must be anticipated.

A good analyst makes a list of possible viable solutions, each defined by feasibility, accurate risk analysis, and constraints, as well as accompanying assumptions underlying its adoption.

The fourth step is about defining the solution scope, meaning the analyst must identify new capabilities to be generated as a consequence of the implementation of a new project.

The stakeholders focus on this step because it helps them to understand the path to the solution's arrival and the tools that will be required to implement it. Items involved in the scope include warehouses, databases, and processes.

It is mentioned in BABOK that "The solution scope will change throughout a project, based on changes in the business environment or as the project scope is changed to meet budget, time, quality, or other constraints."

The final step involves defining the business case, as a tool to represent the practical benefits of the solution proposed, and the true benefits to the organization from a business point of view.

The case is about quantifying the benefits of the solution, so to complete the work initiated with the expected payback analysis in step one. The outcome is a specific amount of revenue, dollars saved, and other quantifiable benefits, including the description of the metrics that helped the analysis.

The key factor in the success of the overall analysis, after completing the five steps, is the involvement and agreement of the stakeholders, together with the help of communication as an efficient tool for implementation.

Performing the enterprise analysis requires specific core competencies that an analyst must possess in order to effectively lead enterprise analysis projects. The competencies required can be listed as follows:

- Ability in creating and maintaining a business structure. This competency requires the analyst to understand the architecture of the business, defined as the company's present and future state, in terms of strategy, goal, and objectives.
- Attention to the feasibility of the options that are on the table, to be exerted by looking at them, assessing whether they are technically possible and whether they will meet the organization's goals.
- Identification and analysis of opportunity, done by identifying new investment opportunities, in collaboration with experts in the field of operation of the target business.
- The business case must be prepared and maintained afterward. The proposed solution comes at a cost in terms of resources, and the costs must be weighed against the tangible benefits that the solution will offer.
- Management of the several types of risk (financial, operational, economic, etc.) to be faced when implementing the selected solution. Comparison with the risk of not implementing the solution is a logic consequence.

Enterprise analysis is commonly offered, as outsourced service, by individuals and companies. They offer the service to companies that are less experienced in the analysis process, thus unable to provide it internally.

The several types of services offered by professionals in the field include the organizational research, aimed at ensuring that all the possible potential solutions are explored.

4.3.2 Business Requirements Analysis

The main reason for developing a new product or services in the various industries is to address a specific need of the consumers. Sometimes, companies find themselves in situations where, despite spending tremendous time and resources, there's a mismatch between what has been designed and what is actually needed.

Sometimes the discrepancy is even between some order of the client and the final product. In other cases, there is a change of mind halfway through a project. Finally, it is even possible that new requirements come at the conclusion of product creation.

All of these problems can be easily avoided by developing a careful and detailed business requirements analysis, which is the process of identification, analysis, and recording of the requirements related to a specific business goal.

By such analysis it is possible to give a clear definition of the scope of the project, in order to provide a careful assessment of important factors such as the timescale and the amount and type of resources needed to complete it (Robertson 2006).

The first step in obtaining the desired outcome, product, or service is the accurate definition of it. Requirements analysis allows to break the business need into specific requirements on which there is common consensus, for a better understanding of it.

What's more, it's usually much quicker and cheaper to fix a problem or misunderstanding at the analysis stage than it is when the "finished product" is delivered.

Modern corporate sector is characterized by a general tendency to establish internal procedures and methodologies to conduct the analysis. The procedures are then specific for different industries and different companies.

As for the enterprise analysis in previous sections, the business requirements analysis is based on five steps. The first one is the identification of the key stakeholders in the organization, who will be affected by the project.

The first thing to do at this stage is to exactly identify the sponsor of the project, who could be either an internal or an external client. In both cases, it is essential to know who has the final say on what will be included in the project's scope and what won't.

It is then important to identify the final users of the proposed solution, product, or service. The needs of the end users are the foundation of the analysis, and their input should be carefully considered.

The end users of the product may belong to several departments in the organization, or they may be all concentrated in one division. The stakeholders' requirements about the product have to be collected and recorded.

When recording preferences, the individual perspectives should be taken into account. The understanding that different perspectives correspond to different requirements allows to build a complete picture of what the project should achieve.

The stakeholders should be interviewed in a format that clarifies the basic scope of the project, without deviating from the main idea. This should discourage the end users from describing functionalities never supposed to be embedded in the project. If users have articulated these desires in detail, they may be disappointed when they are not included in the final specification.

Interviews with stakeholders can be held either individually or in groups. In the first case, there will be a better understanding of the individual positions, while in the second case, the way how information between departments flows can be understood.

Another technique available to the analyst is the use of scenario-based cases, in order to walk through the system or process as a user. The simulated environment gives the idea of how the system or service would work. It is a good technique for gathering functional requirements, but you may need multiple "use cases" to understand the functionality of the whole system.

When implementing the technique, it is useful to check for previous cases about similar services and build a mock model for the system or product to give the idea of the final look of the product.

Using this, users can address feasibility issues, and they can help identify any inconsistencies and problems. You can use one or more of the above techniques to gather all of the requirements.

A third step is the categorization of the requirements, in order to simplify the analysis by grouping the requirements into four main categories. This step simplifies a lot of the analysis.

Functional requirements define what the functioning of the product should be from the point of view of the end user. They describe the features and functions with which the end user will interact directly.

The set of actions to implement in order to keep the product functioning over time defines the operational requirements. Technical requirements refer to the technical issues to consider when trying to successfully implement the process. Finally, transitional requirements define what steps are needed for a smooth implementation of the product creation (Larson 2009).

After all the requirements have been collected, recorded, and categorized, they must be interpreted, in what constitutes the fourth step of the analysis. Once all requirements are gathered, it must be decided which ones are achievable and how the system or product can deliver them.

The interpretation of requirements follows several sub-steps including the following:

- Precise definition of the requirements, that have to be clearly specified and sufficiently detailed, in order to avoid problems coming from non-identified unknowns.
- Establishment of the prioritization of requirements by giving priority to those that are more relevant and more critical.
- Analysis of the impact of change, to make sure that all the consequences related to the impact of project implementation (especially on existing processes and people) are clear and fully understood.
- Resolution of conflicting issues by interacting with stakeholders in order to identify potential conflicts. Scenario analysis can be helpful to solve the task, in that it allows all those involved to explore how the proposed project would work in different possible states of the world.
- Feasibility analysis is also important for determining the reliability and ease of use of the new product or system and identifying the major problems that may be encountered.

The outcome of the analysis is a detailed report to be circulated among the involved stakeholders, end users, and development teams. The feedback received can resolve remaining conflicts and constitute a sort of contract between the analyst and the stakeholders.

The final step is the collection of signed agreement of the key stakeholders, stating that the presented requirements are effective in addressing the specific needs. This formal commitment will play an important part in ensuring that the project does not suffer from scope creep.

4.3.3 Analysis Techniques

Several analysis techniques are available to the analyst, to facilitate the business change. One of them is called PESTLE, and it is used to perform an external environmental analysis by examining the many different external factors affecting an organization.

The "P" of PESTLE stands for political and represents the influences that may come from political pressures. The "E" means economic, mainly the impact of national and world economy.

After that there is the "S" for sociological, which is related to how the society can affect the many aspects of an organization. The "T" stands for technological and refers to the effect of new technology.

The letter "L" is for legal and describes the effects of domestic and global legislation, while the last "E" is for environmental and focuses on the local, national, and world environmental issues.

Another method of analysis is called heptalysis, and it is performed as an in-depth analysis of seven core factors, for early-stage businesses. The factors are market opportunity, product/solution, execution plan, financial engine, human capital, potential return, margin of safety (Sharp et al. 2001).

The MOST analysis is used for internal environmental analysis and defines four different attributes. The purpose of the analysis is to ensure that the project currently under development is aligned to each of the four attributes.

The first MOST attribute is mission, meaning the overall direction of the business. Then there are objectives, identified as the key goals to complete the mission. After that there are strategies, which are the options available to move forward. Finally, the tactics state how the strategies can be implemented.

SWOT analysis is another popular tool for business analysis, with a focus on the areas of strength and of major opportunities. Another side of the analysis concentrates on the weaknesses of the business as well as on the internal and external threats.

The analysis of the strengths relies on understanding what are the advantages of running the business and what are the strong areas of current operations and the best-performing activities of the firm.

Weaknesses include the aspects of the firm that could be improved, aspects of the business that are not run properly, and key areas where the overall performance is quite poor (Jacka and Keller 2009).

Another point is about the opportunities available to the organization, namely, what are the business areas where competition is weaker. Threats are the last issue, and they focus on the obstacles the business has to face, including key areas where competitors are much stronger than the firm.

The CATWOE analysis relies on the prompt thinking about the aimed achievements of the business. The perspectives of the firm over some time interval help analysts to quantify the impact of possible solutions on the agents involved.

The first component of CATWOE are customers that aims at identifying who are the beneficiaries of the business project outcome and how the issue under investigation affects them.

The second component is actors, meaning those who are involved in the process, and will be also involved in the solution implementation, and what factors will affect their success in the process.

Transformation process is another important component, addressing the issue of what processes or systems are affected by the issue. The world view factors identify what is the big picture and the generalized impact of the issue.

The owner component identifies who is the owner of the process under investigation and what is their role in the achievement of a solution. Finally, the environmental constraints define what are the constraints and limitations that will impact the solution and its success.

The Six Thinking Hats (STH) analysis method is a brainstorming approach to business analysis, meant to generate ideas and analyze options. It is useful for fostering thinking and it can have a high motivation power. It basically involves restricting the group to only thinking in specific ways—giving ideas and analysis in the "mood" of the time.

The color combinations are as follows:

- White: Pure facts, logical
- Green: Creative
- Yellow: Bright, optimistic, positive
- Black: Negative, devil's advocate
- Red: Emotional
- Blue: Cold, control

The MoSCoW analysis technique is used for requirement prioritization, by matching tasks with the appropriate level of priority, gauging it against the validity of the requirement itself and its priority against other requirements.

So basically the levels of prioritization start from the highest, namely, the Must have, which means the requirement is needed for the delivery to not be a failure. After that, the Should have level means that in absence of the requirement, the business will have to adopt a workaround.

The Could have level is the level where the possession of the requirement will increase the delivery satisfaction. Finally, at the Would like level, the requirement is maybe needed in the future but not immediately.

4.4 Summary

Business analysis is important to assess the potential contribution of an investment to the overall value of the firm. Capital budgeting and project valuation help in the analysis process.

There are three main rules of capital budgeting that help in identifying the profitable investment opportunities, discarding the unprofitable ones, and concentrating the choice of investment on the best solutions.

The net present value rule states that only projects having a positive NPV should be taken into consideration, and among many possible opportunities, the project with the highest NPV should be selected.

The valuation of projects can be done in several ways, relying on probabilistic approaches. The most common type of approach is the scenario analysis, which relies on the simulation of several states of the world and assesses the potential outcomes.

Decision trees are another tool that helps in the analysis of an investment by drawing on the branches of a tree the many possible outcomes of actions, as represented by the three nodes.

Simulations are a computational tool and involve the use of algorithms and computer power to estimate the parameters of a model that represents the possible future outcomes based on the analysis of historical data.

Business analysis is composed of sub-disciplines that approach the issue from different points of view. Enterprise analysis, for example, relates to the needs of a business, the strategy, and identifications of the actions to put in place to meet the goals of the firm.

The business requirements analysis relates to the development of a careful and detailed process of identification, analysis, and recording of the requirements related to a specific business goal.

The several types of analysis techniques available include methods that challenge the analysis issue from different sides and process the available information to produce outcomes about issues and options.

Problems

1. A project has even cash inflows and requires an up-front investment of 350,000 €. It will generate positive cash flows of 56,000 € per month, for 1 year. The target discount rate is 12% on a yearly basis. There is no salvage value. Calculate the NPV.
2. Assume an initial investment on fixed assets of 1,022,000 €. It is expected to generate cash inflows of 271,100 €, 385,000 €, 442,400 €, and 266,500 € at the end of first, second, third, and fourth year, respectively. At the end of the fourth year, the residual value of assets is 90,000 €. Calculate the NPV.
3. Given the three possible states of the world in the below table, calculate the expected return and standard deviations.

Economy	Probability	Stock A return	Stock B return
Boom	5%	21%	25%
Normal	70%	2%	−1%
Recession	25%	−15%	−20%

4. The expansion of a factory has a cost of 16,000,000 €. If the management decides to not do anything, and the economy state is good, the expected revenues are 24,750,000 €. If the economy is bad, revenues will only be 2,000,000 €. If the expansion is approved, the firm will register revenues of 18,500,000 € and 1,800,000 € if the economy is bad. Assume there is a 53% probability of a good economy and a 47% chance of a bad economy. Should the company expand or not?
5. Following exercise 4, consider an additional alternative to wait and see how the economy goes, bearing the risk that there will not be enough resources available if waiting. What is the right decision in this case?
6. What are the advantages of probabilistic approaches to risk, compared to analytical methods? What about the disadvantages?
7. Describe the steps needed in the implementation of simulations for project and business analysis.
8. Daniele is offered a choice where you can take a certain amount of 15,000 € or take part in a gamble, where he can win 50,000 € with probability 50% and 1000 € with probability 50%. Draw a decision tree to represent the choice.

Case Study: Net Present Value

Voltoncamp Ltd

The Case

Voltoncamp Ltd is a company active in tourism and hospitality, which offices in Europe and operating worldwide. They offer tourism packages around the world and own estate in the form of touristic villages worldwide.

Willing to expand its business operations, the company is willing to invest fresh capital into new ventures. In particular the company wants to build a new village in order to face the high postcrisis demand for tourism.

The company is considering a new investment proposal by a partner in Hong Kong, for building a new village, and to operate it starting early 2018. But there is also an investment opportunity in Europe, which requires to refurbish and renovate an existing village, therefore at a lower cost.

There are catches with both investment opportunities. For instance, Hong Kong charges no taxes on corporate income, while in Europe profits from any investment are charged at an average 30% rate.

On the other hand, the European investment requires a much lower investment of $10,000,000, compared to the $20,000,000 required to build a village from scratch in Hong Kong. Both amounts are available as up-front cash payment.

Cristina, a freelance financial analyst, is hired by the company to analyze the two investment opportunities and recommend the best investment, given the financial environment, the maturity of the investment, and the macroeconomic variables involved.

Tourism in Hong Kong is targeted to various types of tourist. Voltoncamp would offer medium-high level tourist facilities and services, with an expected gross profit of $2,000,000 for the first year, increasing of an average 5% per year for the following years.

A village in Europe would make the company gross a pretax income of $2,000,000 as well. The revenue growth in Europe is supposed to be milder than in Hong Kong, due to the postcrisis economy still to be fixed, at a rate of 3% per year.

Other factors to be taken into consideration are the expected reinvestment rate for average low-risk financial investments in Europe, estimated at a constant 3% per year. The same level of risk grants a return of 5% in Hong Kong.

Building a new village in Hong Kong means the company will have no refurbishing costs for the first 15 years of the investment life. After that, an investment of further $5,000,000 will be required in order to keep running the business for further 15 years.

About the village in Europe, it will have to be demolished and rebuilt in 15 years from now. Rebuilding a new village will cost $15,000,000 and will guarantee further 15 years of operations without refurbishing needs.

Consider that profits are also subject to inflation, which in Hong Kong is currently set at 2%, and supposed to increase by 0.1% per year on average, for the next 30 years. In Europe the inflation is much lower, at 1%, set to increase by 0.05% per year on average, for the next 30 years.

Last but not least, the average cost of capital for similar investments in Europe is 13% per year, while in Hong Kong is 15% per year. As a company, Voltoncamp normally conforms to the competitors in terms of cost of capital, so the above figures are assumed to be good proxies for the company's cost of capital.

Questions

1. Assuming a maturity of 15 years, if you were Cristina, which course of action would you recommend to the company, invest in Hong Kong or in Europe?
2. How would your answer to point 1 change if one considers a 30-year maturity of the investment?
3. What factor could influence the calculation of the amounts involved and therefore change the answers to both points 1 and 2, if taken into consideration?
4. Assume profits can be reinvested at a rate of 1% in Europe and 3% in Hong Kong. Would the answer to points 1 and 2 change? How? Explain.
5. Assume the company decides to borrow the money to invest instead of paying it up front in cash. What factors would then influence the decision of the company?
6. How does the answer to point 1 change if the company can borrow the full amounts needed over the maturity of the investment, at a fixed constant rate of 3% per year in Europe and 5% per year in Hong Kong?

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Debt Valuation

5

Debt is one of the possible sources of capital for the company. When the debt is issued on the market, it takes the form of bonds and complements the capital obtained as debt from banks.

Bond markets are very liquid and generally show high volumes of trading, and bonds of many types are traded every day, in order to satisfy the needs of capital of the corporations and the investment needs of investors.

The main issue when valuing the debt of a corporation is to calculate and interpret the actual value of the bonds issued by the firm and grasp the riskiness of the business and how the market perceives it.

Valuation of bonds goes through valuation of the cash flows generated and can be performed according to various approaches. All of them are meant to successfully compute the market value of debt.

The understanding of debt value for a firm is a primary task when looking at the overall average cost of capital and possibly to a valuation of the company. Mathematical models help with the task.

According to different forms assumed for the short rate at infinitesimal frequencies, the term structure used to discount bond cash flows assumes different shapes and provides several possible actual values of the cash flows involved.

After studying this chapter, you will be able to answer the following questions, among others:

1. What types of bonds are available in financial markets, and how do they differ from each other?
2. What is the relationship between the price and yield of a bond, and what does the yield curve represents?
3. How can duration and convexity measures be used to assess the riskiness of a bond?
4. What types of stochastic models for the short rate are available?

A. Corelli, *Analytical Corporate Finance*, Springer Texts in Business and Economics, https://doi.org/10.1007/978-3-319-95762-3_5

5. How can models for the short rate be used to build a term structure and price a bond?

The first section of the chapter is an introduction to the various types of bonds in the market. The second section deals with the relationship between the price and yield of a bond and presents the term structure and yield curve concepts. The third section gives a description of the stochastic models of the short rate and how they can be applied to the building of the term structure and bond pricing.

5.1 Bond Markets

Learning Outcomes

- Learn the foundations of bond theory.
- Explain the different types of bonds available on the market.
- Understand the role of risk in fixed income investment.

5.1.1 Bond Fundamentals

For a company, the cost of debt is represented by the interest rate it has to pay on new loans, like bank loans or bond issues. In relation to the bond issuance, the rate is clearly not the same as the coupon rate on the firm's existing debt, which reflects the interest rate the firm had to offer at the time the debt was issued.

When a company has already issued bonds, the existing debt trades on the market and the price reflect the normal fluctuations of the economic environment and modifications in the risk level.

A yield to maturity can be implied from bond prices, reflecting the actual valuation of the risk associated to the company, in terms of a yield that adds up a premium to the current risk-free level.

The yield to maturity indicates what debt holders would earn if they held the debt to maturity and received all of the payments as promised. Thus, the yield to maturity is the firm's current cost of debt.

Example 5.1 A company has bonds due in 36 months with a coupon rate of 5% and a market price of 95.72. The yield to maturity corresponding to these characteristics is 3.30%. In reality there is no need to compute the yield to maturity because prices and their implied yields to maturity are always quoted together in the bond market.

As opposed to equity capital, the cost of debt is such that the total expense is always lower than the actual return obtained by debt holders (Cochrane and Piazzesi 2005). This is possible because the interest paid on debt is a tax-deductible expense, whereas dividends paid to equity are not. In case of debt financing, the tax savings partially offset the interest payments.

The tax deductibility of interest lowers the effective cost of debt financing for the firm. More generally, with tax-deductible interest and denoting the corporate tax rate as TC, the effective after-tax borrowing rate is

$$r_{\mathrm{D}}(1-\tau_{\mathrm{C}})$$

This makes the effective cost of debt on a principal L equal to

$$r_{\mathrm{D}}(1-\tau_{\mathrm{C}})L$$

Example 5.2 A company borrows 10,000 € at a rate of 5%, and the corporate tax rate in the economy is 33%. Then its net cost at the end of the year is calculated as in the table.

Interest expense	$0.05 \times 10,000$	500
Tax savings	$(0.05 \times 10,000 \times 0.33)$	(165)
After-tax expense	$0.05 \times (1-0.33) \times 10,000$	335

The effective cost of the debt $\tilde{r}_{\mathrm{D}}$ after taxes is

$$\tilde{r}_{\mathrm{D}} = \frac{335}{10,000} = 3.35\%$$

5.1.2 Types of Bonds

The market of bonds is populated by several types of securities that can be differentiated based on the issuer, the structure, and the cash flows. The main distinction is between government bonds and corporate bonds.

Government bonds are issued by sovereign governments and can be zero-coupon bonds or coupon-paying bonds. As for most bonds, it promises to pay periodic interest payments and to repay the face value on the maturity date.

The currency of denomination of government bonds is generally the domestic currency of the country of issuance. In case the bonds are issued in a foreign currency, they are named sovereign bonds, but the term is often used as a synonymous of government bonds, regardless of other factors.

In terms of the terminology used by the media, it is common when a government or sovereign is close to default, to refer to such a situation as a sovereign debt crisis, again showing no relationship with the currency of issuance of the bonds.

The terms on which a government can sell bonds depend on how creditworthy the market considers it to be. International credit rating agencies will provide ratings for the bonds, but market participants will make up their own minds about this.

Bond markets are very old, with the first known issuance of government securities dating back to 1517, in the Netherlands. At that time, the country was not yet formed, so the city of Amsterdam was the official issuer (Fama 2006).

However, these bonds are considered their predecessor which later merged into Netherlands government bonds. The average interest rate at that time fluctuated around 20% at the time of issuance.

The first national government issuing a bond was England in the late seventeenth century. The war against France, occurring at that time, was the reason for issuing securities in the form of a tontine.

European governments started issuing bonds later, to generally fund public expenditure, mostly in the form of perpetual bonds. They are special bonds with no maturity, but their use was limited in time, with governments issuing only bonds with a limited term to maturity lately.

Another important distinction is between secured and unsecured bonds. The former are backed by a lien on the borrower's assets. If the borrower defaults, those assets can be sold to pay off the bondholders.

Unsecured bonds, also called debentures, are only backed by the general ability of the corporation or other borrower to pay its bills. If the borrower goes bankrupt, debentures can't be paid off until secured bondholders are paid.

Investors in subordinated debentures are a step down in the line for reimbursement and do not get paid until after holders of senior claims get their money. However, they take priority over the equity holders of any type.

Zero-coupon bonds may be secured or unsecured. They are issued at a big discount from face value because they pay all the interest at maturity, with no payments along the way.

Down from the government level, there are bonds that are issued locally by local states or municipalities. They are called municipal bonds and are very popular on US markets. They are issued by state or city governments, or their agencies, and come in two principal varieties.

General obligation bonds are the first type of municipal bonds. It is a type of security that is backed by the full taxing authority of the government. Revenue bonds, the second type, are instead backed by the receipts from a specific source of revenue, from the project that has to be financed by the bond issuance. They are therefore not considered as secure as general obligation bonds.

The interest that is paid to holders of both revenue and general obligation municipal bonds is exempt from federal income taxes and, usually, income taxes of the issuing state as well.

Government bonds in the United States have denominations dependent on the time to maturity. The so-called treasury bills are those that mature in less than 1 year. Bonds with maturity between 1 and 5 years are called treasury notes, and the bonds with maturity longer than 5 years are called treasury bonds. All of them are debentures, backed by the full faith and credit of the federal government.

Another popular type of bonds in the United States are the agency securities, which are issued by various government-sponsored agencies. They are not technically backed by the US government, but still a moral obligation is assumed to exist, which presumably wouldn't let an agency issue fail (Fabozzi 2001).

Callable bonds are a type of bonds that can be called (redeemed) by the issuer before maturity. This is an option that is written on the bond for the benefit of the company or government issuing the bond.

The borrower or buyer of the bond is subject to the issuer's decision of recalling the bond or not at any time. A company might decide to call its bonds if, for instance, interest rates fell so far that it could issue new bonds at a lower rate and thus save money. When bonds are called for an amount higher than paid by the buyer at issuance, the government would charge taxes on the forced sale.

Convertible bonds are the other types of bonds with an option attached. In this case, the option is in favor of the bondholder. Convertible bonds in fact can be swapped for the same company's common stock at a fixed ratio of amount of bonds to number of shares of stock.

The convertible option makes many corporate bonds attractive to the investors. If the price of the stock increases enough after purchase of the bond, the option may be convenient to be exercised, with sometimes a consistent profit on the swap.

Example 5.3 Assume an investor buys ten convertible bonds issued by company A, at 100 € each. Each bond pays a coupon of 5% and can be converted into four shares of the company. At the time of bond purchase, the stock is selling at 20 €. Because break-even conversion price is

$$\frac{100}{4} = 25 \ \text{€}$$

It follows that 5 € is the price paid per share for the conversion privilege. If the stock raises above \$25, the investor can make a profit by converting the bonds to stock. If the price were to go to 30 €, one could turn the 1000 € bond investment into

$$40 \times 30 = 1200$$

worth of stock.

Because their fate is so closely tied to that of the stock price of the issuing firm, convertible bonds tend to be more closely in sync with the stock market than the bond market.

5.1.3 Knowing the Risks

Both government and corporate bonds represent a good source of income, sometimes granting a relatively high rate of return to the investor, while being a safe investment, especially when compared to stocks.

It is still important to know the potential risks and potential downturns associated with holding corporate and/or government bonds. There are several types of risk that are attached to an investment in fixed income.

Credit risk is the risk of loss due to the inability of the borrower to hold its obligations, by failing on its debt. The definition clearly states that whenever someone lends money to someone else, the operation is subject to some degree of credit risk.

Generally, the yield on the government bonds denominated in domestic currency is considered as a proxy of the risk-free rate in the country of issuance. This is because it is assumed that the government can raise taxes or create additional currency in order to redeem the bond at maturity.

The ability to repay its debt is crucial for a company, and it is normally translated into credit ratings by major rating institutions such as Standard & Poor's or Moody's. Ratings range from "AAA" for high credit quality investments to "D" for bonds in default. The investor puts a lot of trust on the judgments expressed by these agencies.

In case a company has a low credit rating, with troubles in repaying its obligations, the banks and other lending institutions may ask for higher interest rates before agreeing to give a loan (Brigo and Mercurio 2001).

This can have an adverse impact on the company's ability to satisfy its debts with current bondholders and will hurt existing bondholders who might have been looking to unload their positions.

The currency risk associated with fixed income investments is the risk that the value of the currency a bond pays out will go down compared to the amount in the holder's domestic currency.

A European investor may invest in US T bonds to get more currency risk exposure than by investing in some European bonds. The same is true for an American investor considering investment in European bonds to have more currency risk than US bonds.

A bond paying in a currency that does not have a history of keeping its value may not be a good deal even if a high interest rate is offered, given that the gain in interest will be most likely offset by the loss on the currency side.

Inflation risk is the risk that the value of the currency a bond pays out will lose purchasing power over time due to an increase in inflation. Investors expect some amount of inflation, so the risk is that the inflation rate will be higher than expected.

Some governments issue inflation-linked types of bonds, protecting investors against upper movements of the inflation rate. These bonds link both the interest payment and the face value to some inflation index.

There is an inverse relationship between interest rates and bond prices, with bond prices raising when the market interest rates go down. It happens because when interest rates are on the decline, investors try to capture or lock in the highest rates they can for as long as they can.

This can be achieved by scooping up the existing bonds that carry an interest rate higher than the market. The surplus of demand implies an increase in bond price. If the prevailing interest rate were on the rise, the reaction of investors would force bond prices down.

Example 5.4 An investor owns a bond that trades at par and has a yield of 5%. If the prevailing market interest rate raises to 6%, the investor will decide to sell the 5% bond to buy the bond at 6%, for a better interest. This drives the price of the 5% bond price down, below par.

A common issue for bond investors is the so-called reinvestment risk, which arises when the proceeds from the sale of some funds have to be reinvested at a lower rate than before, therefore worsening the position of the investor. One of the main ways this risk presents itself is when interest rates fall over time and callable bonds are exercised by the issuers.

The issuer of a callable bond can redeem the bond prior to maturity, by forwarding the principal payment to the bondholder, usually for a value that is slightly above the par value of the bond.

By having the bonds recalled, the bondholder might end up with a lot of cash which is difficult to handle because reinvestment opportunities may present refinancing risk, which can have a major adverse impact on an individual's investment returns over time.

This is why the investors in callable bonds receive a yield that is higher than on a similar noncallable bond. The extra yield compensates for the extra risk in the portfolio.

Active bond investors can attempt to mitigate reinvestment risk in their portfolios by staggering the potential call dates of their differing bonds. This limits the chance that many bonds will be called at once.

Every bond is a certificate of debt, and by buying one, an investor is lending money to the issuer of the bond. This money must be repaid at maturity, with some interest on top. In case of corporate bond, the possibility to cash back the investment relies on the full faith of the issuer.

Among the range of possibilities, the investor must also consider the default of the borrower. If this happens, the debt will not be repaid, and the credit risk involved in the bond purchase will unleash its effects fully.

As one means of analyzing the possibility of default, some analysts and investors determine a company's coverage ratio before initiating an investment. By analyzing the financials of the borrower, they can determine its operating income and cash flow and then weigh that against its debt service expense.

In the case of government bonds, the markets are usually very liquid, with many securities populating the markets and ease of trading. This is not the case of corporate bonds, in some cases.

If there is no sufficient volume of some kind of bonds in the market, investors are at risk of not being able to sell their bonds quickly due to a thin market with few buyers and sellers for the bond.

The low demand for a particular bond issue leads to high volatility of prices, with a potential adverse impact on the return of the bondholder's portfolio. By selling in a thin market, one may be forced to take a much lower price than expected to sell the position in the bond.

There are also minor types of risk that can affect an investment in fixed income. Event risk in particular is the risk that the firm issuing a bond will undertake a process that will change its structure.

Operations like leveraged buyout, merger and acquisitions and debt restructuring can increase the debt ratio, pushing the value of bonds down and increasing the overall riskiness of the firm.

It is possible to talk about event risk also in case of natural and industrial accidents happening to the business, as well as important governmental interventions that produce regulatory changes.

Recall that it is possible to measure the convexity of a bond, as the rate of change of the dollar duration of a bond. Convexity and modified duration combined together improve the estimate of price sensitivity to large changes in interest rates.

By definition, plain bonds with no option attached have positive convexity, while convertible bonds and other bonds with options embedded have negative convexity. It means that the bond value is subject to extension risk when interest rates rise and contraction risk when interest rates fall.

Mortgage-backed bonds present additional risks as well. Prepayment risk is the risk that, in case interest rates go down, the mortgage holders decide to refinance or repay their loans sooner, therefore creating an early return of principal to holders of the loans.

An extension risk on the other hand is the risk that rising interest rates will slow the assumed prepayment speeds of mortgage loans. This will delay the return of principal to the investors, so that they will not be able to reinvest at higher yields.

5.2 The Price/Yield Relationship

Learning Outcomes

- Describe the relationship between bond prices and yields.
- Define fixed income futures and their application to financial risk.
- Explain how yield shifts impact on portfolio immunization.

5.2.1 Bond Prices and Yields

There are several ways to calculate the interest accrued on an investment or loan, with the main distinction being between simple interest and compound interest. The simple interest is just paid at the end of accrual period, without being reinvested.

In this way, there is no extra interest earned on the matured interest, and the equation can be written as

$$I_S = L \times r \times \Delta t$$

where:

I_S is the amount of simple interest generated by the investment.
r is the annual interest rate applied on the nominal amount.
L_0 is the nominal amount on which the interest rate is applied.

In case of compound interest, the accrued interest at the end of compounding period is reinvested together with the capital, increasing the amount earned at the end of later periods. Define the final value of the investment, called the future value, compounded on t years:

$$L_t = L(1 + r)^t \tag{5.1}$$

The above equation is specific for annual compounding but can be generalized to allow for different compounding frequencies. Consider an interest rate that is compounded m times per year. In this case formula (5.1) is modified as

$$L_t = L\left(1 + \frac{r}{m}\right)^{tm}$$

For m tending to infinite, the highest compounding frequency is represented by continuous compounding, which can be approximated by

$$L_{\Delta t} = L\mathrm{e}^{r\Delta t}$$

Some financial applications require knowledge of the present value of an investment, when knowing its final value and the interest rate applied to the nominal amount. The formulas for compounding can then be inverted to get

$$L_t = \frac{L_t}{(1 + r)^t}$$
$$L = \frac{L_t}{\left(1 + \frac{r}{m}\right)^{tm}}$$
$$L = L_t\mathrm{e}^{-r\Delta t}$$

Example 5.5 A consumer wants to buy a car worth 25,900 €, and he is offered two different ways of payment. He can either pay the whole sum cash or go on a zero-interest loan for 1 year. The second choice is obviously the best but does not assume the seller offers a price discount for cash payment. If the discounted price is 23,610 €, this corresponds to a 10% discount on price. Now the convenience of choosing between the two forms of payment depends on the interest rate in the economy. At the 10% rate, it doesn't matter to buy the car from one seller or the other. But other interest rates generate different present values to be compared, therefore making one offer or the other the best one.

Table 5.1 Effective annual rate calculation for different compounding frequencies

Compounding	m	Effective annual rate	$r = 4\%$
Annually	1/1	$r = (1 + r/1)^1 - 1$	4.00%
Semiannually	1/2	$r_S = (1 + r/2)^2 - 1$	4.04%
Monthly	1/12	$r_M = (1 + r/12)^{12} - 1$	4.06%
Weekly	1/52	$r_W = (1 + r/52)^{52} - 1$	4.07%
Daily	1/365	$r_D = (1 + r/365)^{365} - 1$	4.08%
Continuous	$1/\infty$	$r_e = e^r - 1$	4.08%

The last column shows an example of the calculation applied to a nominal rate of 4%

An important concept related to the compounding frequency is the effective annual rate. Based on different frequencies, the same investment at the same rate can give different final values (Merton 1974).

Table 5.1 summarizes the effective rate accrued on some investment for the same nominal rate r compounded at different frequencies m and the calculation for the most common frequencies with an example in terms of numbers.

Clearly, the higher the compounding frequency, the higher the effective return earned on the investment. This is the reason for borrowers to usually prefer paying interest at lower compounding frequencies, while lenders prefer higher compounding frequency.

That is why yield is sometimes referred to as the IRR of an investment in bonds for $n = \frac{T}{\Delta t}$ periods, where T is the maturity.

The IRR is the interest rate r_y that solves the equation

$$B_0 = \sum_{i=1}^{n} \frac{C_i}{\left(1 + r_y\right)^i}$$

where:

C_i are the cash flows paid by the bond at each time i.
$n = t$ in terms of the notation used so far, in case of annual payments.

There is also the continuous compounding version of the IRR which can be expressed as

$$B_0 = \sum_{i=1}^{n} C_i e^{-r_y i} \tag{5.2}$$

For a bond paying periodic coupons to the investor, the payments are the intermediate coupons, plus the sum of last coupon and face value at the end, while for a zero-coupon bond, only the face value at maturity is paid with no intermediate payments.

Example 5.6 Consider now two bond yields above (11%) and below (8%) the average return. For a yield of 11%, the present value of the bond is (continuous compounding

$$B_0 = \sum_{i=1}^{7} 8e^{-0.11i} = 83.25$$

The present value of the bond corresponding to an 8% yield is

$$B_0 = \sum_{i=1}^{7} 8e^{-0.08i} = 98.31$$

The equation has no explicit solution for the yield and therefore cannot be inverted. The way to solve it is computationally or by adopting iterative trial-error procedures in order to identify the yield that gives the market price.

The procedure starts with a guess on the value of r_y as an input in formula (5.2). Then one must check whether the present value from calculation matches the actual price of the bond. The yield is then calibrated to get the match.

There is also an approximation equation that allows for a fair approximation of the yield to maturity of a coupon bond, as closed form solution. The equation is

$$r_y = \frac{C_i + \frac{B_{\mathrm{FV}} - B}{t}}{\frac{B_{\mathrm{FV}} + B}{2}}$$

where:

B_{FV} is the face value of the bond at maturity.
B is the market price of the bond.

As a note to avoid confusion, just have in mind that the notation B_{FV} is used to identify the value at maturity (face value) of the bond, as well as B_T, which is by convention 100 €.

Example 5.7 Consider a 7-year bond pays 8% coupon rate, face value 100 €, currently selling at 90 €. The average return per year is

$$r_y = \frac{8 + \frac{100-90}{7}}{\frac{100+90}{2}} = 9.92\%$$

Back to trial-and-error procedures for yield calculation, this is sometimes performed by bounding the right yield between two extreme values, one higher and the other lower than the actual yield.

The aim is to get closer to market price, just changing the guess on the yield appropriately, from a lower rate r_l, corresponding to a positive NPV_+, and a higher rate r_h, corresponding to a negative NPV_-. Then the yield to maturity is given by interpolation:

$$r_y = r_l + \frac{NPV_+}{NPV_+ - NPV_-}(r_h - r_l)$$

where:

NPV_+ is the positive net present value, from a low guess on the yield.
NPV_- is the negative net present value, from a high guess on the yield.

Example 5.8 Given the two bonds in Example 5.6, the NPV at 11% is

$$-90 + 83.25 = -6.75$$

NPV at 8% is

$$-90 + 98.31 = 8.31$$

The yield to maturity is then

$$r_y = 0.08 + \left(\frac{8.31}{8.31 - (-6.75)}\right)(0.11 - 0.08) = 9.66\%$$

which is a fair approximation of the initial calculation. Computing the yield numerically gives a value of $r_y = 10.06\%$.

Standard calculations for bond price and yield to maturity, as presented till now, imply the yield to maturity is the same for all time periods considered during the life of the bond.

But if one accounts for the fact that the payments happen at different points in time, it can be assumed that they should be discounted at different yields, meaning that each cash flow is discounted by a rate appropriate to its maturity. The present value of a bond in discrete compounding is given by

$$B_0 = \sum_{i=1}^{n} \frac{C_i}{(1 + r_i)^i}$$

and continuous compounding can be written as

$$B_0 = \sum_{i=1}^{n} C_i e^{-r_i i}$$

where:

r_i is the yield to maturity corresponding to time i.

The pricing done by choosing different rates for different maturities is more accurate, without making necessary assumptions on reinvestment rates. The term structure rates can be any current spot rates, expectations of future spot rates, expected inflation, liquidity, and risk premium.

When pricing between two maturities both forward in time, it is appropriate to work with forward yields, which indicate the expected spot yield at some date in the future and can be derived directly from spot rates.

By indicating the spot rate at time 1 by r_1 available for investing for 1 year, and the spot rate r_2 as the rate available now for investing for two periods, the forward rate between periods 1 and 2 is the rate implied for investing for a 1-year period in 1 year's time.

This can be generalized to any forward rate between time t_1 and t_2, which can be written as

$$r_{1,2} = \left(\frac{(1+r_2)^{t_2}}{(1+r_1)^{t_1}}\right)^{\frac{1}{t_1-t_2}} - 1$$

where:

r_1 is the spot rate at time t_1.
r_2 is the spot rate at time t_2.

Finally, it is worthwhile to mention the par yield, the theoretical coupon rate r_{FV} that makes the bond calculated price equal to par value. The calculation is made by reworking the bond pricing equation as

$$\begin{aligned} B_T &= \sum_{i=1}^{t} \frac{r_{\mathrm{P}} B_T}{(1+r_i)^i} + \frac{r_{\mathrm{P}} B_T + B_T}{(1+r_t)^t} \\ &= \sum_{i=1}^{t} \frac{C_{\mathrm{P}}}{(1+r_i)^i} + \frac{C_{\mathrm{P}} + B_T}{(1+r_t)^t} \end{aligned}$$

where:

B_T is the bond face value.
r_{P} is the par yield.
C_{P} is the coupon calculated on the par yield.

Example 5.9 Assume the spot rates in the term structure are

Year	Spot rate
1	6.00%
2	6.75%
3	7.00%

The par yield on a bond such that will be priced at par therefore is

$$100 = \frac{C_{\mathrm{P}}}{(1.06)} + \frac{C_{\mathrm{P}}}{(1.0675)^2} + \frac{C_{\mathrm{P}} + 100}{(1.07)^3}$$

The solution to the equation is $C_{\mathrm{P}} = 7.21$. Therefore, the par yield is $r_{\mathrm{P}} = 7.21\%$.

Once the par yield curve is obtained, it can be used for calculation on the coupon to be set on new bond issues and the relative value assessment.

5.2.2 The Yield Curve

Many theoretical models in financial economics are based on the assumption of constant interest rates. But in reality this is not true and interest rates may vary through time. This occurs primarily because inflation rates are expected to differ through time.

One way to find the variability of rates over time is to compare bonds with the same risk and plot on a graph their yields for different maturities. The relationship between interest rates (yields) and maturities is called term structure of interest rates, and the graph plotting it is called yield curve.

The curve can take different shapes according to the structure of the stochastic model for the short rate that is used to generate it. In general, it can take four different shapes, so as to be upward sloping, downward sloping, flat, or humped (see Fig. 5.1).

The upward shape is the most common and most observed on the market. It implies that short-term rates are lower than long-term rates, due to expectations that macroeconomic factors will push future rates to be higher (Litterman and Scheinkman 1991).

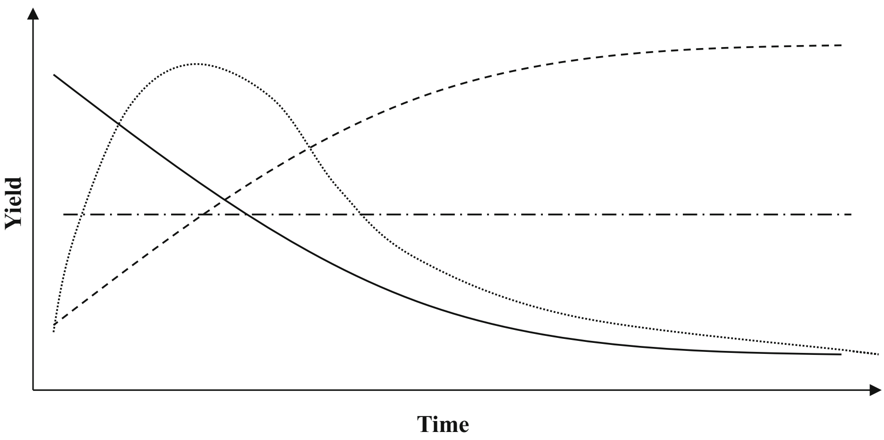

Fig. 5.1 The yield curves, as resulting from most common models of the interest rates, and observed empirically, can take different forms

The mechanism underlying such a result is that the investor has expectations of higher inflation and a tighter monetary policy to compensate that. As a consequence, investors demand a higher-risk premium. Such a risk is priced in the yield curve by demanding higher yields for longer maturities.

In case of a downward-sloped curve, interest rates imply expectations of future downturn in the economy. A flat yield curve indicates expectations on the economy have a negligible effect on the risk premium demand of investors.

A humped curve shows the demand of capital on intermediate maturities, a phenomenon observed in reality, where in recent years, on some markets, investors have demanded high volumes of long maturity instruments, leaving the intermediate maturities at lower demand, and consequently higher yields.

A complete term structure of interest rates must cover all the maturities between the shortest and the longest represented on the yield curve. Therefore, it is necessary to complete the structure by inferring the rates for the other maturities.

In general, fixed income markets are very liquid, and instruments are available to cover all maturities. However, in order to complete for missing maturities, a bootstrapping approach can be used.

The method consists in constructing some longer-term zero-coupon bonds by forming portfolios of traded coupon bonds. Zero-coupon yields can be derived from the prices of these bonds (Longstaff and Schwartz 1995).

Consider a simplified market with T bonds, one for each available maturity. As another simplification, let's assume that all bonds make payments at each maturity with identical payment dates. It is then possible to construct zero-coupon bonds for each maturity 1, 2, . . .T.

The bootstrapping of new rates over the maturity T goes through construction of appropriate discount factors $v_{t+1}, v_{t+2}, \ldots, v_{t+T}$ for all maturities considered. The discount factors are defined as the present value of a bond with face value 1, expiring at each maturity involved.

The next step is to derive yields from the discount factors, by building the yield curve up to time $t + T$. Recall the price equation for a coupon-paying bond:

$$B = \sum_{i=1}^{T} C_i e^{-y_i t_i} \simeq \sum_{i=1}^{T} \frac{C_i}{(1 + y_i)^i}$$

where:

C_i is the payment made by the bond at time i.
y_i is the yield corresponding to maturity i.
t_i is the time corresponding to maturity i. For annual compounding $t_i = i$.

Indicating the payment of bond $i = 1, \ldots, T$ at time $t + j$ $(j = 1, \ldots, T)$ with $C_{i,j}$ and the price of bond i with $B_{i,t}$, the discount factors must then satisfy the pricing kernel, as above, given by equations

$$\begin{pmatrix} C_{1,1} & C_{1,2} & \cdots & C_{1,T} \\ C_{2,1} & C_{2,2} & \cdots & C_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ C_{T,1} & C_{T,2} & \cdots & C_{T,T} \end{pmatrix} \begin{pmatrix} v_{t+1} \\ v_{t+2} \\ \vdots \\ v_{t+T} \end{pmatrix} = \begin{pmatrix} B_{1,t} \\ B_{2,t} \\ \vdots \\ B_{T,t} \end{pmatrix} \tag{5.3}$$

The nature of the bonds ensures that the matrix on the left-hand side of Eq. (5.3) is non-singular, with only one solution possible.

The next step consists in forming a portfolio of the T bonds, with elements zero-coupon bonds paying 1 at $t + j$. Denoting by $\alpha_{i,\ j}$ the units of bond i needed to replicate the zero-coupon bond maturing at $t + j$, it must hold

$$\begin{pmatrix} C_{1,1} & C_{2,1} & \cdots & \cdots & \cdots & C_{T,1} \\ C_{1,2} & C_{2,2} & \cdots & \cdots & \cdots & C_{T,2} \\ \vdots & \vdots & \ddots & & & \vdots \\ C_{1,j} & C_{2,j} & \cdots & \cdots & \cdots & C_{T,j} \\ \vdots & \vdots & & & \ddots & \vdots \\ C_{1,T} & C_{2,T} & \cdots & \cdots & \cdots & C_{T,T} \end{pmatrix} \begin{pmatrix} \alpha_{1,j} \\ \alpha_{2,j} \\ \vdots \\ \alpha_{j,j} \\ \vdots \\ \alpha_{T,j} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \tag{5.4}$$

Equation (5.4) gives a unique solution associated with the maturity $t + j$. By comparing it to Eq. (5.3), the equality

$$v_{t+j} = \begin{pmatrix} \alpha_{1,j} & \alpha_{2,j} & \cdots & \alpha_{T,j} \end{pmatrix} \begin{pmatrix} B_{1,t} \\ B_{2,t} \\ \vdots \\ B_{T,t} \end{pmatrix} = \sum_{i=1}^{T} \alpha_{i,j} B_{i,t} \tag{5.5}$$

can be derived, yielding the discount factor corresponding to the j-th entry.

By running Eq. (5.4) for all $j = 1, \ldots, T$, the values for all the zero-coupon bonds are obtained. Equation (5.5) can then be applied to get the corresponding discount factors.

Example 5.10 Two bonds are traded on the market. Bond A is a 2-year 4% coupon bond; 8% provides payments of 4 € in the first year and 104 € in the second year. Bond B is a 1-year zero-coupon bond providing a payment of 104 €. Assume the price of bond A is 97 €, while the price of bond B is 90 €:

$$\begin{pmatrix} 91 \\ 96 \end{pmatrix} = \begin{pmatrix} 5 & 105 \\ 104 & 0 \end{pmatrix} \begin{pmatrix} v_{t+1} \\ v_{t+2} \end{pmatrix}$$

The solution is $v_{t+1} = 0.96$ and $v_{t+2} = 0.91$.

5.2.3 Duration and Convexity

The cash flows of a bond follow a pattern that depends on the sensitivity of bond prices to changes in bond yields. There is an inverse relationship since more risky bonds, with higher yield, are cheaper to buy.

Another factor is the maturity of the asset. In general, the longer the time to maturity, the greater the sensitivity to changing interest rates, and the greater the yield to maturity, the lower the sensitivity to changing interest rates.

The sensitivity of the bond price to the yield is generally implemented into the duration measure. Duration measures the average time length for the cash flows of the bonds, and it is a good estimate of the sensitivity.

It is a time-weighted measure of the length of life of a bond, and a high duration implies a high volatility with respect to changes in the yield to maturity. Duration can be measured in some ways for different purposes: Macaulay duration, modified duration, and effective duration.

In continuous compounding, the Macaulay duration is defined as

$$D = -\frac{dB_y}{dr_y} \times \frac{\mathrm{e}^{r_y}}{B_y}$$

where:

r_y is the yield to maturity of the bond.
B_y is the price of the bond corresponding to the yield r_y.

The first-order derivative can be expressed as

$$\frac{dB_y}{dr_y} = -\mathrm{e}^{-r_y} \sum_{t=1}^{T} tC_t \mathrm{e}^{-r_y \Delta t}$$

where:

C_t is the payment made by the bond at time t.
Δt is the time interval between payments.

The equation for the duration follows as

$$D = \frac{1}{B_y} \sum_{t=1}^{T} tC_t \mathrm{e}^{-r_y \Delta t}$$

which, for annual coupon, in discrete notation becomes

$$D = \frac{1}{B_y} \sum_{t=1}^{T} \frac{tC_t}{\left(1 + r_y\right)^t}$$

If price normalization is taken out of the formula, the result is the so-called dollar duration, defined as

$$D_{\$} = \sum_{t=1}^{T} tC_t \mathrm{e}^{-r_y \Delta t}$$

The discounted payments divided by the bond price sum up to one as defined by

$$\sum_{t=1}^{T} \frac{C_t \mathrm{e}^{-r_y \Delta t}}{B_y} = 1$$

Therefore, each payment can be seen as a weight w_t, and one can write

$$D = \sum_{t=1}^{T} tw_t$$

Example 5.11 A 5-year bond with face value 100 €, selling at par, pays semiannual coupon rate 5%. If the yield of the bond is 5% as well, the Macaulay duration of the bond (discrete compounding) is given by

$$D = \frac{1}{100} \times \left(\frac{5}{1.05} + \frac{10}{1.05^2} + \frac{15}{1.05^3} + \frac{20}{1.05^4} + \frac{525}{1.05^5} \right) = 4.55$$

The duration of the bond is 4.55 years.

Putting it in words, each weight in the duration equation indicates the relative importance of that specific payment, with larger payments having a higher weight and smaller payments having a lower weight.

Through the Macaulay duration, it is possible to measure the percentage change in the price of a bond, for a minimal change in its yield. It is therefore a measure of elasticity.

A further step as a measure of riskiness of a bond is the modified duration. It can be directly derived by the Macaulay duration and is defined as

$$\mathrm{MD} = -\frac{dB_y}{dr_y} \times \frac{1}{B_y}$$

The equation describes the negative of the first-order derivative of the price, normalized by the bond price. As for the duration, modified duration can be expressed analytically in terms of cash flows of the bonds modified duration, as

$$\mathrm{MD} = \frac{\mathrm{e}^{-r_y}}{B_y} \sum_{t=1}^{T} tC_t \mathrm{e}^{-r_y \Delta t}$$

which can be discretized as

$$\text{MD} = \frac{D}{\left(1 + r_y\right)}$$

Modified duration informs about the percentage change in prices due to any specific change in percentage of the yield.

Example 5.12 Using the data in Example 5.11, the modified duration of the bond is given by

$$\text{MD} = \frac{4.55}{1.05} = 4.33$$

The modified duration of the bond is 4.33 years.

Duration measures are first-order approximations, therefore more accurate for small changes in the yields, and less accurate for large changes, due to the curvature of the bond price-yield relationship.

Both duration and modified duration work for changes of the yields in the context of flat yield curves. When the curve is not flat, or does not shift parallely, they are not efficient measures.

For more complex bonds, for example, those embedding an option, other measures must be used. Effective duration allows to analyze a bond with embedded option, given that Macaulay and modified duration are not applicable because of nonlinearity of the price changes:

$$\text{ED} = \frac{B_\text{l} - B_\text{h}}{2B_y \Delta r_y}$$

where:

Δr_y is change in yield.
B_h is price of the bond corresponding to the yield going up by Δr_y.
B_l is price of the bond corresponding to the yield going down by Δr_y.

One of the main properties of a zero-coupon bond is that its duration coincides with maturity, given there are no intermediate payments. It follows that the modified duration instead will be slightly lower than the zero-coupon bond's maturity.

In case of coupon bonds, the coupons act as partial intermediate repayments, so to decrease the overall risk and shorten the effective maturity of the cash flows. It follows that the greater the coupon, the lower the duration, because the bond is repaying back the investment faster than a similar bond with lower coupons.

Following the same logic, it is clear that the duration is lower for greater yield, given that payments are discounted at higher rate, with consequently a lower present value, that impacts on duration.

As a first-order derivative measure, the duration can capture a linear relationship between yield and bond price. But the relationship turns out to be nonlinear in practice. Therefore, in order to capture the sensitivity of bond prices to larger changes in yield, a second-order approximation is needed.

Convexity is a measure of the curvature of the value of a security or portfolio as a function of interest rates. Convexity is related to the second-order derivative of the price function.

By combining duration and convexity when measuring the relationship between bond prices and yields, it is possible to reduce substantially the approximation error in presence of the curvature.

The expression for the second-order derivative is

$$\frac{d^2 B_y}{dr_y^2} = \sum_{t=1}^{T} t^2 C_t \mathrm{e}^{-r_y \Delta t}$$

Convexity is defined as

$$\mathrm{CV} = \frac{d^2 B_y}{dr_y^2} \times \frac{1}{B_y}$$

which can be discretized as

$$\mathrm{CV} = \frac{1}{\left(1 + r_y\right)^2 B_y} \sum_{t=1}^{T} \frac{t(t+1) C_t}{\left(1 + r_y\right)^t}$$

Example 5.13 Using the data in Examples 5.10 and 5.11, the convexity of the bond is given by

$$\mathrm{CV} = \frac{1}{100 \times (1.05)^2} \times \left(\frac{5 \times 2}{1.05} + \frac{10 \times 3}{1.05^2} + \frac{15 \times 4}{1.05^3} + \frac{20 \times 5}{1.05^4} + \frac{525 \times 6}{1.05^5} \right) = 23.93$$

The convexity of the bond is 23.93.

By performing a Taylor series expansion on the second-order derivative, the approximation of the change in yield is given by

$$\frac{\Delta B_y}{B_y} = -D \Delta r_y + \frac{1}{2} \mathrm{CV} \left(\Delta r_y\right)^2$$

Effective convexity is the equivalent of effective duration for duration and can be calculated with the formula

$$\mathrm{ECV} = \frac{B_\mathrm{l} + B_\mathrm{h} - 2B_y}{2B_y \left(\Delta r_y\right)^2}$$

5.3 The Term Structure of Interest Rates

Learning Outcomes

- Describe the term structure of interest rates and the yield curve.
- Define and comment single-factor models of the short rate.
- Define and comment multifactor models of the short rate.

5.3.1 The Brownian Motion

A stochastic process can be defined to be Markovian when its value for next time step only depends on the current value, without taking into account the history of the process.

The past is not relevant to the future realizations of the process. The condition can be mathematically formulated as

$$\Pr(X_n = x_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2}, \ldots, X_0 = x_0) = \Pr(X_n = x_n | X_{n-1} = x_{n-1})$$

Example 5.14 There is an urn containing two black balls and one white ball. A ball is drawn in each of the three consecutive trials. There is no information about the color of the ball in the first trial, and the second trial yielded a black ball. It follows that there is a 50% chance to withdraw a black ball in the third trial, given that one black ball and one white ball are left. If there is information provided about the color of the balls in both the first and second trial, then it would be a 100% surety to guess the color of the ball in the third trial. It is clear that the probability distribution for tomorrow-ball's color is affected by the information about the past. This stochastic process of observed colors does not have the Markov property.

The stochastic movement of particles in a system was first observed by R. Brown in the nineteenth century. He let grains of pollen move suspended in the water, and, using a microscope, he observed their strange but regular zigzag path.

After the first formulation of the theory of Brownian motion, developed by Bachelier in the early 1900s, Einstein used a probabilistic model to explain the basic properties of the process.

He could in fact observe the behavior of water molecules concluding that the kinetic energy of fluids was acting on it, so that any molecule receives a random number of impacts from random directions, from other molecules, in any short period of time.

The Brownian motion, or Wiener process, can be conveniently seen as a limit of the random walks with infinitesimal steps (Musiela and Rutkowski 1997).

Recall that standard random walk is a sum process in which each step is a Bernoulli variable, getting values +1 or −1, with equal probability:

$$S_T = X_1 + X_2 + \ldots + X_T$$

where

$$X_t = \begin{cases} +1 & \text{with probability } 1/2 \\ -1 & \text{with probability } 1/2 \end{cases}$$

with expected value and variance given by

$$E(S_T) = 0$$
$$\mathrm{Var}(S_T) = T$$

It is possible to show that, in the limit for $n \to \infty$, the Brownian motion can be defined as the stochastic process W_t, such that

$$\Delta W = \varepsilon\sqrt{\Delta t}, \quad \varepsilon \sim N(0,1)$$

where

$$E(\Delta W) = 0$$
$$\mathrm{Var}(\Delta W) = \sqrt{\Delta t}$$

The basic properties of the Brownian motion are

$$W_0 = 0$$

$$W_t \sim N(0,t)$$

- The paths of the process are continuous function of time $t \in [0,\infty)$.
- For any set of time points s and t, such that $0 \leq s \leq t$, it holds that

$$\begin{aligned} E\left(W_t^2\right) &= \mathrm{Var}(W_t) + [E(W_t)]^2 = t + 0 = t \\ E(W_t - W_s) &= E(W_t) - E(W_s) = 0 - 0 = 0 \\ E\left[(W_t - W_s)^2\right] &= E\left(W_t^2\right) + E\left(W_s^2\right) - 2\mathrm{Cov}(W_t, W_s) \\ &= t + s - 2\min(W_t, W_s) = |t - s| \end{aligned}$$

- The covariance between the process at two different points in time is $\mathrm{Cov}(W_t, W_s) = \min(s,t)$
- Process increments on nonoverlapping intervals are independent, meaning:
  - The distribution of $W_t - W_s$ depends on $t - s$ only.
  - If $0 \leq u \leq s \leq t$, the random variables $W_t - W_s$ and W_u are independent.

The process path over a long interval T is such that $W_T - W_0$ is the sum of n small changes in Δt of the process, and it is equal to

$$W_T - W_0 = \sum_{i=1}^{n} \varepsilon_i \sqrt{\Delta t}$$

so that

$$\begin{aligned} &E(W_T - W_0) = 0 \\ &\mathrm{Var}(W_T - W_0) = n\Delta t = T \end{aligned}$$

From the basic properties above described, it is then possible to derive other properties allowing for calculus simplifications, when handling the Brownian motion. Consider the model expressed in continuous time, dW, and recall that, for $\Delta t \rightarrow 0$, the time interval becomes very small and equal to dt.

Also recall from basic properties that

$$\begin{aligned} &E(dW) = 0 \\ &E(dWdt) = E(dW)dt = 0 \\ &E\left(dW^2\right) = dt \end{aligned}$$

It follows that

$$\begin{aligned} &\mathrm{Var}\left(dW^2\right) = E\left(dW^4\right) - \left[E\left(dW^2\right)\right]^2 = 3(dt)^2 - (dt)^2 = 0 \\ &E\left[(dWdt)^2\right] = E\left[(dW)^2\right](dt)^2 = 0 \\ &\mathrm{Var}(dWdt) = E\left[(dWdt)^2\right] - [E(dWdt)]^2 = 0 \end{aligned}$$

The argument to derive the final important properties of the Brownian motion is that, if the above equations hold, and all the elements have a zero second-order moment, their expected and actual values coincide.

Moreover, since the square of dt is supposed to converge to zero even faster than dt, it is possible to conclude that

$$\begin{aligned} &dW^2 = dt \\ &dWdt = 0 \\ &dt^2 = 0 \end{aligned}$$

Now consider introducing a drift term in Δt, so to obtain a new Δx process, that can be defined as

$$\Delta x = a\Delta t + b\varepsilon\sqrt{\Delta t}$$

where a and b are constants, and

$$E(\Delta x) = a\Delta t$$
$$\mathrm{Var}(\Delta x) = b^2\Delta t$$

The model can obviously be expressed in continuous time and takes the form

$$dx = adt + bdW_t$$

which is called generalized Brownian motion.

An Ito process is defined as a generalized Wiener process in which a and b are not constant but dependent on x and t, respectively:

$$dx = a(x, t)dt + b(x, t)dW_t$$

The generalized Wiener process is the leading process in defining stock price movements in the markets but can also be used to model the behavior of the short rate of interest, as clarified in the next section.

5.3.2 Single-Factor Models

The classical interest rate models start from a process describing the behavior of the short rate, at infinitesimal time steps. The state variable is the instantaneous forward rate, and the short rate is modeled using a Brownian motion (Duffie and Singleton 1999).

The stochastic differential equation describing the dynamics of short rate follows an Ito process:

$$dr_t = m_{r,t}dt + s_{r,t}dW_t$$

where:

$m_{r,\, t}$ is a drift parameter dependent on both r_t and t.
$s_{r,\, t}$ is a diffusion parameter dependent on both r_t and t.

The single-factor models are characterized by having a single driving factor which is modeled in multiple possible forms, according to the form chosen for the drift and the diffusion. In fact, by varying the choice of the coefficients $m_{r,\, t}$ and $s_{r,\, t}$, the process can lead to different dynamics of the instantaneous rate.

Equilibrium models are those where the short rate is fully endogenous, and generated by the model, through equilibrium conditions of no-arbitrage, and without any external input.

The Vasicek model is a popular equilibrium model which is founded on a short rate modeled as

$$dr_t = a(b - r_t)dt + \sigma dW_t$$

As from the formula, the process is characterized by mean reversion, meaning that the rate aims in the long run at a fixed constant level, b, floating around it at a reverting speed a, which is constant as well. All parameters are nonnegative constants.

The main advantage of such an equilibrium model is the explicit form taken by the short rate, which is normally distributed. One downturn of the model is that in its form, it allows for negative rates as outcome.

In order to derive the analytical expression of the short rate, it is necessary to introduce the variable change:

$$x_t = r_t - b$$

The process in dx_t becomes

$$dx_t = -ax_t dt + \sigma dW_t$$

Consider another change of variable:

$$z_t = e^{at} x_t$$

The process becomes

$$\begin{aligned} dz_t &= ae^{at} x_t dt + e^{at} dx_t \\ &= ae^{at} x_t dt + e^{at}(-ax_t dt + \sigma dW_t) \\ &= e^{at} \sigma dW_t \end{aligned}$$

Integration on both sides gives

$$\begin{aligned} z_t &= z_0 + \int_0^t e^{as} \sigma dW_s \\ \rightarrow e^{at} x_t &= e^{a0} x_0 + \int_0^t e^{as} \sigma dW_s \\ &= x_0 + \int_0^t e^{as} \sigma dW_s \end{aligned}$$

so that

$$x_t = e^{-at}\left(x_0 + \int_0^t e^{as}\sigma dW_s\right)$$

Recall that the equality $x_t = r_t - b$ holds, so that the short rate can be calculated as

$$\begin{aligned} r_t - b &= e^{at}(r_0 - b) + e^{-at}\sigma\int_0^t e^{as}dW_s \\ \rightarrow r_t &= e^{at}(r_0 - b) + \sigma\int_0^t e^{-a(t-s)}dW_s + b \\ &= e^{at}r_0 + b(1 - e^{-at}) + \sigma\int_0^t e^{-a(t-s)}dW_s \end{aligned}$$

Generalizing the solution to any time interval $\Delta t = t - s$ gives

$$r_t = e^{a\Delta t}r_s + b\left(1 - e^{-a\Delta t}\right) + \sigma\int_s^t e^{-a\Delta t}dW_u$$

Through some complicated calculus, it is possible to show that every model for a short rate implies a specific solution for the price of the bond and term structure.

For example, in the case of the Vasicek model, the solution for the price at time t of a bond with face value equal to one, maturing at time T ($\Delta t = T - t$), is given by the formula

$$P_{t,T} = \Lambda_{t,T}e^{-\Theta_{t,T}r_t}$$

where

$$\Theta_{t,T} = \frac{1 - e^{-a\Delta t}}{a}$$

and

$$\Lambda_{t,T} = e^{\frac{\left(\Theta_{t,T} - \Delta t\right)\left(a^2 b - \frac{\sigma^2}{2}\right)}{a^2} - \frac{\sigma^2\Theta_{t,T}^2}{4a}}$$

Starting from the bond price is then possible to derive the term structure. In particular, the continuously compounded interest rate at time t for a term of $\Delta t = T - t$ can be extracted from the formula

$$P_{t,T} = e^{-r_{t,T}\Delta t}$$

So that the point $r_{t,\ T}$ which determines the term structure in that specific time interval is

$$r_{t,T} = \frac{1}{\Delta t} \ln P_{t,T}$$

In order to overcome the possibility of negative rates implied by using the Vasicek model, Cox, Ingersoll, and Ross (CIR) proposed a model incorporating a different form for the diffusion part, with the short rate defined as

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dW_t$$

The CIR equation allows only for rates that are bounded in the region of nonnegative numbers, by multiplying the diffusion term by the square root of the short rate itself. Bond prices in the CIR model have the same general form as in Vasicek's model, but the functions $\Lambda_{t,\ T}$ and $\Theta_{t,\ T}$ are given as

$$\Theta_{t,T} = \frac{2(e^{\gamma\Delta t} - 1)}{(\gamma + a)(e^{\gamma\Delta t} - 1) + 2\gamma}$$

and

$$\Lambda_{t,T} = \left[\frac{2\gamma e^{\frac{(\gamma+a)\Delta t}{2}}}{(\gamma + a)(e^{\gamma\Delta t} - 1) + 2\gamma}\right]^{\frac{2ab}{\sigma^2}}$$

where

$$\gamma = \sqrt{a^2 + 2\sigma^2}$$

The bond price and interest rate are again dependent on the short rate at time t. The value of the latter determines the level of the term structure but does not define the shape, which is only dependent on time.

The main disadvantage of equilibrium models is that there is no link between the output structure and the actual situation of the rates in the real market. The model parameters can of course be chosen and calibrated, but they can never fit reality very closely.

In the fixed income market, when large amounts of capital are employed, very small differences in the rates can generate very high differences in profits or losses. Therefore, for advanced investor requiring more accurate estimates, equilibrium models are not good.

The problem of fit can be overcome by letting some parameters vary deterministically with time. No-arbitrage models are designed to be exactly consistent with today's term structure of interest rates.

Ho and Lee (1986) described the simplest form of no-arbitrage model and the first to be proposed. The model uses the current term structure of rates as an input to the model, making a step further than equilibrium models.

The Ho-Lee process for the short rate is given by (Hull and White 1993)

$$dr_t = \theta_t dt + \sigma dW_t$$

where θ is nonrandom function of t and σ is, as usual, a nonnegative constant.

The analytical solution for the rate is

$$r_t = r_s + \sigma(W_t - W_s) + \int_s^t \theta_u ds$$

and the expression for the price of the bond is

$$P_{t,T} = \Lambda_{t,T} e^{-r_t \Delta t}$$

where

$$\ln \Lambda_{t,T} = \ln\left(\frac{P_{0,T}}{P_{0,t}}\right) + \Delta t F_{0,t} - \frac{1}{2}\sigma^2 t \Delta t^2$$

The model is interesting because of its intrinsic simplicity. However, the simplicity is accompanied by some disadvantages. The main drawback of the model is that it does not take into account for mean reversion of rates.

Mean reversion is defined as the tendency of the interest rates to turn around some average in the long term, at a speed and frequency that is variable. It is an empirically verified important feature of interest rates.

Therefore, for a model to be accurate, it must incorporate it. The solution has been given by the Hull-White model. It is also known as the extended Vasicek model, in that it is basically the no-arbitrage version of it. The process is

$$dr_t = (\theta_t - a r_t)dt + \sigma dW_t$$

In this case, the analytical solution to the model is

$$r_t = e^{-a(t-s)} r_s + \int_s^t e^{-a(t-u)} \theta_u du + \int_s^t e^{-a(t-u)} \sigma dW_u$$

and the price of the bond for the given short rate becomes

$$P_{t,T} = \Lambda_{t,T} e^{-\Theta_{t,T} r_t}$$

where

$$\Theta_{t,T} = \frac{1 - e^{-a\Delta t}}{a}$$

and

$$\ln \Lambda_{t,T} = \ln \left(\frac{P_{0,T}}{P_{0,t}} \right) + \Theta_{t,T} \frac{\partial f_{0,t}}{\partial t} - \frac{1}{4a^3} \sigma^2 \left(e^{-aT} - e^{-at} \right)^2 \left(e^{2at} - 1 \right)$$

5.3.3 Multifactor Models

One important limitation of one factor models is that, given their nature, they are accurate only for parallel shifts of the yield curve. The reality is that curves like the LIBOR, for example, often move steepening, so that more complicated models are required.

The use of a parallel shifting setting is, for example, not adequate to price fixed income derivatives maturing in a long term. The same works for derivatives written on long-term assets.

It is often observed that yields at opposite ends of the term structure move in opposite directions. This is a behavior that single-factor models cannot explain, and more factors are needed for the analysis.

The solution to the above problems is given by introducing multifactor models, as pioneered by Merton (1974) and Vasicek (1977), among others. The efficiency of these models is given by allowing for instantaneous correlation between bond returns to be not perfect, leading to higher flexibility of the term structure.

Using a multifactor model allows to establish the dependence between short rate and a vector of factors $\mathbf{x}$, as given by

$$r_t = \sum_{i=0}^{n} x_{i,t} \tag{5.6}$$

where

$$dx_t = \mathbf{A}(b - x_t)dt + \mathbf{\Sigma} dW_t, \quad x_t, b \in \mathbb{R}^n \tag{5.7}$$

where:

$\mathbf{A}$ and $\mathbf{\Sigma}$ are $n \times n$ matrices.

The solution to Eq. (5.6) is a Gaussian process in the form

$$x_t = e^{-\mathrm{A}t} x_0 + \int_0^t e^{-\mathrm{A}(t-s)} \mu ds + \int_0^t e^{-\mathrm{A}(t-s)} \mathbf{\Sigma} dW_s$$

As for the single-factor models, the bond prices are expressed in affine form, according to the functional form of the parameters. The high tractability makes Gaussian multifactor models a very good choice to capture different empirical aspects of the data.

A popular multifactor model is the multifactor CIR model, which is based on the independency among factors. The short rate satisfies Eq. (5.7), and the dynamics of each $x_{i,\ t}$ are derived from the single-factor CIR model and are given by

$$dx_{i,t} = a_i(b_i - X_{i,t})dt + c_i\sqrt{x_{i,t}}dW_{i,t}, \quad a_i, b_i, c_i > 0$$

There exist again functional forms for parametrized $A_{i,\ t}$ and $B_{i,\ t}$ that help to express the expectation of the discounting factor in an affine form, as defined by

$$E\left(e^{-\int_t^T X_{i,u}du}\right) = e^{A_{i,\Delta t}+B_{i,\Delta t}X_{i,t}} \tag{5.8}$$

It is then possible to derive the term structure, recalling that the short rate is given by the sum of the single factors, so that

$$\begin{aligned} E\left(e^{-\int_t^T r_u du}\right) &= E\left(e^{-\int_t^T \sum_i X_{i,u}du}\right) \\ &= \ldots \\ &= \prod_{i=1}^{n} E\left(e^{-\int_t^T X_{i,u}du}\right) \end{aligned}$$

Substituting for Eq. (5.8) yields the ZCB price as defined by

$$P_{t,T} = \prod_{i=1}^{n} e^{A_{i,\Delta t}+B_{i,\Delta t}X_{i,t}} = e^{\sum_i e^{A_{i,\Delta t}+B_{i,\Delta t}X_{i,t}}}$$

Another popular model is the Hull and White (2001) which represents the instantaneous forward rate as a linear combination of the stochastic variables x_t and y_t, each driving a specific feature in the shape of the yield curve.

The two stochastic variables follow the processes

$$\begin{aligned} dx_t &= -\lambda_1 x_t dt + \sigma_1 dW_1 \\ dy_t &= -\lambda_2 x_t dt + \sigma_2 dW_2 \end{aligned}$$

where:

σ_1 and σ_2 are the instantaneous volatilities.

The Brownian motions are correlated by some typically largely negative coefficient (close to -1), so that parallel moves of the yield curve are inversely correlated with the steepening of the curve.

5.4 Summary

Bond valuation is an important task for the calculation of the cost of debt of the firm, given that a large part of the debt of big corporations is nowadays issued in the form of bond issuance.

The fundamentals of bond valuation rely on the proper identification and understanding of the features that make a bond a unique type of debt security. Features like maturity, face value, and coupon payment are components that identify a bond.

There are several types of bonds, namely, government (treasury) bonds and corporate bonds. Government bonds can be sovereign bonds, issued by a country or federation, or municipal bonds, issued by local municipalities and states.

When investing in bonds, it is very important to be fully aware about the risk attached to them. Such a risk reflects the creditworthiness of the issuing party, and it is related to the return the bonds offer to the investor.

There is an inverse relationship between a bond's price and yield, since the latter is used to discount the cash flows of the bond at present value so to obtain the former. Furthermore, the relationship is convex, defined by a curve.

The yield curve describes the relationship existing on the real market between the maturity of the traded bonds and the yield they offer. The term structure of interest rates is the mathematical description of such a relationship.

Duration and convexity are risk measures that define the intrinsic riskiness of a bond, due to its maturity and the intermediate payments it offers, when present. The convexity adjustment allows for a better approximation of the bond price given the yield.

The term structure of interest rates is mathematically defined as the relationship between the bond yield and the maturity. It is therefore a complex relationship that depends on the assumptions made on the short rate (continuous).

Several models use the above assumptions to derive a closed form solution for the yield and the price of the bond associated to some level of the short rate. Such a derivation can be obtained through either a single-factor or a multifactor model.

Problems

1. Explain what is meant by basis risk when futures contracts are used for hedging.
2. Which bond's price is more affected by a change in interest rates, a short-term bond or a longer-term bond, being all the other features fixed? Why?
3. Provide the definitions of a discount bond and a premium bond. Give examples.
4. All else equal, which bond's price is more effected by a change in interest rates, a bond with a large coupon or a small coupon? Why?
5. What is the difference between the forward price and the value of a forward contract?
6. Someone argued that airlines have no point in using oil futures given that the chance of oil price being lower than futures price in the future is the same as the chance of it being lower. Discuss this.

7. A futures price can be assimilated to a stock paying a dividend yield. What is the dividend yield in the futures case?
8. The annual effective yield on a bond is 7%. A 5-year bond pays coupons of 5% per year in semiannual payments. Calculate the duration.
9. Calculate the modified duration and convexity of the bond in Exercise 8.
10. Prove that the duration of a portfolio of many assets is the weighted average of all durations of the single assets.
11. Consider the following portfolio:

Bond	Coupon	Maturity	Par value	Price value	YTM
1	7.0	5	10,000,000	9,209,000	9.0%
2	10.5	7	20,000,000	20,000,000	10.5%
3	6.0	3	30,000,000	28,050,000	8.5%

Determine the yield to maturity of the portfolio.

12. Consider the two bonds in the following table:

	Bond A	Bond B
Coupon	8%	9%
Yield to maturity	8%	8%
Maturity	2 years	5 years
Par	100.00 €	100.00 €
Price	100.00 €	104.055 €

(a) Compute the duration and modified duration for the two bonds.
(b) Compute the convexity for the two bonds.

13. Recall the two bonds in Exercise 4.

(a) Repeat the calculations of duration, modified duration, and convexity, using shortcut formula, by changing the yields by 0.2%.
(b) Compare the results with those found in Exercise 4 and comment.

14. An investor holds 100,000 units of a bond whose features are summarized in the following table. He wishes to be hedged against a rise in interest rates.

Maturity	Coupon rate	Yield	Duration	Price
18 years	9.5%	8%	9.14	114.18 €

Characteristics of the hedging instrument, which is here a bond are as follows:

Maturity	Coupon rate	Yield	Duration	Price
20 years	10%	8%	9.49	119.79 €

Coupon frequency and compounding frequency are assumed to be semiannual. YTM stands for yield to maturity. The YTM curve is flat at an 8% level.

(a) What is the quantity of hedging instrument that the investor has to trade? What type of position should the investor take on the hedging instrument?
(b) Suppose that the YTM curve increases instantaneously by 0.1%. Calculate the corresponding new price for the two bonds.

15. Consider the two bonds in Exercise 7.
    (a) When YTM curve increases instantaneously by 0.1%, what happens to the portfolio in terms of profits or losses when the portfolio is not hedged? What if it is hedged?
    (b) If the curves shift by 2% instead, how does the answer to point a changes?
16. A bank is required to pay 1,100,000 € in 1 year. There are two investment options available with respect to how funds can be invested now in order to provide for the 1,100,000 € payback. First asset is a noninterest-bearing cash fund, in which an amount x will be invested, and the second is a 2-year zero-coupon bond earning the 10% risk-free rate in the economy, in which an amount y will be invested.
    (a) Develop an asset portfolio that minimizes the risk that liability cash flows will exceed asset cash flows.
17. What position is equivalent to a long forward contract to buy an asset at K on a certain date and a put option to sell it for K on that date?
18. How can a forward contract on a stock with a particular delivery price and delivery date be created from options?

Appendix: Principal Component Analysis of the Term Structure

The term structure can be alternatively described by using a principal component analysis (PCA). The changes in the term structure (ΔTS), by means of principal components x_i, can then be defined by

$$\Delta TS = (\Delta x_1, \Delta x_2, \ldots, \Delta x_n)$$

Knowledge of matrix calculus is needed in order to apply the method. There is a unique change in the key rates for each realization of the principal components, where the latter are linear combinations of changes in the interest rates, given as

$$\Delta x_i = \sum_{i=1}^{n} \eta_{ij} \Delta y_j, j = 1, 2, \ldots n$$

where:

η_{ij} are the principal component coefficients.
y_j is the yield corresponding to maturity j.

Each component explains the maximum percentage of the total residual variance not explained by previous components. The matrix of zero-coupon rates is symmetric with m independent eigenvectors, corresponding to m nonnegative eigenvalues. Looking at eigenvalues in order of size, the highest eigenvalue corresponds to a specific eigenvector, whose elements are identified the coefficients of the first principal component.

The second highest eigenvalue corresponds to another specific eigenvector, whose elements are identified as the coefficients of the second principal component. And so on, for all eigenvalues.

So the variance of each component is given by the size of the corresponding eigenvalue, and the proportion of total variance of the interest changes explained by the i-th principal component is

$$\sigma_{y,i}^2 = \frac{\lambda_i}{\sum_{i=1}^{m} \lambda_i} \tag{5.9}$$

From the condition of independency of eigenvectors, it follows that the matrix of coefficients η_{ij} is orthogonal, so that its inverse corresponds to the transpose. Equation (5.9) can then be inverted, to get the interest rates, as

$$\Delta y_j = \sum_{i=1}^{n} \eta_{ij} \Delta x_i, j = 1, 2, \ldots n$$

From how the model is built, it is clear that lowest eigenvalues play a very little role in determining the changes in interest rates. Therefore, it is possible to reduce the dimensionality of the model to the m highest eigenvalues, as given by

$$\Delta y_j = \sum_{i=1}^{m} \eta_{ij} \Delta x_i + \varepsilon_i$$

where:

ε_i is an error term due to the approximation from reduced dimensionality.

The first k components are then able to give a sufficiently accurate approximation of the changes in interest rates. The portfolio sensitivity to these components can be used to define the IRR profile.

Difference variances for each principal component are implied by the model, with corresponding even (i.e., unitary) shift in all components making them not equally likely.

A further step involves giving to each factor a unit variance, so to make changes in each factor comparable. Again from matricial calculus, the unit variance is obtained by multiplying each eigenvector by the square root of the corresponding eigenvalue, so that the model gets the form

$$\Delta y_j = \sum_{i=1}^{m} \left(\eta_{ij} \sqrt{\lambda_i} \right) \frac{\Delta x_i}{\sqrt{\lambda_i}} + \varepsilon_i$$

so that in an equivalent equation, the product of eigenvalue and eigenvector is isolated. The new factor loading in parenthesis measures the impact of one standard deviation move in each principal component.

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6 Equity Valuation

Together with the debt capital, another important part of the financing of a firm is the equity capital, a source of financing that is based on cash inflows provided by the ownership of the company or by retention of profits.

When the company cannot have access to debt, as a cheaper source of capital, or in general the management is not keen to increase the leverage of the capital structure, equity becomes the primary source of funding for every business.

The analysis of equity capital starts from the understanding of how equity markets work and how the equity capital is priced, in order to make a first distinction with the debt capital.

The valuation of equity capital is one of the most challenging tasks in corporate finance and involves different types of models, according to what is the information available at the time of valuation.

Like in the case of debt, equity capital can take many forms, from the simplest common stocks to more structured instruments like preferred stocks and more. The differences among those are important to be understood, to take advantage of them.

The case of common stock valuation is a foundation element of correct company valuation and equity pricing, and that makes the chapter of major importance for understanding the following parts of the book.

After studying this chapter, you will be able to answer the following questions, among others:

- What is market microstructure, and how does it affect the price formation process?
- How can the price formation model be described mathematically?
- What are the differences between different types of liquidity measurable on the market?
- What are the main models for common equity pricing? In what way do they differ from each other?

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- What are the other sources of equity, and how do they differ from common stocks?

The first section of the chapter is dedicated to the market prices, analyzing the price formation process, and describing liquidity. The second section deals with the valuation of equity capital by introducing the dividend discount model and the free cash flow model. The last section focuses on the non-common sources of equity, including preferred stocks, warrants, and convertibles.

6.1 The Stock Market

Learning Outcomes

- Define market microstructure.
- Describe the price formation process in financial markets.
- Explain the difference between funding liquidity and market liquidity.

6.1.1 Market Microstructure

In finance, the branch analyzing price formation mechanisms and market kernels is called market microstructure. As a field, it focuses on how the working process of a financial market affects the level of prices.

The analysis of market features, and their impact on the various market variables, like prices and volumes of transactions, is the core aim of the field and allows for a better understanding of how markets process the available information.

Standard transactions on regulated markets involve the transfer of an asset from one investor to another. The operation involves trading costs having an impact on the final price of the asset (Ball et al. 2011).

One of the main contributions of market microstructure is to establish a connection between transaction costs associated with trading and the bid-ask spread on the market, which itself depends on the asset liquidity.

Depending on how the transactions take place, it is possible to identify several types of market structure. Sometimes different types are mixed, and the differences depend on how transactions take place. A first important distinction is between auction markets and dealer markets.

Pure auction markets are those where the investors trade directly with each other without the intervention of dealers. In a call auction market, on the other hand, trading happens at specific times, when the securities are called for trading. In such a market, investors place orders, specifying desired price and quantity of the target security, and trading happens at specific times following specific rules.

Another type of auction market is the continuous auction market. As opposed to call auction, such a market is characterized by investors trading on orders placed at an earlier time by other investors.

The second type of market, the dealer market, is made of major dealers placing bid and offers to be answered by investors. It is therefore a market characterized by market makers, setting the initial prices that will adapt to the price formation process.

Besides the distinction among markets, it is also possible to distinguish between different types of orders to be input in the market, depending on the mechanism of order execution and satisfaction (Almgren and Chriss 2000).

A limit order sets minimum and maximum price for the trade to take place. A limit order to buy sets a maximum price, above which the investor will not accept to buy. A limit order to sell, on the other hand, sets a minimum price, below which the investor is not willing to sell the security.

The best limit order to buy results in being the lowest higher price in a centralized continuous auction market, while the best limit order to sell is the higher price. Both establish the market level, and the quantities at those prices represent the depth of the market.

Example 6.1 Consider an investor who wants to sell 18,500 shares of some stock A at no less than €30 dollars per share, but only if the whole quantity can be traded at once, in the current session. Such an investor should then issue an all-or-nothing, day order to sell the shares.

A particular type of order is the stop-loss, which can be placed to buy or sell when the asset price reaches a predetermined threshold. In this way the trader experiencing a bullish trend will manage to limit the losses.

The technique is to usually set the stop-loss price at a level right below the price paid for the asset. Such an order gives the advantage that monitoring of the asset price is not needed, once the limit has been set. It is therefore an efficient way to trade in case of long absence from the market, at the desired price (Handaa et al. 2003).

The disadvantage is that sometimes the stop-loss price could be activated by a sudden and temporary shock on the asset price, which creates a short-term fluctuation not reflected by the real trend of price.

Example 6.2 An investor can buy a stock for €40.00 and immediately after that enter a stop-loss order for €38.50, so that the stock will be sold at market price in case it falls below the stop-loss price. This limits the loss on the position to

$$40.00 - 38.50 = €1.50$$

with no need of further action after the stop order has been sent.

The stop-loss order automatically turns into a market order when the threshold is reached. Technically the final selling price could be different from the stop price, especially in fast-moving markets, due to the sudden changes.

Regarding the orders they place, it is also possible to set distinctions among investors. There are active traders, generally placing market orders and interested in continuous trading. Passive traders, on the other hand, normally issue limit orders, to make the best out of limited transactions, therefore earning higher margins from active traders.

A popular type of trading is buying on margin, which involves borrowing cash from some broker and using it to buy assets. The strategy clearly leverages the position of the investor, increasing the risk, but allows to trade on assets that would not be affordable otherwise (Bangia et al. 1998).

When opening a margin account, an investor is required to place a minimum initial investment, as minimum margin. The account becomes operational, and the investor can borrow a certain percentage (initial margin) of the purchase price of a stock.

The percentage of stock price that can be borrowed is usually 50%, with maturity chosen by the investor. When the position is closed, and the stock is sold on the market, the proceedings go to the broker up to full payment of the loan.

The marginable securities on the account are collateral of the loan, and some interest must be paid to the broker. Interests are accrued on the part of the loan which is not reimbursed, and increase as debt increases.

Example 6.3 An investor deposits an amount of €1000 in a 40% margin account, so that an investment of €1800 can be afforded. If the investor buys €500 value of stocks, the amount left on the margin account is €1300. The investor has used only a fraction of the equity in the account, meaning that the loan is not active yet, so no interest is due. When securities are bought for more than the equity of €1000, the loan position starts, and interests are accrued on it.

A margin agreement also includes maintenance margin, to be paid on top of the initial margin, as the equity required on the margin account after each trade. If the equity in the margin account falls below the maintenance margin, a margin call is issued, so that the investor must close the position in the stock or add more cash to the account.

Example 6.4 Referring to Example 6.3, assume the full €1800 available to the investor is used to buy stocks, so that the whole loan is used. If the market value of the securities goes to €1300, the value of the equity falls from the original €500 to

$$1300 - 1000 = €300$$

Assuming a maintenance margin is 25%, the amount of required equity to be maintained is given by

$$1000 \times 0.25 = €250$$

Since the equity is higher than this, the situation is fine. If the maintenance margin was 40% instead, the amount of maintained equity would be

$$1000 \times 0.40 = €400$$

which is higher than the €300 equity. In this case, a margin call will be issued by the broker, and the investor will have to restore an appropriate level of equity.

The structure of the market is such as to accommodate the needs of the many different types of investors. From the small investor to the big player, the market must specify the order in which transactions are executed.

To do so, some markets, for example, adopt price criteria, giving priority to orders with the best price and secondary priority to the order posted first at a given price. Some other markets modify secondary priority rules to accommodate large transactions.

Example 6.5 The current trading price of a futures contract is bid at €134.10 and asks €134.35 cents. There is a trader A who is the bidder with time precedence at that price. In order to buy at €134.10, a trader B must wait until investor A trades, or he can gain precedence by improving the bid to €134.15. Trader B would then have price priority over his bid and time precedence over all subsequent bids at 134.15. If trader A then wants to reclaim his precedence, he would have to bid higher at €134.20. Time precedence encourages traders to jump over each other's prices with improved prices.

The whole trading process can be divided in four steps, from information processing to clearing. In the first step, the information about past and current prices is made available, followed by brokers taking orders from investors and forwarding them to the market.

The third step is the actual execution of the orders through kernel processing, while the last step is the clearing of orders which takes place comparing the transaction orders of buyers and sellers in order to clear them.

The whole process results in a bid-ask spread on the market, representing the difference between asking price and offer price. It shows the trading prices available to investors, and it is a reliable proxy of the liquidity of the market.

When dealers place bid or ask prices, they give an option to the market investors to trade at this actual bid and ask, before it changes according to new information hitting the market.

Example 6.6 Consider an actively traded asset A and a rarely traded asset B. The closing price of the most liquid asset A is more likely to reflect new information hitting the market, rather than the price of asset B. The returns of asset B will reflect the new information by showing autocorrelation, due to the fact that asset B has a zero return in periods of non-trading, reverting to the cumulative mean return when trading is active. Thus, negative autocorrelation arises due to the mean reversion.

Market structure is affected and determined by several important factors, mainly regarding liquidity. When a market is liquid, the elasticity is such that there are no large changes in price due to shifts in demand. When a market is less liquid instead, the bid-ask spread is much more sensitive to shifts in demand.

Transparency is another factor related to liquidity, in that it resembles the amount of information held by market agents. If a market can communicate prices and spreads in real time to both buyers and sellers, then it is considered highly transparent.

The features of a microstructure are generally defined by econometric issues, given that the trading kernel most commonly consists of a series of discrete events in continuous time, thus defining a point process.

In a perfect market, the price of an asset would be determined at the equilibrium between demand and supply. The reality is not that straightforward because every market has frictions that must be taken into consideration.

Market price of an asset normally reflects all the expenses involved in its trading, so as to carry all the information available at the time of trading. Consequently, the bid-ask spread reflects all these additions, therefore widening, as they increase.

When natural players are not present in the market, investors are obliged to deal with market makers in order to get a quick transaction. That generates illiquidity on the market.

In fact, the market makers bear the risk of price fluctuations while holding the asset in inventory, so they must be compensated, widening the bid-ask spread and reducing the profitability for the small investors.

The role of information for market liquidity is also crucial, and big players generally tend to benefit from the most complete information about fundamental values on the markets.

Since the price in equilibrium reflects all the available information, the asymmetry in information generates a mismatch in the marginal prices for different types of investors, giving rise to disequilibrium.

6.1.2 Price Formation

In the past years, up till recent ones, financial markets have set new rules for the management of trading orders on the exchange. Thus order-driven markets are the focus of recent attention from practitioners.

The development of electronics limits order book trading platforms in virtually all of the market centers in the world. And that has improved the efficiency of market transactions in terms of speed and effectiveness.

The matching of order on the two sides of the order book is not an easy task, given that in general the high-valuation investors are willing to buy from the low-valuation shareholders, in order to gain from the price difference.

Therefore, low-valuation investors have a problem from not gaining anything from the deal. And on the opposite side, the same concept holds for low-valuation investors trying to buy from low-valuation shareholders.

Among the several models describing price formation in stock markets, a popular one is the Foucault's model. The assumptions underlying the model are continuous trading with a single risky asset, with investors trading one share of it sequentially through market or limit order.

The market is populated by two groups of investors, giving a different value to the asset. One group assigns a high value V_h while the other assigns a low value $V_l < V_h$. High-value and low-value investors populate the market with proportions h and l, respectively.

Investors are risk-neutral and maximized their expected utility. For a buy order processed at price b, it is given by

$$E(u) = \eta(V_i - b), \quad i = h, l$$

where:

η is the probability of execution of the order.

Similarly, the expected utility from a sell order at a specified price is

$$E(u) = \eta(b - V_i), \quad i = h, l$$

In the absence of a trade, the utility is normalized to zero.

Private information is scarce and only available to a share δ of the investors. This type of information on the asset value is worth to the investors a high value H_+ or a low value H_- with 50% probability each.

Consider a game of mutual strategies. Traders are given an optimal strategy, given the strategies of other traders. Each trader must choose a type of order (market vs. limit) and (for limit orders) the bid or ask order placement price.

Optimal bid price b^* and offer price a^* define the equilibrium in which a counterparty in the next period is induced to trade at these prices via a market order. The expected utility of a limit buy order placed at bid price b can then be rewritten as

$$E(u) = (1 - k)[(1 - p)(V_h - b) - pH]$$

where:

$p = \frac{\delta}{2}$ is the proportion of informed investors weighted by the 50% probability.

When a buyer is not informed and the utility from the two different trading methods is the same, it makes no difference to make a market order or a limit order, as defined by

$$V_h - a_m = (1 - k)[(1 - p)(V_h - b) - pH]$$

where:

A_m is the market ask price.

The optimal ask price A^*, sufficient to induce an uninformed coming buyer to trade via a market order, can be then expressed as

$$a^* = V_h - (1 - k)[(1 - p)(V_h - b) - pH]$$

Specular to it, it is possible to write the optimal bid price B^* that will induce an uninformed coming seller to trade via market order, by the formula

$$b^* = V_l + k[(1 - p)(a - V_l) - pH]$$

There exist parameter values for V_h, V_l, H, δ, and k for which equilibrium bid and ask prices are given by

$$\begin{aligned} a^* &= \lambda V_l + (1-\lambda)(V_h - qH) \\ b^* &= \mu V_h + (1-\mu)(V_l + qH) \end{aligned}$$

where

$$\begin{aligned} q &= \frac{p}{1-p} \\ \lambda &= \frac{1-k(1-p)}{1-k(1-k)(1-p)^2} \\ \mu &= \frac{1-(1-k)(1-p)}{1-k(1-k)(1-p)^2} \end{aligned}$$

6.1.3 Funding vs. Market Liquidity

An important distinction to be made when talking about liquidity is between funding liquidity and market liquidity. The distinction is important to understand the link between corporations and financial markets at full.

Funding liquidity can be defined as the ability to settle obligations with immediacy, and a financial institution is said to be liquid when it can face obligations in time. Funding liquidity risk is the possibility that an institution will not be able to meet its obligations over a specific time horizon (Brunnermeier and Motohiro 2009).

There is a difference between the point in time when a liquidity event happens and the risk associated with that event. Risk outcome in fact can take infinitely many values according to the distribution of future outcomes.

Consider banks as an example. A bank in fact is considered liquid if at each point in time the outflows of central bank money are smaller than the sum of inflows and stock held by the bank.

The net liquidity demand (NLD) indicator measures the fulfilment of the above conditions, to monitor liquidity, and can be calculated as

$$D_{\mathrm{NL}} = C_{\mathrm{OUT}} - C_{\mathrm{IN}} - M$$

where:

C_{OUT} is the sum of the outflows at a specified time.

C_{IN} is the sum of known inflows.

M is the stock of central bank money.

In order for a bank to be liquid, the money demand must be completely filled, which is allowed by the existence of an interbank market, together with deposits and other liabilities. That means the following inequality must hold

$$D_{\mathrm{NL}} \leq w_D L_D + w_{\mathrm{IB}} L_{\mathrm{IB}} + w_A A + w_{\mathrm{CB}} L_{\mathrm{CB}}$$

where:

L_D is the amount borrowed from new depositors.
w_D is the price of new deposits.
L_{IB} is the amount borrowed from interbank market.
w_{IB} is the price of interbank market funds.
A is the amount of assets sold.
w_A is the price of assets sold.
L_{CB} is the amount borrowed from central bank.
w_{CB} is the price of central bank funds.

As from the model, there are basically two stochastic elements driving the funding liquidity risk. One is the future evolution of the NLD, and the other is the future level of liquidity priced (Drehmann and Nikolaou 2010).

In particular, when NLD is negative, there is an excess of liquidity supply, to be sold out to the market. On the other hand, when NLD is positive, and cannot be funded in any way, the bank becomes illiquid.

The theory of market microstructure is useful in the analysis of liquidity and illiquidity sources. There are three transactional properties governing the relationship between market microstructure and liquidity, namely, tightness, depth, and resiliency (see Fig. 6.1).

Tightness is the cost of closing a position over a short period of time, and it is defined as the difference between the bid and the ask prices. Depth is the volume of trades that is possible to process without affecting the price of assets. Resiliency is an elasticity measure, defined as the speed at which the price converges to the liquidation value of the underlying commodity.

There are specific values for the properties in case a market is liquid, and that helps in identifying illiquidity issues, when present. In a liquid market, for example, tightness approaches zero, meaning the bid-ask spread is minimized (Ernst et al. 2009).

The depth of a liquid market is small enough to not affect asset prices, and resiliency is high enough to ensure that prices eventually approach the underlying value, so as to avoid arbitrage opportunities.

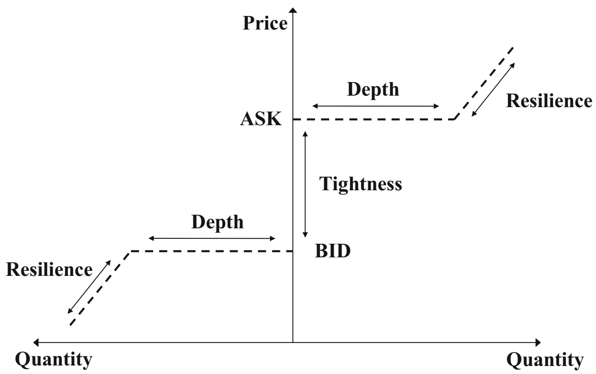

Fig. 6.1 The scheme resembles the properties of a market, in terms of liquidity

6.2 Common Equity

Learning Outcomes

- Learn how to use dividends for equity valuation.
- Apply the most common types of dividend discount models.
- Apply the free cash flow models to equity valuation.

6.2.1 Dividend Discount Models

The cost of equity is essentially represented by the returns demanded by the investors in order to invest in the company. It is the reward granted in exchange to the capital inflows provided by the buyers of corporate shares.

It is therefore an important part of the corporate capital structure, and the estimation of the cost of equity represents a major stake of corporate finance literature. The cost of equity can be estimated using models based on either dividend payments or the cash flows.

The first method is the dividend growth model that measures the stock price and capital to be raised by using the information about the dividends paid out to shareholders by the company.

The model can then be used to estimate the cost of equity, and this model can take into account the dividend growth rate. The equation sheet for the Paper F9 exam will give the following formula:

$$P_0 = \frac{D_0(1+g)}{r_e - g} \tag{6.1}$$

where:

- D_0 is the current dividend.
- g is the expected dividend future growth rate.
- r_e is the cost of equity.

As can be observed, the equation aims to predict the ex dividend market price of the share by processing the dividend today, the expected future dividend growth, and the appropriate cost of equity.

The numerator indicates the expected dividend in one period, due to the growth g. It is possible to rework Eq. (6.1) in order to isolate the cost of equity. The result is the formula

$$r_e = \frac{D_0(1+g)}{P_0} + g$$

Since all the terms on the right-hand side are known at time zero, for a listed company, the calculation of the cost of equity is straightforward. When other information is not available, the future dividend growth is assumed to continue at the recent historical rate.

Example 6.7 A company is about to pay a dividend of €2.50 on a stock whose market price is €27 (cum dividend). The historical dividend growth rate, which is expected to continue in the future, is 3%. The estimated cost of capital is

$$r_e = \frac{2.50(1 + 0.03)}{27.00} + 0.03 = 0.1254 = 12.54\%$$

To improve the accuracy of the measurement, one must however use the ex dividend price of the stock that can be calculated as the cum dividend market price less the impending dividend.

Example 6.8 The exdividend market price in the previous example is

$$P_{0,Ex} = 27.00 - 2.50 = €24.50$$

The cost of equity is therefore given by

$$r_e = \frac{2.50(1 + 0.03)}{24.50} + 0.03 = 0.1351 = 13.51\%$$

Stock valuation is based on the general assumption that if a market is populated by rational investors, the value of the stock today represents the present value of all future cash flows to be accrued in the future.

By recalling the time value of money, one can discount the future cash flows to present value and obtain the price today of an asset. Such a price is the intrinsic value of the stock because it is the value of the stock that is perceived based on all available information.

The buyer of a stock usually expects to get two types of cash flows from it. First of all, there is a dividend paid out during the holding period; plus there will be an expected price to be cashed when the stock is sold back on the market. Since this expected price is itself determined by future dividends, the value of a stock is the present value of dividends through infinity:

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + r_e)^t}$$

where:

D_t is the expected dividends per share.

r_e is the cost of equity.

The model is based on two main inputs, namely, the expected dividends and the cost of equity. The expected dividends can be obtained by making some assumptions about expected future growth rates in earnings and payout ratios.

The discount rate depends on the riskiness of the asset and can be measured with the already seen models of asset pricing like CAPM or arbitrage and multifactor models. The model is flexible enough to allow for time-varying discount rates, where the time variation is caused by expected changes in interest rates or risk across time.

The issue with projecting future dividends is complicated because it is not possible to make such a projection through infinity. Therefore, several versions of the dividend discount model are available, making different assumptions about future growth.

The simplest model is designed to value stocks in an environment of stable growth, with the firm paying out as much of dividends as it can afford. When the firm is in such a steady state, the Gordon growth model is the good choice for valuation.

The model can be used to value a firm that is in "steady state" with dividends growing at a rate that can be sustained forever and relates the value of a stock to its expected dividends in the next time period, the cost of equity, and the expected growth rate in dividends:

$$P_0 = \frac{D_1}{r_e - g}$$

where:

D_1 is the dividend expected in the next period.

It is worth understanding what is meant for stable growth rate, and there are two insights worth keeping in mind when estimating a "stable" growth rate. When considering the growth rate in the firm's dividends, one can expect them to last forever.

As a consequence, all performance financial measures are expected to grow at the same rate. If a company is growing stably at some rate, while the dividends grow at a higher rate, the dividends will exceed earnings over time. If the opposite holds, and the earnings grow at a higher rate than dividends, the latter will then converge toward zero, which is different from a steady state.

It follows that for a firm to grow at a steady state, analysts should be able to substitute in the expected growth rate in earnings and get precisely the same result, if the firm is truly in a steady state.

There is also an issue related to how to judge whether a growth rate can be considered reasonable for a stable growth. The rule of thumb is that the growth rate has to be less than or equal to the growth rate of the economy in which the firm operates.

The analysts are generally not in agreement on the rate to be adopted, but the common agreement in the academia and research is that there are three reasons why a firm is in stable growth.

First of all, there is usually uncertainty about expected inflation, which affects the real growth in the economy. Different analysts may have different expectations about inflation, which in the long term may project a nominal growth rate in the economy that is higher.

The growth rate of the company can be much less than the growth rate of the overall economy, and in some cases the firms become smaller over time, in proportion to the economy.

Finally, if a firm is likely to maintain a few years of “above-stable” growth rates, an approximate value for the firm can be obtained by adding a premium to the stable growth rate, to reflect the above-average growth in the initial years. Even in this case, the flexibility that the analyst has is limited.

When the growth of the company is not assumed to be stable, but it varies over time, often from initial high growth to a final stable growth, the analyst will be better served using a two-stage or a three-stage model.

It is very difficult in reality for a firm to meet the requirement of a constant growth rate. Earnings are in fact usually quite volatile, but the model can still be applied when there is an average over cyclical periods.

Thus, a cyclical firm that can be expected to have year-to-year swings in growth rates, but has an average growth rate that which is defined, can be valued using the Gordon growth model, without a significant loss of generality.

An intuition that strengthens this point is that dividends are usually smoothed overtime, therefore not linked to the cyclicality of earnings. Moreover, the mathematical effects of using an average growth rate rather than a constant growth rate are small.

By assuming that dividends are constant forever, the stock valuation is given by the present value of dividends per share in perpetuity:

$$P_0 = \frac{D_1^*}{r_e}$$

where:

D_1^* is the constant dividend per share expected from the next period.

The rate of discount (return) is proportional to the time value of the money tied up in the investment and also reflects the risk associated with the uncertainty about the amount of future cash flows.

Example 6.9 Assume a stock paying a current dividend of €2.00 per share and a required rate of return of 10%. The value of a share of stock is therefore

$$P_0 = \frac{2.00}{0.10} = €20.00$$

Therefore, by paying €20.00 per share and assuming the dividends remain constant at €2.00 per share, the investor will earn a 10% return per year on the investment every year.

By assuming that the dividend grows at a constant rate, one can decompose the D_1 into the previous dividend multiplied by the growth rate g. The present value of the common stock is the present value of all future dividends. The specific case of dividends growing at the constant rate g is commonly referred to as the dividend valuation model (DVM):

$$P_0 = \frac{D_0(1 + g)}{r_e - g} = \frac{D_1}{r_e - g}$$

This model is also referred to as the Gordon model, and it is one of a general class of models referred to as the dividend discount model (DDM).

Example 6.10 A stock pays a current dividend of €7.00, and the dividends per share are expected to grow at a rate of 3% per year. If the required rate of return (cost of equity) is 12% in perpetuity, the value of the share is

$$P_0 = \frac{7.00(1 + 0.03)}{0.12 - 0.03} = €80.11$$

The case is different when the growth is expected to change over time, representing the common scenario that most firms encounter in real life. Companies in fact experience life cycles up and down turns with initial rapid growth when they start, a decreased growth in the intermediate phase of operations, and a situation of declining growth in their final stage. Further, companies may experience changes in their growth due to acquisitions and divestitures.

A modification of the dividend growth model allows for a two-stage growth representation, with an initial phase of unstable growth and a subsequent steady state where the growth rate is stable and is expected to remain so for the long term.

The standard setting of the model usually accounts for an initial stage of higher growth and a second stage of much lower stable growth, but it can be adapted to cases where the firm is expected to post low or even negative growth rates for a few years and then revert back to stable growth.

The model is built on two stages of growth, with initial growth lasting for a set period of T years and the stable growth phase starting immediately afterward. So there are two growth rates, one for the first phase and one for the stable infinite growth. The value of the stock according to the model is given by

$$\begin{aligned} P_0 &= \sum_{t=1}^{T} \frac{D_t}{1 + r_e^*} + \frac{P_T}{\left(1 + r_e^*\right)^T} \\ &= \sum_{t=1}^{T} \frac{D_t}{1 + r_e^*} + \left[\frac{1}{\left(1 + r_e^*\right)^T} \times \frac{D_{T+1}}{(r_e - g)}\right] \end{aligned} \tag{6.2}$$

where:

r_e^* is the cost of equity in the high growth stage.

r_e is the cost of equity in the stable growth stage.

If the initial growth is stable, and the dividend payout ratio is stable too, over the n years of extraordinary growth, Eq. (6.2) can be simplified as

$$P_0 = \frac{D_0(1 + g^*)\left[1 - \frac{(1+g^*)^T}{\left(1+r_e^*\right)^T}\right]}{\left(r_e^* - g^*\right)} + \frac{D_{T+1}}{(r_e - g)(1 + r_e)^T}$$

where:

g^* is the growth in the high growth stage.

Example 6.11 A share of common stock currently pays €1.70 per share and is expected to grow at a rate of 5% per year for 2 years and afterward at a rate of 3% per year. The required rate of return is 7%. The stock price can be calculated as

$$P_0 = \left[\frac{1.70(1.05)}{1+0.07} + \frac{1.70(1.05)^2}{(1+0.07)^2}\right] + \frac{P_2}{(1+0.07)^2}$$

The term P_2 is calculated as

$$P_2 = \frac{1.70(1.05)^2(1.03)}{0.07-0.03} = €48.26$$

so that

$$P_0 = \frac{1.785}{1.07} + \frac{1.874}{1.145} + \frac{48.26}{1.145} = 1.668 + 1.637 + 42.148 = €45.453$$

The result is a typical two-stage model, which looks similar to the standard dividend valuation model. But in this case, the DVM is used to determine the price beyond which there is constant growth. The dividends in the first growth phase are instead discounted using basic cash flow discounting.

A further step in the analysis is to introduce an extra stage and account for a three-stage dividend growth. It allows for an initial period of high growth, a transitional period where growth declines, and a final stable growth phase. It is the most general of the models because it does not impose any restrictions on the payout ratio.

The first stage of the model is of stable growth, but higher than classical stable growth, followed by a decline in growth in the second stage and a new stable growth at a lower level, in the last stage.

The value of the stock is given by discounting the expected dividends during the high growth and the transitional periods, plus the terminal price at the start of the final stable growth phase.

$$P_0 = \sum_{t=1}^{T_1} \frac{\text{EPS}_0(1+g_{\text{HG}})^t \theta_{\text{HG}}}{(1+r_{\text{HG}})^t} + \sum_{t=1}^{T_2} \frac{\text{DPS}_0}{(1+r_{\text{TR}})^t} + \frac{\text{EPS}_{T_2}(1+g_{\text{SG}})^t \theta_{\text{SG}}}{(r_{\text{SG}} - g_{\text{SG}})(1+r_f)^t}$$

$$P_0 = \sum_{t=1}^{T_1} \frac{\text{EPS}_0 \times (1+g_{\text{HG}})^t \times \theta_{\text{HG}}}{(1+r_{\text{HG}})^t} + \sum_{t=T_1+1}^{T_2} \frac{\text{DPS}_t}{(1+r_{\text{TR}})^t} + \frac{\text{EPS}_{T_2} \times (1+g_{\text{SG}})^t \times \theta_{\text{SG}}}{(r_{\text{SG}} - g_{\text{SG}})(1+r_f)^t}$$

where:

EPS_0 are the earnings per share in year *t*.

DPS_t are the dividends per share in year *t*.

g_{HG} is the growth rate in high growth phase, lasting T_1 years.

g_{SG} is the growth rate in stable phase.

θ_{HG} is the payout ratio in high growth phase.
θ_{SG} is the payout ratio in stable growth phase.
r_{HG} is the cost of equity in high growth phase.
r_{TR} is the cost of equity in transition phase.
r_{SG} is the cost of equity in stable growth phase.

The model is more flexible than the other dividend discount models, but it requires more inputs to run. The advantages from the additional flexibility must be compared to the disadvantages of having higher noise in the estimation due to possible errors in the inputs.

It is quite common to find companies that experience an actual three-stage growth process during their life. Most companies go through a development stage with high growth, a maturing phase with moderate growth, and a declining phase with little, no, or negative growth.

This is exactly the pattern to be described by a three-stage dividend model. The practical application of the model is more cumbersome than classical DVM, and it runs through six steps, as illustrated in the following example.

Consider the valuation of a stock that has a current dividend of €3.00 per share. Dividends are expected to grow at a rate of 12% for the next 5 years. Following that, the dividends are expected to grow at a rate of 8% for 5 years.

After 10 years, the dividends are expected to grow at a rate of 3% per year, forever. If the required rate of return is 15%, it is possible to calculate the value of the stock by breaking the calculation in to six steps:

First of all, the dividends for the years from 1 to 11 must be calculated as

Ear	Dividend growth rate	Dividend
1	12%	3.360
2	12%	3.763
3	12%	4.215
4	12%	4.721
5	12%	5.287
6	8%	5.710
7	8%	6.167
8	8%	6.660
9	8%	7.193
10	8%	7.768
11	3%	8.001

Year	Dividend	Present value
1	3.360	2922
2	3.763	2845
3	4.215	2771
4	4.721	2699
5	5.287	2629
6	5.710	2469
7	6.167	2318

(continued)

8	6.660	2177
9	7.193	2045
10	7.768	1920
11	8.001	1720

Present value of dividends after year 10 can be calculated as the present value of a growing perpetuity:

$$\mathrm{PV}_{\infty} = \frac{8.001}{0.15 - 0.03} = €66.67$$

The discounted value at time zero of the growing perpetuity is then given by

$$\mathrm{PV}_0 = \frac{66.675}{(1 + 0.15)^{10}} = €16.48$$

The sum of the present value of the dividends in the first 10 years is given by

$$\mathrm{PV}_{10} = \sum_{t=1}^{10} \frac{\mathrm{DIV}_t}{(1 + 0.15)^t} = €24.79$$

The value of the stock at present is given by the sum of the present value of the growing perpetuity and the present value of the dividends in the first 10 years, as for

$$\mathrm{PV}_{\infty} = 16.48 + 24.79 = €41.27$$

It is possible to compare the model to the CAPM in order to grasp the differences among them. The dividend growth model gives a measure of cost of equity through the analysis of empirical data publicly available for most companies.

The calculation comes straight through an algorithm that involves measuring the dividends, estimating the dividend growth, copying the market value of the shares, and using the amounts in the equation to estimate the cost of equity.

The model is however limited in that it does not give any information about why different shares have different costs of equity. This is due to the fact that dividend growth models ignore the risk aspect of valuation.

That model simply measures what's there without offering an explanation. Note particularly that a business cannot alter its cost of equity by changing its dividends. The equation

$$r_e = \frac{D_0(1 + g)}{P_0} + g$$

might suggest that the rate of return would be lowered if the company reduced its dividends or the growth rate.

The reality is different, and normally a dividend cut or growth decrease would result in a lower market value of the company. The value would decrease until the level corresponding to the point where investors obtain the required return.

The CAPM is a more complete model, making a step further by introducing systematic (market) risk in the equation for valuation. Other returns in the economy as well as the relationship among the various risks translate into beta and asset return.

Another important feature of CAPM is to offer several ways to measure the inputs, as the risk-free rate, the market return, and the beta. They can be estimated from empirical data or are normally available as public information.

6.2.2 Discounted Free Cash Flow Models

The models of free cash flows aim at establishing how much cash the firm can return to the shareholders through its operations. It is therefore a direct derivation from the net income, which is the first entry to be considered.

Recall that net income is the accounting measure of the stockholders' earnings during the period. The task is to convert it to a cash flow by subtracting out a firm's reinvestment needs.

The net income must be further netted of some types of expenses, in order to get to the free cash flows. For example, one must subtract any capital expenditures, defined broadly to include acquisitions, since they represent cash outflows.

On the other hand, depreciation and amortization must be added back to the net income, because they do not represent cash flows. Net expenditure is then the difference between capital expenditures and depreciation.

It is usually a function of the growth characteristics of the firm. High-growth firms tend to have high net capital expenditures relative to earnings, whereas low-growth firms may have low, and sometimes even negative, net capital expenditures.

When working capital increases, the firm's cash flows are reduced, and vice versa, with lower working capital translating into higher cash flows available to equity investors. Companies experiencing a fast growth are subject to large increases in working capital, especially in industries characterized by high levels of working capital, like retailing. In order to capture cash flow effects of working capital, one must focus on the noncash working capital.

The last point in determining cash flow is debt. Equity holders are typically subject to changes in the level of debt that impact on their cash flows. Repaying the principal on existing debt represents a cash outflow; but the debt repayment may be fully or partially financed by the issue of new debt, which is a cash inflow.

By accounting for the cash flow effects of net capital expenditures, changes in working capital and net changes in debt, it is possible to define the net cash flows after all netting, named as free cash flow to equity (FCFE), as from the formula

$$\begin{aligned}\text{FCFE} = {} & \text{Net Income} - (\text{Capital Expenditures} - \text{Depreciation}) \\ & - (\Delta \text{ in non-cash working capital}) + (\text{New debt issued} - \text{Debt repayments})\end{aligned}$$

This remaining net cash flow is then available to be repaid to shareholders in the form of dividends or become retained earnings that constitute additional equity for the company.

In order to simplify the calculation, one can assume that net capital expenditures and working capital are financed by a mix of equity and debt capital. The proportion of net capital expenditures and working capital raised in the form of debt can be indicated as *d*, and the effect on cash flows to equity of these items can be represented as

$$\begin{aligned}\text{FCFE} = {} & \text{Net Income} - (\text{Capital Expenditures} - \text{Depreciation}) \\ & - (\Delta \text{ in non-cash working capital}) + (\text{New debt issued} - \text{Debt repayments})\end{aligned}$$

The equity cash flows associated with capital expenditure needs (FCFE_{CAP}) can be defined as

$$\text{FCFE}_{\text{CAP}} = -(\text{Capital Expenditures} - \text{Depreciation})(1 - d)$$

and the equity cash flows associated with working capital needs (FCFE_{WC}) can be defined as

$$\text{FCFE}_{\text{WC}} = -(\Delta \text{ in non-cash working capital})(1 - d)$$

Following the above formulation, it is possible to express the cash flow to equity, net of capital expenditure, and working capital, for a constant debt-to-equity ratio, as

$$\begin{aligned}\text{FCFE} = {} & \text{Net Income} - (\text{Capital Expenditures} - \text{Depreciation})(1 - d) \\ & - (\Delta \text{ in non-cash working capital})(1 - d)\end{aligned}$$

The net debt payment does not appear in the equation anymore, given that the new debt issue is financing the former debt, in order to keep the debt-to-equity ratio constant, as per assumption of the model.

It is particularly useful to assume that a specified proportion of net capital expenditures and working capital needs will be financed with debt if the target or optimal debt ratio of the firm is used to forecast the free cash flow to equity that will be available in future periods. For past periods, an alternative approach is to use the firm's average debt-to-equity ratio over the period to arrive at approximate free cash flows to equity.

The traditional formulation of the cash flow calculation assumes there are no preferred dividends paid. The model values common equity, so that the formulas need to be modified in order to account for the existence of preferred stock and dividends.

The preferred dividends must be subtracted from the net income as well, in order to arrive at the FCFE, according to the modified formula

$$\begin{aligned}\text{FCFE} = {} & \text{Net Income} - (\text{Capital Expenditures} - \text{Depreciation}) \\ & - (\Delta \text{ in non-cash working capital}) - (\text{Pref. dividend} + \text{New pref. stock issued}) \\ & + (\text{New debt issued} - \text{Debt repayments})\end{aligned}$$

which, in the short form, becomes

$$\text{FCFE} = \text{Net Income} - \text{Pref.dividends} - (\text{Capital Expenditures} - \text{Depreciation})(1-d) \\ - (\Delta \text{ in non-cash working capital})(1-d)$$

The non-equity financial ratio (d) would then have to include the expected financing from new preferred stock issues.

The FCFE model is not that dissimilar from the dividend discount model, if one considers that, in all above forms, it represents a model where we discount potential dividends rather than actual dividends.

The various versions of the model based on cash flow are therefore all variants of the dividend discount model, with a major change given by the fact that the dividends are replaced by the free cash flows to equity.

Such a replacement goes beyond changing one type of cash flow with another, given that it entails the assumption that the FCFE will be paid out to stockholders. Two logical consequences follow.

The first consequence is that there will be no future cash buildup in the firm, since the cash that is available after debt payments and reinvestment needs is paid out to stockholders each period.

Another consequence is that the part of growth in income due to current assets will be excluded from the expected growth in FCFE, leaving only the contribution of the income from operating assets.

A further complication of the model is the inclusion of stock buybacks into dividends to be discounted. It is useful to understand how the FCFE model compares to the modified dividend model in this case.

Stock buybacks can be seen as accumulated cash due to the strategy of not paying out the FCFE as dividends for some time. Thus, FCFE represent a smoothed out measure of what companies can return to their stockholders over time in the form of dividends and stock buybacks.

The growth of cash flows can be estimated by using the same approach used for the dividends, given they both represent cash flows to equity investors. The expected growth rate can then be expressed as

$$E(g) = R_R \times r_e$$

where:

R_R is the retention ratio.

The amount of cash flow that is not paid out as dividends is reinvested into the firm as retained earnings. This is not consistent with the assumption that free cash flows to equity are paid out to stockholders which underlies FCFE models.

In order to make the model more realistic, it is therefore necessary to replace the retention ratio with the equity reinvestment rate. This is a measure of the share of net income to be reinvested into the company. The model can be then formulated as

$$R_E = 1 - \frac{\text{Net capital expenditure} + \Delta \text{ in working capital} - \text{Net debt issues}}{\text{Net income}}$$

Please note that in the FCFE model, there is no extra cash left in the firm after reinvestment. It is therefore not appropriate to use a return on equity that includes interest income from current assets.

The same argument works for the book value of equity, which includes the value of cash and other marketable securities. The best solution is to construct a modified version of the return on equity that measures the noncash aspects.

$$\text{Non-cash ROE} = 1 - \frac{\text{Net income} - \text{After tax income from cash \& marketable securities}}{\text{Book value of equity} - \text{Cash and marketable securities}}$$

The product of the equity reinvestment rate and the modified ROE will yield the expected growth rate in FCFE as

$$\text{Expected Growth in FCFE} = \text{Equity reinvestment rate} \times \text{Non-cash ROE}$$

There are several versions available of the FCFE model, depending on the type of company to valuate. For example, for a company that is growing at a stable rate, the best model to use is the constant growth FCFE model.

The model assumes a value of the equity that is a function of expected FCFE in the next period, the stable growth rate and the required rate of return.

$$P_0 = \frac{\text{FCFE}_1}{r_e - g}$$

where:

FCFE_1 is the free cash flows to equity in period 1.

The underlying assumption of the constant growth FCFE model is similar to those of the Gordon growth model. A reasonable growth rate must be used in the model, in relation to the nominal growth rate in the economy.

In order to represent a realistic proxy, the chosen long-term growth must not exceed the growth rate of the economy in which the firm operates by more than one or two percent. Additional assumptions relate to other features possessed by the firm that are shared by stable firms in general.

For example, the risk of the company is average, with capital expenditures not too large, relative to depreciation. In terms of asset pricing, this means that if CAPM is used, for example, the beta of the equity should not be significantly different from one.

The reinvestment rate for a stable growth firm can be estimated in two ways. A first approach is to use the typical reinvestment rates for firms in the industry of operation. Alternatively, the relationship between growth and fundamentals can be used to estimate the required reinvestment.

The expected growth in net income can be written as

$$\begin{aligned}\text{Expected growth rate in net income} = {} & \text{Equity Reinvestment Rate} \\ & \times \text{Return on equity}\end{aligned}$$

which turns in an estimation of the equity reinvestment rate as

$$\text{Equity Reinvestment Rate} = \frac{\text{Expected growth rate in net income}}{\text{Return on equity}}$$

To illustrate, a firm with a stable growth rate of 4% and a return on equity of 12% would need to reinvest about a third of its net income back into net capital expenditures and working capital needs. Put another way, the free cash flows to equity should be two thirds of net income.

6.2.3 Relative Valuation Models

Relative valuation models assess the value of the assets of a company based on comparable assets in the industry. So, for example, the valuation of assets of an IT company will be run based on the values observed in the IT industry.

After the reference asset values are identified, relative valuation moves on in scaling them into a standard variable that can generate comparable prices. This standardization makes sense when the assets to be compared differ significantly from each other.

An example of how the standardization differs from case to case is corporate stocks. It has been observed that higher growth companies should trade at higher multiples than low growth companies.

As from Larsen et al. (2011), relative valuation (as opposed as the FCF and DDM models) does not estimate the intrinsic value of the assets based on the cash flows they generate. It rather estimates the value of an asset by considering how much the market is currently willing to pay for it.

The convergence or divergence of the result of relative valuation with the result of intrinsic valuation is therefore determined by how correctly the market is currently pricing the target asset.

There are several multiples that can be used for relative valuation, which are summarized in Table 6.1, and the following part of this section analyzes some of them.

For example, one can value the asset as a multiple of the earnings generated by it. This is the case when stock prices are calculated as a multiple of the EPS of the company.

Table 6.1 Most commonly used multiples

	Price-over-earnings (PE) ratio
	Value over EBIT
	Value over EBITDA
Earnings multiples	Value over cash flow
Book value multiples	Market value over book value of equity
	Market value over book value of assets
	Market value over replacement cost
Revenues multiples	Market price over sales per share
	Market Value over Sales

Current EPS allow to estimate the price-over-earnings (PE) ratio, so that a current PE can be used to estimate a projected PE, which is the basis for the valuation of the stock.

Another commonly used earning multiple is EBITDA, and it is rather obvious that from the point of view of the buyer, lower multiples are preferable in that they lead to a lower valuation. Anyway all multiples are sensitive to the growth potential of the company.

Another class of multiples relates to the book value of the assets. One should recall that the accounting value of a business may differ significantly from its market value, thus generating confusion among the analysts.

Ratios like the price-to-book value of the equity of a firm are often used by the investors to realize how the market is overvaluing or undervaluing a business. These measures can vary across industries.

Some analysts like to use the ratio of the value of the company over the replacement value of the assets as multiple, instead of the price-to-book ratio. This ratio is commonly known as Tobin’s Q.

The third class of multiples is related to the revenue variables and departs from the accounting choices. The ratios of the corporate value to the generated revenues can be used as a multiple in relative valuation.

One of these ratios is the price over sales ratio that divides the market value of the corporate equity by the revenues. Another one takes into account the overall enterprise value of the firm, represented by the value of the operating assets of the firm, dividing it by the sales.

Revenue multiples are very useful in that they allow to compare companies across different markets. As other multiples, they vary across industries in that they heavily depend on the profit margins normally realized in each industry.

Some multiples are specific to some sector and do not make sense when used in valuating firms in other sectors. These multiples rely on very specific entries to calculate ratios based on firm-specific value or cost measures.

In the Internet sector, it is common, for example, to use the webpage hits generated by the websites of the company, and the corporate value per hit becomes an interesting and specific multiple to be used in that sector.

Sector-specific multiples can be misleading in that they rely on the value that is associated to the variable considered (in the above example the webpage hit). If there is no sense of what is a high, a medium, or a low price for a hit, then the resulting valuation may be hard to interpret and compare to some benchmark value.

In order to be sure the method in use is well defined and the measurements are uniform across the companies in the same sector, one must analyze several aspects of the multiple.

For example, it is very important to analyze the multiple with cross section in order to determine its distribution across companies in the same sector and also across different sectors in the same market.

Moreover, the sensitivity of the multiple to changes in its determinants must be analyzed, and the selection of the right comparable firms is at the basis of the correct analysis of the similarities and differences existing across firms in the industry of reference.

The correct definition of multiples is very important, and in most cases it differs from one analyst to another. Whichever definition is used, a correct relative valuation is possible when the definition of the multiples used in the analysis is consistent over the different companies analyzed and over time.

Consistency of definition of the multiples also includes the way the ratios are calculated. Some of them in fact are controversial in that the numerator and denominator not always represent the same type of financial.

In the price over EBITDA indicator, for example, the numerator is clearly a measure related to the company's equity, while the denominator includes the part of revenues that belong to other stakeholders, other than equity holders.

However, most experts agree that these inconsistencies are easily solved when the ratios used are the same for all the companies compared in the analysis, independently of being a mix of equity and firm measures.

An aspect that is very important is the time span used in the calculation of the various elements involved in the analysis. All the variables involved in the ratio and multiples calculation should be calculated or averaged over the same time period. This ensures consistency in the valuation, especially when the uniform time span is applied to all the companies included in the analysis.

In terms of consistency, it also helps to have a descriptive analysis of the multiple of reference, in order to be able to judge whether a value is high, medium, or low or to establish a benchmark value for the market or industry.

This can be achieved by analyzing the distributional characteristics of the multiple, especially across the market. While in fact it is very common for analysts who are expert of some specific sector to have the sense of the ranking of companies in that sector, in terms of multiple, it is not equally straightforward to get the sense of this ranking in the overall market.

Standard statistics like average, standard deviation, median, maximum, and minimum, help the analyst to build a map of the situation in terms of some multiple, in some market. All supported by a scientific approach.

6.3 Special Issuances

Learning Outcomes

- Understand the features of preferred stocks.
- Learn what warrants are.
- Learn what convertibles are.

6.3.1 Preferred Stocks

Other than common stocks, a company can also issue preferred stocks to be offered to investors. It is a type of stock with preference over common stocks in dividend payments and firm liquidation.

The particular features preferred stocks make them a very special type of hybrid security, halfway between stocks and bonds. Stock-like features include taxability of dividends, no specific maturity, and the fact that failure to pay dividends does not constitute bankruptcy.

In terms of bond features, preferred stocks come with the promise of a fixed dividend, like for the coupon of a bond. Moreover, they have no voting rights attached, and the liquidation value attached to them is usually pre-stated.

Other bond features are the sensitivity to interest rate change and the seniority for repayment in case of liquidation of the company, just after the bond holders. Moreover, most preferred dividends are cumulative.

Another important feature of preferred stocks is that companies owning preferred stocks can deduct 70% of the preferred dividend from their taxable income. As a consequence, dividend yields are often at or below yields on bonds of similar risk.

The market for preferred stocks has been dominated by corporations and institutional investors for a long time. But lately, individual investors also started finding it attractive and decided to go for the regular income preferred stocks can give.

All of the above mentioned features make preferred stocks a peculiar type of financial security and must be analyzed one by one in order to grasp the sense of having such a type of hybrid equity in the capital structure.

The seniority of preferred stock is an important feature. Dividends to be paid to them are senior to the dividends received by common stocks. However, as a form of equity capital, the owners of preferred stocks are junior compared to bondholders.

In case of cumulative preferred stocks, the dividends accrue in case a payment is missed, and no dividend is paid to common shareholders until the total of the dividends to the preferred stocks is paid out. One should keep in mind that unlike common stockholders, preferred shareholders do not participate in the potential for increased dividends.

Preferred stocks are an attractive investment to those investors who want to improve their yields and current income. The yields of preferred stocks in fact are normally higher than those on bonds from the same issuer.

The volatility of price is strongly related to their bond-like features, and preferred stocks are subject to interest rate risk, even if not to the same extent as bonds. In general, prices will rise as interest rates fall, and prices will move lower as rates rise. Also market factors and industry situation contribute to the change in price.

Attached to preferred stocks, there is also a credit rating, similarly to what happens with bonds. The rating measures the likelihood of the company to pay dividends regularly and in time. Generally, an issuer's preferred stock ratings are two notches below an issuer's senior debt, which is reflective of the junior claim of the preferred.

An advantage of preferred stocks is that they normally come at a price which is lower than the standard price, therefore making it possible for the investor to build up a diversified portfolio with a limited investment.

Given the lower par value, preferred stocks are appealing to the individual investors, as mentioned above. This enhances liquidity, given that as opposed to other fixed-income instruments, preferred stocks are listed on all major stock exchanges. The benefits in terms of liquidity are huge, and also the visibility and strong marketability of the shares are significantly improved.

Given the lower purchasing price, compared to standard stocks, preferred stocks offer a high opportunity for capital appreciation. The potential capital gains implied by the investment are therefore interesting.

When preferreds trade at a discount to par, investors can invest in these securities relatively inexpensively and earn a favorable yield on the dividends received, and should the outlook for the issuer become more favorable or interest rates fall, they can earn a capital gain on the shares.

The historical reason for companies to invest in preferred stocks has been the very favorable tax treatment of the dividends. In the United States, for example, domestic companies are allowed a tax relief on the 70% of the dividend amount. Therefore, the corporate taxation only hits a small part of the dividends from preferred stocks, and the procedure is called dividend received reduction (RDR).

Securities must be held by the company for at least 46 days in order to qualify for tax relief. Moreover, the issuing company must be a domestic one, and preferred stocks must be straight, meaning they are not a fixed-rate capital security structure.

Several structures are possible for preferred stocks. Traditional preferred stocks are known as straight and represent the equity of a publicly held domestic corporation, while sharing features with bonds.

The issuer of straight preferred stocks pays quarterly dividends set at a fixed amount, similar to the coupon payment on a bond. With the attraction of well-recognized issuers of preferred stocks that are typically listed on markets like the American NYSE, individual investors also benefit from higher yields than fixed-rate corporate bonds.

Preferred stocks enjoy a higher stability than common stocks given the fact that they enjoy the high liquidity of the regulated stock exchange market, with the comfort of the fixed income stream attached.

An innovative type of preferred stocks available to investors is the fixed-rate capital securities (FRCS). They have similar characteristics to preferred stock while allowing the issuing company to deduct the interest.

FRCS are available in two different types. The Junior Subordinated Debentures (JSD) are the simplest form of fixed-rate capital securities. The issuing company in fact directly issues the securities without the need to set up a separate company for it.

The second type of FCRD is called Trust Originated Securities (TOS), representing preferred stocks issued by a separate trust. The proceeds from the sale of TOS are then invested by the trust in junior subordinated securities issued by the parent company.

Most FRCS carry a dividend deferral option, which allows the issuing company to defer the payment of dividends for a term of up to twenty quarters, equivalent to 5 years. In case of such a type of issuance, the company is not allowed to declare any common dividends until accumulated dividends on FRCS have been paid in full.

Foreign preferred stocks (FPS) are issued in one country in a foreign currency. In the case of the American market, they are called Yankee preferred stocks and are issued by foreign companies in the United States.

FPS normally carry a higher yield than domestic issuances, given that the former do not have access to the DRD system, and have a lower reputation compared to domestic preferred stocks. It must be taken into account that in some cases, the dividend for some FPS may be subject to a withholding tax imposed by the issuer's country.

One may consider the example of the United Kingdom, where the regulators impose a 15% withholding tax on US shareholders. Investors can then reclaim this tax provided that the shares are not held in a tax-deferred account.

A more sophisticated type of preferred stock is the adjustable rate preferred stock (ARPS), for which the dividends are reset quarterly to reflect market rates. The benchmark for setting the new rate is the higher of the 3-month Treasury bill discount rate, the 10-year constant maturity Treasury rate or the 30-year constant maturity Treasury rate.

For this type of securities, both a minimum and a maximum level are set for the dividend yields, known as collars. Moreover, they qualify for the DRD standard, in case they are owned by corporations.

The ARPS have a lower yield than straight preferred stocks, in that the investors benefit from owning a security that potentially insulates them against adverse changes in the shape of the yield curve, in addition to changes in the absolute levels of interest rates.

Example 6.12 An investor buys preferred stocks at a price of €25. The stock pays a dividend yield of 7.5%, for a euro amount per share of

$$\mathrm{DIV} = 25 \times 0.075 = €1.875$$

It is possible to perform standard calculations about preferred stocks. The following are definitions and examples of some common price and yield calculations related to this flexible instrument.

Preferred stocks trade at a clean price, meaning that the price does not account for any interest earned since the last dividend payment. However, theoretically, some interest can be accrued on the stocks from the last dividend to the time of purchase. This represents an implied accrued dividend.

Example 6.13 Consider a share that paid a dividend of €1.875, 60 days before, is purchased today. The price must then include accrued interest of

$$I_{\mathrm{ACC}} = \frac{60}{360} \times 1.875 = €0.31$$

The current yield is a measure of the expected annual return on the investment, obtained by dividing the annualized dividend by the current preferred stock price. It gives the sense of the profitability of investing in preferred equity.

Example 6.14 By inverting the relationship in Example 6.11, the dividend yield can be obtained as

$$r_{\mathrm{DIV}} = \frac{1.875}{25} = 0.075 = 7.5\%$$

Another way to label the clean price, so the price of the preferred share minus the implied accrued dividend, is the stripped price.

Example 6.15 For the stock in the previous examples, the stripped price is given by

$$P_{\mathrm{STR}} = 25 - 0.31 = €24.69$$

Starting from the stripped price of the stock, it is possible to calculate a stripped yield, which is given by the ratio of the annualized dividend over the stripped stock price.

Example 6.16 Shares paying an annualized dividend of €1.875 with a stripped price of €24.69 offer a stripped yield of

$$r_{\mathrm{DIV}} = \frac{1.875}{24.69} = 0.0759 = 7.59\%$$

Sometimes investors are looking for monthly income in their investment. This can be easily done by investing in three different issues of preferred stocks, so that each stock will pay quarterly in different months, to cover the whole calendar.

Many preferreds pay dividends quarterly in one of three cycles: (1) January, April, July, and October; (2) February, May, August, and November; or (3) March, June, September, and December.

It comes naturally to ask why investors like to hold preferred stocks, given that they have a lower yield than bonds, and they are also subordinate to them. The main reason is the tax advantage of DRD to corporations.

Among the several types of preferred stocks available on the market, there are adjustable-rate preferred stocks, with a dividend yield equal to some reference rate plus a spread. The market value of this type of shares is stable, and there are an upper bound and a lower bound to the dividend rate.

Convertible preferred stocks can be converted into certain number of shares of common stock, representing an option granted to the preferred stock holder. Callable preferred stocks on the other hand can be called before the maturity date at a pre-specified price at the option of the issuer and represent an option granted to the issuer.

6.3.2 Warrants

Warrants are a type of financial options that give holders the right to buy shares of a company at a fixed price for a given period. Each warrant specifies the number of shares of equity that the holder can buy, the exercise price, and the expiration date.

Even if from the definition they look very similar, the main difference between a warrant and a financial option is that the former has longer maturity periods, with some warrants having an infinite maturity, so that they never expire.

In many cases, warrants are attached to privately placed bonds issued by some company. In this case, the loan agreement will state whether the warrants are detachable from the bond and potentially be sold separately. Normally, the warrant can be detached immediately.

Warrants have been issued infrequently in the last years, and companies issuing them are mostly based in the United States. More recently, governments have purchased warrants from banks as part of the financial rescue plans put in place to combat the global credit crunch in 2008.

A warrant is a security that gives the holder the right to buy an underlying stock of the issuing company at a fixed exercise price, until the expiry date. As one can see, this is similar to the definition of a financial option.

Warrants usually come as an attachment to bonds or preferred stocks of a particular company. It helps the issuer to sell the bond at a lower yield that would have to be offered if nothing were attached to it. Warrants can also be used in private equity deals. Frequently, these warrants are detachable and can be sold independently of the bond or stock.

When the warrants are attached to preferred stocks, it might be the case that they need to detach and sell the warrant before any dividend can be cashed. This is why in most cases the warrants attached to preferred stocks are quickly sold separately.

Warrants have similar characteristics to that of standard equity options. They can in fact be exercised when the holder informs the issuer their intention to purchase the shares underlying the warrant. Right after the bond issuance, the parameters of the warrant, including the strike price, are fixed.

The premium of a warrant is the difference in price between buying the shares through the warrant and buying them directly on the market. The gearing or leverage shows how much exposure on the underlying is given by holding the warrant compared to having bought the shares directly.

The expiration date of a warrant follows the same rules and standard of an option, with the value of the warrant getting lower as the time to maturity reduces. Therefore, the expiry date is the date on which the right to exercise ceases to exist.

Like the options, the warrants can be European style or American style. They are longer-dated options and are normally traded on the over-the-counter markets, rather than on regulated exchanges.

Warrants are issued by private corporations and, when issued by the company itself, are dilutive, because if the warrant is exercised, the company issues new shares of stock, so the number of outstanding shares increases.

Warrants do not carry any voting rights, and the owner receives an existing share at exercise time. In case of employee stock options, new shares are created and issued by the company.

Warrants usually have a longer life than standard options, and their life is measured in years. Upon expiration, the warrants are worthless unless the price of the common stock is greater than the exercise price.

There are traditional warrants that are attached to a bond and simply carry the right to buy shares in the issuing entity. The writer of a traditional warrant is therefore also the issuer of the underlying instrument.

One way to calculate the value of the warrant is to subtract the clean price for the bond from the overall price paid for it with the warrant attached as

$$\begin{aligned} P_w &= B_w - B \\ &= B_w - \left[\sum_{t=1}^{T} \frac{C}{(1+r_r)^t} + \frac{F}{(1+r_r)^T} \right] \end{aligned}$$

where:

B_w is the price paid for the bond with warrants.

C is the coupon payment.

r_r is the required rate of return.

Another type of warrant is named naked, and it is issued stand-alone, without being attached to a bond of another instrument. Like traditional warrants, these are traded on the stock exchange.

Warrants are typically issued by banks and securities firms, in which case they are called covered warrants, not involving the firm issuing the outstanding shares. Covered warrants are very popular and generate more volume compared to uncovered ones.

From a financial point of view, they are generally bought by retail investors, rather than investment funds or banks. The latter in fact normally prefer financial options, given that they are keenly priced and trade on a different market.

When warrants are issued by the holders of the underlying asset, they are called third-party warrants. Companies and mutual funds in particular can issue warrants on their assets.

Example 6.17 A company issues 1,000,000 warrants, giving the right to convert each warrant into a share at €250. Such a warrant is issued by the company. Assume a mutual fund holds 100,000 shares of the company and sells the warrants attached to them. If the stock does not cross €250, the buyer will not exercise the warrant. The seller will therefore keep the warrant premium.

The main advantage of issuing third-party warrants is that they signal the value of the underlying asset by tracking its value. A mutual fund, selling warrants at some exercise price, signals that the value of the fund shares in the short-medium term may tend to that value.

If volumes in such warrants are high, the price discovery process will be that much better, for it would mean that many investors believe the stock will trade at that level in 1 year.

Third-party warrants are long-term call options issued in the form of a covered call or write, given that the issuer of the warrant is the holder of the underlying asset and the option is sold against it.

The name warrant is also used for the issuance of checks by government agencies. The checks are in fact not immediately redeemable due to lack of funds, but they will be in the future, with interest compensation.

In some states, a warrant is a demand draft drawn on a government's treasury to pay its bills. Checks or electronic payments have replaced these warrants, but in Arkansas, some counties and school districts use warrants for nonelectronic payments.

It is possible to express in general terms the profit associated to the exercise of a warrant. The gain from a single warrant can be expressed as

$$V_w = \frac{E + K_w m_w}{m + m_w} - K \tag{6.3}$$

where:

E is the firm's value net of debt.

K_w is the exercise price of the warrant.

m is the number of shares outstanding before the exercise of the warrant.

m_w is the number of new shares from the exercise of the warrant.

The first term on the right-hand side of Eq. (6.3) is the value of a share of equity after the warrant is exercised. The numerator of the left term is the firm's value net of debt after the warrant is exercised.

The sum $m + m_w$ is the amount of shares outstanding after the exercise of the warrants. By rearranging terms, we can rewrite the equation as

$$V_w = \frac{m_w}{m + m_w}\left(\frac{E}{m} - K\right)$$

The term in the parenthesis on the right-hand side of the equation is the gain on a call option written on the equity of the firm, before the warrants are exercised. It follows that the gain from exercising a warrant is a proportion of the gain from exercising a call in a firm without warrants.

We can therefore value a warrant using the Black-Scholes model, adjusted for the dilution effect:

$$P_w = \frac{c_w}{\left(1 + \frac{m_w}{m}\right)}$$

where

c_w is the value of a call option written on the equity of a firm without warrants.

6.3.3 Convertibles

In the case of a warrant attached to a bond, it is always possible to separate the two securities. This is not possible in the case of convertible bonds, which gives the holder the right to exchange it for a given number of shares any time up to and including the maturity date of the bond.

The concepts of convention ratio, the amount of stocks to be received for every bond converted, and the conversion price to be paid for every stock purchased through conversion of the bond are the key features of a convertible bond.

In fact, both concepts implicitly assume that the bond is selling at par. If the bond is selling at another price, the terms have little meaning. By contrast, conversion ratio can have a meaningful interpretation regardless of the price of the bond.

The value of the convertible bond is determined by three factors, namely, the value of the naked bond, the conversion value, and the value of the embedded option. All of the three components deserve to be analyzed and understood.

The straight (naked) bond value is what the convertible bonds would sell for if they could not be converted into equity. It will depend on the general level of interest rates and on the default risk.

Example 6.18 Company TXL raises €10,000,000 by issuing 5.25% convertible bonds. Each bond is convertible into four shares, and the share price at the time of bond issuance is €18.75. The conversion price of

$$K = \frac{100}{4} = €25.00$$

is much higher than the actual equity price. Suppose that straight debentures issued by the same company are rated AA, and AA-rated bonds are priced to yield 3.25% per 6 months. The straight bond value of the convertible bonds can be determined by discounting the €2.625 semiannual coupon payment and principal amount at 3.25%:

$$B = \sum_{t=1}^{16} \frac{5.25}{(1 + 0.0325)^t} + \frac{100}{(1 + 0.0325)^{16}} = 61.56 + 63.09 = €124.65$$

The straight bond value of a convertible bond is a minimum value. The price of the convertible could not have gone lower than the straight bond value.

The value of convertible bonds depends on conversion value. Conversion value is what the bonds would be worth if they were immediately converted into equity at current prices.

The conversion value is equal to the share of equity that can be received upon conversion of the bond, multiplied by the current price of the stocks, as determined by the market.

Normally, the value of the convertible bond is higher than the sum of the value of the naked bond and the conversion value. This is due to the fact that the bond does not have to be converted immediately.

The option of waiting for exercising the option embedded in the bond is assumed to have a value, and it raises the value over both the straight bond value and the conversion value.

In case of a low value of the firm, the value of the convertible bond is mostly made by the naked bond, while in case of a high value of the company, most of the value is in the conversion value.

$$B_{\mathrm{CV}} = \max(B_C, K_{\mathrm{CV}}) + c_{\mathrm{CV}}$$

where:

B_C is the straight bond value.

K_{CV} is the conversion value.

c_{CV} is the option value.

Compared to a normal naked bond, a convertible bond pays a lower interest rate, given that the investor will accept a lower interest rate on a convertible because of the potential gain from conversion.

According to the specific situation, issuing straight bonds can be better or worse than issuing convertible bonds. First of all, consider a scenario of an increase of the share price. This is of course beneficial to the company, but it also enhances conversion.

The firm would have benefited even more had it previously issued straight debt instead of a convertible. The gain in terms of the lower interest paid on the convertible bonds is offset by the loss of selling equity at a cheap price to the convertible bond holders.

On the other hand, in case of a fall in price of the share, the company will be upset on one side but will also benefit from having issued convertible bonds compared to straight bonds.

Because conversion does not take place, our comparison of interest rates is all that is needed. It can be concluded that it is not optimal for the firm to issue convertible debt if a future raise in price is expected. The firm is better off having issued convertible debt if the underlying equity subsequently does poorly.

It is not possible in an efficient market to predict the future price of the shares. Therefore, any consideration about the a priori convenience of issuing convertible bonds instead of naked ones is purely speculative.

Another type of dilemma consists in the choice between issuing convertible bonds or equity for financing. Assuming that convertibles are issued, a subsequent raise of the stock price would fully benefit the company.

In case the stock price falls after convertible bonds are issued, the scenario is flipped, and the company suffers from both the loss in equity value and regretting the choice of having issued convertibles instead of equity.

The firm in fact would have benefited by issuing equity above its later market price. That is, the firm would have received more than the subsequent worth of the equity.

The value of the convertible is not severely affected because the straight bond value serves as a floor.

To conclude, the firm is better off having issued convertible debt instead of equity, if the stock price subsequently increases. The firm is worse off having issued convertible debt if the underlying equity subsequently does poorly.

6.4 Summary

Together with debt, equity is the other important source of capital for every company. The stock market resembles lots of investors trading very high volumes of equity shares every day.

The main feature of an equity market is its microstructure, defined by the kernel that processes the orders of the investors translating them into transactions that drive the price of the stocks.

The price formation goes through a process of supply and demand where the information hitting the market at random times plays a crucial role in determining the strategies of the investors.

The availability of the deals and the formation of a fair price are guaranteed by the liquidity of the market. In order for a market to be liquid, there must be sufficient volumes of tradable assets available to the investors to satisfy their demand.

Common stocks are the main junk of equity capital in the structure of a company, and they come with voting rights and other features that result from the ownership of a part of the firm.

Several models are available for common equity valuation including discount dividend models, based on the dividends paid out by the firm, and free cash flow models, based on the cash flows generated by the firm. Also relative valuation models have become of common use among practitioners, in that they are more simple and accounting-based.

On top of the common stocks, there are other forms of equity capital that are relevant to the capital structure of the firm. Preferred stocks, for example, are very popular among investors, in that they guarantee the debt features of a bond, with the advantage of acquiring ownership of the firm.

In the same way, warrants are also very important, as a tool for distributing the chance to buy extra stocks to already existing shareholders and also to give the chance to new investors to enter the capital.

Convertible bonds are a specific type of security with an option embedded. They result in the chance for a debt holder to become a shareholder at some time in the future, so as to change the profile of the investment portfolio.

Problems

1. Explain the difference between illiquidity and insolvency. Does the difference matter?
2. Explain why if the government announces it is abolishing insurance on deposits, a typical bank is likely to face liquidity problems.
3. If yield curves, on average, were flat, what would this say about the liquidity premiums in the term structure?
4. Would you expect the bid-ask spread to higher on actively or inactively traded stocks?
5. Discuss the moral hazard aspects created by deposit insurance.
6. Describe and compare the different types of trading orders available in the markets.
7. A bill has a bank discount yield of 6.65% based upon the asked price and 6.75% based upon the bid price. The maturity of the bill (already accounting for skip-day settlement) is 90 days.
   (a) Find the bid and asked prices of the bill.
   (b) Calculate the bond equivalent yield of the bill as well as its effective annual yield based upon the asked price. Confirm that these yields exceed the discount yield.
8. The table below provides some price information on Marriott:

Bid price	Ask price
37.55	38.33

You have placed a stop-loss order to sell at €37.80.

(a) By placing this order, what are you in effect asking your broker to do?
(b) Given the market prices, will your order be executed?

9. Consider the following limit order book of a specialist. The last trade in the stock occurred at a price of €45.55.

Limit buy orders		Limit sell orders	
Price	Shares	Price	Shares
35.50	5000	35.75	1000
35.25	6000	35.90	2000
35.00	8000	36.00	5000

(a) If a market buy order for 3000 shares comes in, at what prices will it be filled?
(b) What will happen if a market order to sell 5000 shares comes in?

10. Consider the following limit order book of a specialist. The last trade in the stock occurred at a price of €45.55.

Limit buy orders		Limit sell orders	
Price	Shares	Price	Shares
59.75	4000	55.75	1000
59.50	5000	55.80	2500
59.25	7000	56.00	4500

(a) If a market buy order for 1000 shares comes in, at what prices will it be filled?
(b) At what price would the next market buy order be filled?
(c) You are the specialist: do you wish to increase or decrease your inventory of this stock?

11. You have borrowed €20,000 on margin to buy shares in Disney, which is now selling at €80 per share. Your account starts at the initial margin requirement of 50%. The maintenance margin is 35%. Two days later, the stock price falls to €75 per share.
    (a) Will you receive a margin call?
    (b) How low can the price of Disney shares fall before you receive a margin call?
12. Explain why banks hold more liquid assets than most other business.
13. Explain the difference between illiquidity and insolvency. Does the difference matter?
14. Explain why if the government announces it is abolishing insurance on deposits, a typical bank is likely to face liquidity problems.
15. If yield curves, on average, were flat, what would this say about the liquidity premiums in the term structure?
16. Would you expect the bid-ask spread to higher on actively or inactively traded stocks?
17. Discuss the moral hazard aspects created by deposit insurance.

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Capital Structure

7

The capital structure of a company is the founding stone for the daily development of its operation and for an adequate planning of the business. It says in fact how many resources are available and where they come from.

In the previous chapters, it has been pointed out how the corporate capital can be financed either through debit or equity and what are the main differences between the two sources of financing.

The purpose of the chapter is to give an insight into the rationale and meaning of capital structure in terms of costs and benefits for the company. Traditional and modern theories in fact have given different answers to the issue.

The aim is to answer the questions about what is the right mix of debt and equity for a corporation, how can the riskiness be balanced through choosing appropriate leverage, and many more.

Starting with the classical theory of capital structure, and since the 1950s, with the work of Modigliani and Miller, several scientists have tried to explain the reasons behind specific choices of capital structure.

Modern theories still rely on that work, which represents a milestone in the theory of corporate finance. The bottom line is that finding the right balance between debt, and equity is crucial for the profitability of the corporation.

After studying this chapter, you will be able to answer the following questions, among others:

- What do Modigliani and Miller claim about the relevance of capital structure for cost of capital calculation?
- What is the Weighted Average Cost of Capital and how does it change when considered in the case of a levered or unlevered firm?
- What is the role of taxation in the choice of the right level of debt?
- How does the payout policy of the company impact on the cost of capital?
- What are the other main theories of capital structure and how do they differ from the Modigliani–Miller theorems?

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The first section of the chapter focuses on the Modigliani–Miller theorems and their applicability to the calculation of the cost of capital. The second section is devoted to the payout policy and its impact on the capital structure of the firm. The final section is an introduction to the other most popular theories of capital structure.

7.1 The Modigliani–Miller Theorems

Learning Outcomes

- Describe the Modigliani–Miller theoretical framework.
- Explain the irrelevance of capital structure for the value of the company.
- Learn how to calculate and use the Weighted Average Cost of Capital.

7.1.1 The Irrelevance of Capital Structure

A company can raise funding from either equity or debt sources or combinations of both. The money represents the capital, raised from stockholders (its equity) and funds borrowed (its debt).

The left side of the balance sheet lists the firm's assets, and the right side describes the firm's capital. The relative proportions of debt, equity, and other securities that a firm has outstanding constitute its capital structure.

The work of Modigliani and Miller (MM) (1958, 1963), revised by Stiglitz in 1969, results in two propositions about optimal corporate capital structure. The first proposition relates to the invariance of firm value to its capital structure, and the other concerns its invariance to dividend policy.

They issued the first proposition as an answer to the classical theories of capital structure, which did not take into account the risk associated with differentiating the capital sources. After receiving criticism for it, they also issued the second proposition, in order to justify their findings in light of the received criticisms.

The first proposition states that the choice between debt and equity to finance some level of investments is not relevant. The value of the firm is not affected by the debt-equity mix, and there is no optimal leverage ratio.

In the second proposition, the authors state that under the same conditions as for the first proposition, the dividend policy is irrelevant in terms of the value of the firm, which is not affected.

It is clear that both theorems were acclaimed for carrying surprising results known in economics as irrelevance propositions. The MM theorems made a revolution in the theory of corporate finance.

Both propositions in fact state the irrelevance of capital structure choices that would at first glance sound very important for the value of the firm. Such a result is important not just because it shows that the specified choice is truly irrelevant but also because it pushes to understand under what circumstances it is relevant.

The theorems stand as a benchmark for the capital structure choice. When concepts like optimal leverage and optimal payout ratio enter into play, it is always important to understand why in the specific case the MM theorems do not apply and detect the assumption or the set of assumptions that took us away from the benchmark case.

Merton Miller himself in the late 1980s said that the main message of MM theory is that "the view that capital structure is literally irrelevant or that 'nothing matters' in corporate finance, though still sometimes attributed to us (and tracing perhaps to the very provocative way we made our point), is far from what we ever actually said about the real world applications of our theoretical propositions.

Looking back now, perhaps we should have put more emphasis on the other, more upbeat side of the 'nothing matters' coin: showing how what doesn't matter can also show, by implication, what does."

In order to understand this point, one should recall that the MM result is obtained under very strict assumptions, like absence of taxes and liquidation costs, perfect financial markets, free of frictions, and information asymmetry.

The value of the company is given by the present value of its cash flows, where the discount rate is the required return for firms of the same "risk class." Therefore, the firm's value is determined by its cash flows and discount rate and totally independent from the composition of the liabilities used to finance those assets.

As a consequence of the theorem, the average cost of capital does not depend on the level of leverage and its structure. The return simply matches the equivalent demanded by investors for investments of the same risk class.

Debt is in fact cheaper than equity on average, in that it does not include the risk premium. However, the average cost of capital is not reduced because adding debt to the capital structure increases the risk, therefore increasing the cost of equity too and leaving the balance unchanged.

The separation theorem then comes into force, stating that the investment decision part can be totally separated from the financing issue. The investment decisions should aim at value maximization only.

The cost of capital to be used in rational investment decisions is its total cost, as measured by the required rate of return on fully equity-financed firms of the same risk level.

Starting from the MM work, the following decades have seen the focus of research on corporate finance switched on progressively relaxing the above assumptions. For example, it was straightforward to include taxation in the model, given that debt is normally treated preferentially compared to equity in terms of taxes.

The so-called tax shield is the benefit to the debt holder coming from holding a large amount of debt, due to tax advantages. The relief is given by a shield giving benefits to the payer of interests on a loan.

The first adjustments to the theory were done by the authors themselves, by refining the basic foundation and finding a path for inclusion of the issues related to taxation, market imperfections, and transaction costs.

The conclusions of MM have been therefore revised and opposed in the last years, especially due to the empirical evidence of the high growth potential of American firms, due to higher leverage.

Some other scientists worked specifically on relaxing the taxation assumption by introducing the cost of bankruptcy as a downturn of high leverage, therefore offsetting the benefits of the tax shield.

The modern theory of debt states that the benefits of the tax shield are balanced by the presence of bankruptcy costs. The curve generated by the balance of the two has an interior optimum.

The optimal point represents the amount of leverage that maximizes the value of the firm, corresponding to the point where the marginal benefit from tax shield equates the marginal cost from the increased likelihood of bankruptcy.

The last assumption to be relaxed has been the third one, related to the frictions in the market, that are assumed to be absent in the MM framework. The most widely analyzed friction comes from asymmetric information in the form of adverse selection and moral hazard between investors and company managers.

The introduction of asymmetric information into the bunch of realistic assumptions has been crucial and the focus of most corporate finance research in the last decades, at both the theoretical and empirical level.

In relationship to that, there has also been a huge work on clarifying the issues related to the different incentive properties of the various financial instruments that firms can issue to finance their investment.

The first MM theorem states that, if the firm's total return is not affected by the corporate financial decisions, and borrowing terms are the same for investors and companies, then in equilibrium the firm's debt-equity ratio does not affect the corporate value.

The proof of just a statement relies on the cash flows analysis. Consider two firms, A and B, both having earnings represented by a random variable \$. Define:

- V_A: Total value of firm A
- V_B: Total value of firm B
- E_A: Market value of equity in firm A
- E_B: Market value of equity in firm B
- D_A: Market value of bonds in firm A
- D_B: Market value of bonds in firm B
- r_D: Interest rate paid to the debt holders by firm B

Assume that company A is fully equity financed. The first obvious relationship is that the total value of each firm is equal to the sum of debt and equity capital, as defined by

$$V_A = E_A$$

and

$$V_B = E_B + D_B$$

Assuming all profits I are distributed to shareholders, the investors in company A get a total payment of

$$I_A = I$$

and the shareholders of company B get

$$I_B = I - r_D D_B$$

where:

$r_D D_B$ is the part of income that goes to repay the debt.

In order to prove the theorem, consider the case of a deviation from it, with levered firm B having a higher value, so that $V_B > V_A$. If an investor initially owns a share α of firm B, the return from such an investment is

$$\alpha(I - r_D D_B)$$

Assume the investor holding the above portfolio decides to borrow an amount αD_B and uses the proceedings to buy a fraction of equity of firm A equal to

$$\frac{\alpha(E_B + D_B)}{E_A}$$

Such a new portfolio generates a cash flow equal to

$$\frac{\alpha(E_B + D_B)}{E_A} I - \alpha r_D D_B = \alpha\left(\frac{V_B}{V_A} I - r_D D_B\right) > \alpha(I - r_D D_B)$$

The result is obviously a contradiction.

In order to complete the proof, the opposite case of $V_A > V_B$ must also be analyzed. Starting from an investor owning a fraction α of the equity of firm A, the cost of the portfolio is

$$\alpha V_A$$

and the return is

$$\alpha I$$

The investor can leverage his position by buying a fraction

$$\frac{\alpha V_B E_B}{V_A}$$

of firm B, and a quantity

$$\frac{\alpha V_A D_B}{V_B}$$

of bonds. This replicates the capital structure of a leveraged firm and costs

$$\frac{\alpha V_A E_B}{V_B} + \frac{\alpha V_A D_B}{V_B} = \frac{\alpha V_A V_B}{V_B} = \alpha V_A$$

The return guaranteed by the newly formed portfolio is

$$\frac{\alpha V_A}{V_B}(I - r_D D_B) + \frac{\alpha V_A r_D D_B}{V_B} = \frac{\alpha V_A I}{V_B} > \alpha I$$

The comparison of cost and return shows that the investor is able to make a higher return at the same cost, by choosing to leverage his position. This is again a contradiction which proves the MM theorem 1.

After issuing the proposition together with Modigliani, Miller later argued that debate on the theorem was controversial, given the lack of understanding of the limitations and validity of the model.

In order to make the model more realistic, Modigliani and Miller (1963) later modified their model by relaxing one of the assumptions. According to them, taxation was the primary reason for the capital structure to actually matter in reality.

They support this deviation from their own theory claiming that leverage does matter because interests on debt may be deducted from the firm's income and thereby reduce the net taxable earnings.

The tax savings represent an advantage in using debt capital instead of equity capital, reducing the overall cost of capital to the firm. The effect of taxation can be graphically illustrated as in Fig. 7.1.

7.1.2 The Weighted Average Cost of Capital

Calculation of a discount rate can be usually done in several ways. When it comes to the cost of capital, a common strategy is to use the method of Weighted Average Cost of Capital (WACC).

The WACC equation gives an average discount rate for the cash flows of the firm. It can be expressed as the weighted average of the cost of equity and the cost of debt based on the proportion of debt and equity in the company's capital structure

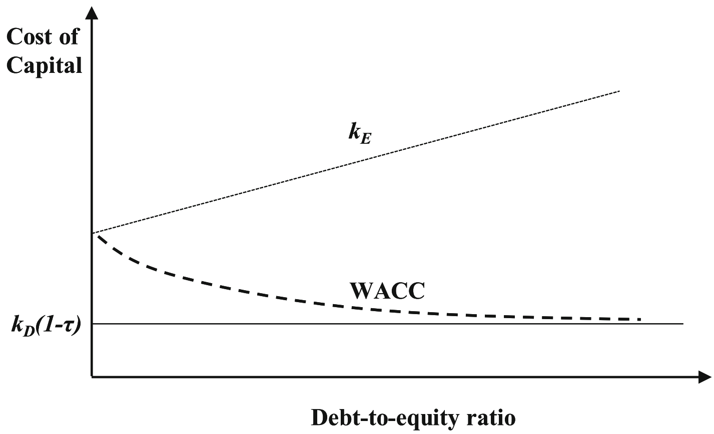

Fig. 7.1 The WACC can be represented as a weighted average of cost of equity and cost of debt

$$r_{\mathrm{WACC}} = r_E\left(\frac{E}{V}\right) + r_D\left(\frac{D}{V}\right)$$

where:

V is the firm's total value.

$\frac{D}{V}$ is the proportion of debt (leverage ratio).

$\frac{E}{V}$ is the proportion of equity.

The WACC varies according to the variations of the debt-to-equity ratio. Given the model, both the cost of equity and cost of debt vary according to the capital mix, with the cost of debt increasing for higher levels of leverage, and the cost of equity increasing too due to the higher riskiness of capital due to high leverage.

The costs of debt and equity track each other because equity holders are always taking more risk than debt holders and therefore require a premium return above that of debt holders.

The WACC equation shows that the overall cost of capital of the firm is given by the weighted average of its debt and equity costs, as from the initial assumption. However, in its simplest form, it does not take into account the effect of tax relief of interest payments on debt.

The firm can in fact benefit from a tax shield which is given by the savings made on taxes by paying the interests on the loan to the bank or the yield on the bond. It turns out that the debt part of the cost of capital is affected by the relief (see Fig. 7.2).

It is then possible to calculate the cost of capital of the firm's assets by computing the weighted average of the firm's equity and debt cost of capital and also including the tax shield effect. The equation can be written as

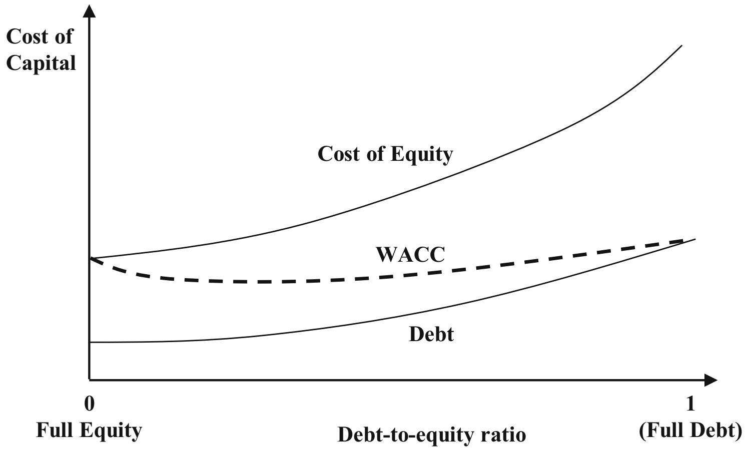

Fig. 7.2 The graph shows the WACC line as a result of combining the cost of equity and cost of debt in different proportions without tax effects

$$r_{\text{WACC}} = r_E\left(\frac{E}{V}\right) + r_D\left(\frac{D}{V}\right)(1 - \tau_C)$$

where:

τ_C is the effective marginal corporate tax rate.

Example 7.1 The equity of a company *Z* is valued €31,200 million, and the debt has a value of €7350 million. The total value is

$$31,200 + 7350 = €38,550$$

Given a cost of equity of 13.25%, a cost of debt of 6.5% and a tax rate of 33%, the corresponding WACC is

$$r_{\text{WACC}} = 0.1325\left(\frac{31,200}{38,550}\right) + 0.0655\left(\frac{7350}{38,550}\right)(1 - 0.33) = 11.56\%$$

When the cost of equity is not directly available, it is sometimes necessary to calculate it by using the CAPM formula.

Example 7.2 A company as a capital structure consisting of a 35% debt and 65% equity. The tax rate is 33% and the cost of debt for the company is 6%. The risk-free rate in the economy is 2%, and the beta of the company stock is 1.1. The risk premium calculated on the market is 7%. It is possible to use the parameters to calculate the WACC as

$$r_E = 0.02 + 1.1(0.07) = 9.70\%$$

The WACC is therefore

$$r_{\text{WACC}} = 0.0977 \times (0.65) + 0.06 \times (0.35)(1 - 0.33) = 7.76\%$$

Actual values of WACC for companies vary widely. It is not uncommon for WACCs to range from 3 to 4% up to 20% or more. Various websites provide WACC estimates for publicly traded companies.

One of the main controversies of WACC application is that the rate is calculated without any time dependence factor, as a static measure valid at an instant in time, but it is then used to model the time value of money.

Another problem is that while it may accurately reflect what a company believes its cost of money is at the current time, the dynamics of the broader economy and the company's capital structure change with time.

The WACC is therefore not static, but dynamic in time, for several reasons. First of all, the company debt ratio changes over time, with only few companies adopting a policy of fixed debt ratio.

Moreover, both the cost of equity and cost of debt may change over time, with a tax rate correlated to the profitability of the company, with some companies getting different breaks due to their location.

It is important to calculate the WACC for a future period of time, even if it is a very difficult task. The standard assumption of constant WACC in the future could in fact lead to severe mistakes in the calculation of the firm's value.

Several factors determine the WACC for every specific company, given the industrial sector of operations. Some industries become so mature that the investors in companies of that sector demand lower risk premiums.

Also, the situation of the global economy affects the cost of debt, through the level of interest rates in the economy. The dynamics also involves the debt ratio, which will vary according to the ability of the firm to reduce their risk, therefore taking on large shares of debt. Finally, the corporate tax rate will change because the company becomes profitable and the expiration of tax breaks granted by local and national governments.

Practical solutions to make the WACC dynamic are to look at the inputs from a dynamic point of view. For example, the trends over time in the cost of debt can be modelled with a yield curve, and the cost of equity can be modelled using a CAPM model with a beta trending over time.

The bottom line is that the WACC is basically a probability distribution given that all the parameters affecting it are probability distributions. The appropriate distribution can be estimated through numerical simulations, including Monte Carlo. In addition, the WACC is a nonstationary process.

The overall corporate cost of capital is determined by each component of the firm's capital. Financial managers take into account the impact of all types of capital which represent a cost to the firm.

Investors who put money in the stock and bonds of a company are locking money that could be used elsewhere. The expected return from those alternative investments constitutes an opportunity cost to them.

The corporate cost of capital should be calculated as a mix of the cost of different sources of capital. The most common way is to calculate a Weighted Average Cost of Capital, defined as the weighted average of equity and debt cost of capital.

The weights are represented by the proportions of debt and equity in the firm's capital mix. It is then possible to calculate the weights just by looking at the right-hand side of the balance sheet.

But an important modification must be made to it, because the real proportions are not given by the book values. The calculation involves the market values of the debt and equity. Proportions are then determined by using those proportions.

Recall that the book values in fact reflect historical costs, as opposed to market values, which are forward-looking and based on the cash flows that the assets are expected to produce in the future.

The investors value a company not for its book value but for the value assigned by other investors to it, due to market conditions and available information. Of course, the market value balance sheet must still balance.

Therefore, the total market value of the firm's assets must equate the market value of the firm claims (debt and equity). This equality clearly confirms that the equity and debt issued by the firm derive their value from the underlying assets they claim.

As a consequence, the risk and required return of debt and equity are determined by the risk of the firm assets they employ. The WACC derivation is based on this important concept.

In order to derive the WACC, a good approach is to consider two different financing scenarios of a firm with debt and without debt. This allows to understand the use of market value weights.

If the firm has no debt, therefore fully financed with equity, all the free cash flows are paid to equity. The free cash flows to the equity are the same as those from the assets so that according to the valuation principle, the market value, risk, and cost of capital for the firm's equity are equal to the corresponding amounts for its assets.

In the case of full equity financing, it is then possible to calculate the cost of capital by simply applying the CAPM and estimate the cost of equity, which in turn is the cost of capital for the firm as a whole. For example, both Cisco and Apple do not issue debt, so the cost of capital for Cisco's or Apple's assets is the same as the firms' costs of equity.

In the case of a firm issuing debt, on the other hand, the issue is about how to incorporate the cost of debt in the calculation of the overall cost of capital of the firm as a whole.

In order to do so, recall the market value balance sheet, and consider the leveraged firm and a corresponding replicating portfolio of the firm's debt and equity. Holding the portfolio is the same as holding the firm's debt and equity.

Holding that portfolio yields the same cash flows as those generated by holding the firm's assets directly. Recall that the return of a portfolio is given by the weighted average of the returns of the securities in it. It follows that the relationship between the required returns (costs) of equity, debt, and assets is given by the WACC.

The WACC is driven by the risk of a company's line of business and, because of the tax effect of interest, its leverage. As a result, WACCs vary widely across industries and companies.

When estimating WACC in practice, financial managers have to face some issues. For example, in some cases, the weights are calculated by using an adjusted value for the debt part. Many practitioners now use net debt, the total debt outstanding minus any cash balances as defined by

$$D_N = D - S_{\mathrm{RF}}$$

where:

D_{NET} is the net debt.

D is the debt.

S_{RF} is the cash and risk-free securities.

The reason to subtract cash from debt is that, assuming interest is paid and earned on both, the two cash flows will offset each other, just as if the firm held no cash and no debt. In fact, we can view cash as being equivalent to negative debt.

When the firm has a huge amount of excess cash on its balance, assessing the risk and cost of capital of the asset employed in some line of business is complicated. Separating the debt from any cash holdings therefore requires measuring the leverage of the firm in terms of its net debt and measuring the market value of a firm's business assets using its enterprise value.

The sum of the market value of the equity of a company plus its net debt is called enterprise value. It can be used to calculate the WACC through a new equation with weights given by

$\frac{E}{V_E}$ and $\frac{D_N}{V_E}$ where:

$V_E = E + D_N$ is the enterprise value of the equity.

The adjustment is very important for companies that have a large excess of cash reserves; otherwise it will not change much the WACC estimate for companies with low levels of cash.

Back to the use of CAPM for estimating the cost of equity, recall that the model involves the risk-free rate in the calculation. It is usually determined using the yields of governmental securities like US Treasury securities, which are free from default risk.

The main issue is about the time horizon to use and which maturity should be observed. Following the rules of CAPM, one should use the risk-free interest corresponding to the investment horizon of the firm's investors. This is why, when it comes to valuing companies, usually the chosen maturities are very long, ranging from 10 years to 30 years.

Another important piece of the CAPM equation is the market risk premium, which can be estimated in several ways. One way is to look at historical data, but being interested in the future market risk premium, there is an issue regarding accuracy and the amount of data we use.

In fact, even if a large amount of data ensures statistical accuracy, going backward for a too long period of time may include data that are very old and that may have little relevance for investors' expectations of the market risk premium today.

7.2 Payout Policy

Learning Outcomes

- Understand the role of dividends in the capital structure.
- Understand the role of share repurchase in the capital structure.
- Learn how to reconsider the Modigliani–Miller framework.

7.2.1 Dividends vs. Share Repurchase

Dividends are distributions of a share of the earning made by the company, to some class of shareholders. The dividend payment is decided by the board of directors and can be issued as cash payments, as shares of stock, or other property.

There are two ways to quote a dividend of a company. It can be either quoted in terms of the actual amount of money received by the shareholders (dividend per share) or can be quoted as a percentage of the current market price of the share (dividend yield).

Dividends are commonly paid out every 3 months and give stockholders a steady return, regardless of what happens to the stock price. It is a cash return of substantial importance for the investors.

Another way to employ the company's earnings is through retained earnings, so keeping them in the equity of the company. Also, it is possible for the company to use net profits to repurchase their own shares in the open markets in a share buyback.

Both dividends and share repurchases are payout policies that do not change the fundamental value of the outstanding shares. Dividend must be approved by the shareholders and may be structured as a one-time special dividend or as an ongoing cash flow to owners and investors.

The expected dividends are a variable of interest in determining the value of the corporate shares. Besides being a cash inflow for the shareholders, they provide information about the profitability of the company.

Retained earnings stay in the company in the form of equity, therefore altering the capital structure of the firm. This is why the dividend policy can impact on the external financing requirements of the firm.

So if the firm needs to raise capital, the amount to be raised from external sources will be higher, for larger cash dividend paid to the shareholders. The company will then have to borrow debt or issue new shares.

The issue and payment of dividends follow a very specific procedure that can be summarized in several steps. The decision about issuing dividends is made by the board of directors at quarterly or semiannual meetings.

The dividend decision is based on the past financial performance and on the future outlook on the profitability. Moreover, recent dividend payments are taken into account for the decision. The payment date of the cash dividend, if one is declared, must also be established.

The dividend policy of the firm sets the amount of dividend and other important decisions. The management of the company can change the policy on the basis of significant changes in earnings.

The decision of issuing a dividend is commonly followed by a statement indicating the information related to the decision and the execution of it. It therefore reports the record date and payment date. The statement is usually published on major financial magazines.

The record date is the time when existing shareholders at that time are recorded to receive the dividend. The recorded shareholders are called holders of record and are eligible to receive the dividend payment at a specified future time.

The trading of a stock on the financial market involves bookkeeping which is time consuming. It takes some time to register the purchase of a stock, and that is why the stock begins selling ex dividend in 2 days prior to the date of record.

Purchasers of a stock selling ex dividend do not receive the current dividend.

A simple way to determine the first day on which the stock sells ex dividend is to subtract 2 days from the date of record.

In case of a weekend, 4 days must be subtracted instead. In normal market conditions, the price of the stock in ex dividend days is expected to drop by the amount of the dividend, as declared.

After the record date, the management also sets a payment date, when the firm is supposed to send the dividend payments to the holders of record. It normally takes a few weeks after the record date for it.

Many companies nowadays offer to the shareholder dividend reinvestment plans, a policy that allows the shareholder to purchase additional shares using the proceedings from the dividend.

The advantage is that the purchase can even be fractional and there are no transaction costs involved (or they are very small). Some companies even allow investors to make their initial purchases of the firm’s stock directly without the need of a broker.

There is usually a small discount on the market price, when buying shares through a dividend reinvestment plan. It is also a cheaper way to issue new shares, avoiding the typical issues of a public sale. Clearly, the existence of a DRIP may enhance the market appeal of a firm’s shares.

The dividend policy is a plan of action, and the management follows it after the decision about issuing dividends is made. The policies developed by the companies are logically consistent with their corporate goals.

The factors affecting a dividend policy can be legal, contractual, internal, and economical (related to the firm's growth potential). There are also issues related to the market and the shareholders.

Legal constraints are those related to the current regulation in the country of the firm. For example, it is a widely developed principle that a company cannot pay out as dividends any portion of its legal capital.

Legal capital is generally defined as the par value of the common stocks, but the definition changes geographically. Some countries include in the legal capital also paid in capital beyond the par value. These capital impairment restrictions are generally established to provide a sufficient equity base to protect creditors' claims.

Sometimes there are limitations on the earnings required to issue a certain amount of dividends, which must be adequately high. The firm is not allowed to pay out dividend amounts beyond the sum of recent retained earnings.

Other regulatory systems prohibit the payment of cash dividends in case the company is not solvent, with overdue liabilities in the record. On the other hand, some authorities also punish an excessive accumulation of earnings, usually meant at reducing the shareholders' taxation.

As mentioned above, there are also contractual constraints to the dividend payment. Generally, these constraints allow the firm to pay cash dividends only when a certain level of earnings has been achieved. It could be also the case that dividends are limited to a certain dollar amount or percentage of earnings.

There are also internal constraints, mainly due to the amount of liquid assets available to the firm at the time of dividend issue. In fact, it is possible for a company to borrow funds to pay dividends, but the lenders are not keen to give this type of loan in that they are not productive, making the loan itself very risky.

The dividend policy must be designed in order to take into account two major issues. First of all, there must be sufficient funds, and secondarily the dividends must be such to give sufficient financing and maximizing the wealth of the firm's owners.

As mentioned above, several dividend policies can be applied, and in some cases, the policy is customized so as to contain one or more element from each of the following policies.

The constant-payout-ratio dividend policy implies there is a constant-payout-ratio in the dividend policy of the firm. It means that the firm pays out to the shareholders a constant percentage of each euro earned.

The ratio is calculated by dividing the cash dividend per share by earnings per share. A constant-payout-ratio dividend policy means that the firm establishes a certain percentage payment on earnings to the shareholders in each dividend period.

This policy has a downturn related to the fluctuation of earnings overtime. It could happen in fact that the dividends may become very low or null. Given the signaling effect of dividends, the firm's stock price may thus be adversely affected.

Another popular dividend policy is the so-called regular dividend policy, which is based on the payment of a fixed-dollar dividend in each period. The shareholders usually get positive information from that policy, thereby minimizing their uncertainty.

This policy is subject to changes, and the amount of the regular dividend is usually increased when there is a stable and proven increase in earnings. The opposite in general never happens and dividends are not decreased.

The dividend following a regular payment policy is often built around a target dividend-payout ratio. The dividend is set at a target ratio and kept regular at the beginning. If any fluctuation occurs, the ratio is adjusted to the target payout.

Some companies adopt a policy on a low-regular-and-extra basis, thus paying a regular dividend, and attaching an extra dividend to it in times of increased earnings. This policy allows to always be realistic and avoid making unreliable promises to the shareholders. This policy is especially common among companies that experience cyclical shifts in earnings.

Having a regular dividend, even of a low amount, is useful for the company to gain the confidence of the shareholders. On top of that the extra dividend allows the shareholder to share the benefits of good business cycles.

The companies adopting such a policy usually are entitled to pay out the extra dividend only if and when earnings are proven to increase. The use of a target dividend-payout ratio in establishing the regular dividend level is advisable.

Finally, it is important to also analyze the stock split strategy. It is a strategy commonly used to lower the market price of some stock, by increasing the number of shares belonging to each shareholder.

The most common type of stock split is the 2-for-1 split, where two new shares are exchanged for each old share. Each new share is worth half the value of each old share. As for dividends, a stock split has no effect on the firm's capital structure.

The reason for a stock split is usually related to the perception that a stock is priced too high and that lowering the market price will enhance trading activity, making the investors more attracted to it.

The right moment for a stock split is generally right before issuing new stocks, in that the marketability is enhanced and the market gets properly stimulated. Sometimes stock splits generate a small increase in the stock price, given the information implied in it and the general increased in dividend payments after a split. Stock can be split in any way desired. Sometimes a reverse stock split is made: a certain number of outstanding shares are exchanged for one new share.

The practice of share repurchase has gained popularity and increased application in recent years. A company can in fact buy back its own shares on the market, thus reducing the number of shares outstanding.

There are several reasons behind the choice of repurchasing own shares, mostly related to having shares available for employee stock option plans and retiring shares. Stock repurchases enhance shareholder value and help to discourage an unfriendly takeover.

The value for the existing shareholder is enhanced in that the number of shares outstanding is reduced, therefore increasing the earnings per share. Moreover, the strategy sends a positive signal to investors (undervalued share). It also provides a temporary floor for the stock price, when declining for some reason.

As a tool for preventing and discouraging hostile takeovers, the share repurchase is based on the belief that a hostile investor is less likely to gain control of the firm if there are fewer publicly traded shares available.

As a signal, the decision of repurchasing shares shows the confidence of the management in the profitability of the company, therefore attracting the interest of investors on the market.

If the retained earnings are constant, the share repurchase decreases the amount of outstanding shares, thus positively affecting the earnings per share and the market price per share.

The repurchase of common stock results in a type of reverse dilution, because the EPS and the market price of stock are increased by reducing the number of shares outstanding. The net effect of the repurchase is similar to the payment of a cash dividend.

There are also tax benefits for shareholders, given that if no dividend is distributed, there is no ordinary income tax to be paid on it. The repurchase instead increases the market value of the share of an amount equal to the dividend not paid, and the value increase is not taxed until the shareholder sells the stock on the market.

Also when the stock is sold, the capital gain taxation on it is usually much more favorable than the taxation on dividends. However, the monitoring authorities in some countries are supposed to issue a penalty when it is believed repurchases have been made to delay the payment of taxes by stockholders.

From an accounting point of view, the stock repurchase reduces the cash and adds up an entry on the other side of the book, commonly named treasury stock, which is shown as a deduction from stockholders' equity.

The process of repurchasing share involves informing the shareholders about the intention of the management. In particular, the purpose of the repurchase should be communicated, as well as the use to be made of the repurchased shares.

There are commonly three methods of repurchase that can be used. One method is the purchase of the stocks on the open market, which puts pressure on the price to increase, when the quantity repurchased is reasonably large in comparison with the total number outstanding.

Another method is the tender offer, which is a formal proposal of purchase at a specified price, issued to the shareholders. The price at which a tender offer is made is normally set above the current market price to attract sellers.

Sometimes, in case the tender offer does not allow to buy back the full amount of shares desired, the two methods can be complemented, and the additional shares can be bought on open market.

Tender offers are preferred when large numbers of shares are repurchased, because the company's intentions are clearly stated and each stockholder has an opportunity to sell shares at the tendered price.

A third method is through negotiation of large blocks of shares between major shareholders. The firm in this case must make sure that the purchase price is fair and equitable in view of the interests and opportunities of the remaining shareholders.

7.2.2 Modigliani–Miller Revisited

Under the assumptions of homogeneous expectations and perfect market, the Miller and Modigliani (MM) dividend irrelevancy proposition asserts.

While dividends are relevant, the dividend policy is irrelevant. The proof of just a statement relies on the cash flows analysis. Consider again two firms, A and B, both having earnings represented by a random variable I. Define:

- d_t: dividend at time t
- D_t: borrowings of the firm at time t
- L: investment at time 1
- I_1: returns at time 1
- I_2: returns at time 2

Following the logic of the above proof of the MM theorem, suppose that the total return I of a firm is unaffected by financial decisions, and buying and selling of securities happen at the same terms for all investors.

First of all, the sum of the borrowing at time 1 and the returns at time 1 must be equal to the sum of the dividends at time 1 and the investment at time 1, as from

$$I_1 + D_1 = d_1 + L$$

or

$$D_1 = d_1 + L - I_1$$

The returns at time 2 are given by the sum of the dividend at time 2, and the amount borrowed at time 1 increased of the interest at the risk-free rate r, as defined by

$$I_2 = d_2 + D_1(1 + r)$$

where the amount of dividends at time 2 is given by

$$d_2 = I_2 - (d_1 + L - I_1)(1 + r) \tag{7.1}$$

Consider an investor owning a share α of the equity. The budget constraint of the investor is given by

$$c_1 + = \frac{c_2}{(1 + r)} = e_1 + \alpha d_1 + \frac{e_2}{(1 + r)} + \alpha \frac{d_2}{(1 + r)} \tag{7.2}$$

where:

c_1 is the consumption at time 1.
c_2 is the consumption at time 2.
e_1 is other income at time 1.
e_2 is other income at time 2.

By substituting from 7.1 into 7.2, one obtains

$$c_1 + = \frac{c_2}{(1+r)} = e_1 + \alpha d_1 + \frac{e_2}{(1+r)} + \alpha \frac{I_2 - (d_1 + L - I_1)(1+r)}{(1+r)}$$

which can be rewritten as

$$c_1 + = \frac{c_2}{(1+r)} = e_1 + \alpha d_1 + \frac{e_2}{(1+r)} + \alpha \frac{I_2}{(1+r)} - \alpha \frac{(d_1 + L)(1+r)}{(1+r)} + \alpha \frac{I_1(1+r)}{(1+r)}$$

It follows that

$$c_1 + = \frac{c_2}{(1+r)} = e_1 + \frac{e_2}{(1+r)} + \alpha \frac{I_2}{(1+r)} + \alpha I_2 - \alpha L$$

The result shows that the shareholder's budget constraint is independent of the dividend, since neither d_1 or d_2 appear in the final formula. Therefore the consumption is totally independent of dividend policy.

Example 7.3 Suppose a firm has 100,000 shares of stock and cash flow of €100,000 in perpetuity. The discount rate is 10%. Three dividend policies are possible.

– The first policy is to pay €10 dividend per year. In this case the stock price should be

$$P_0 = \frac{10}{0.1} = €100$$

– Another policy is to pay a €20 dividend in the next period and the remainder afterward. To do this, the company must go on a debt of €100,000, thus paying a passive interest of

$$P_0 = \frac{20}{(1+0.10)} + \frac{10}{(1+0.10)} \times \frac{0.90}{0.10} = €100$$

– A final option is to pay each shareholder 1 share of stock today so that the firm has 20,000 shares outstanding, each giving the right for €5 dividend. It follows that

$$P_0 = \frac{5}{0.10} = €50$$

Since all existing shareholders have now double number of shares, their total wealth remains unchanged also in this last case.

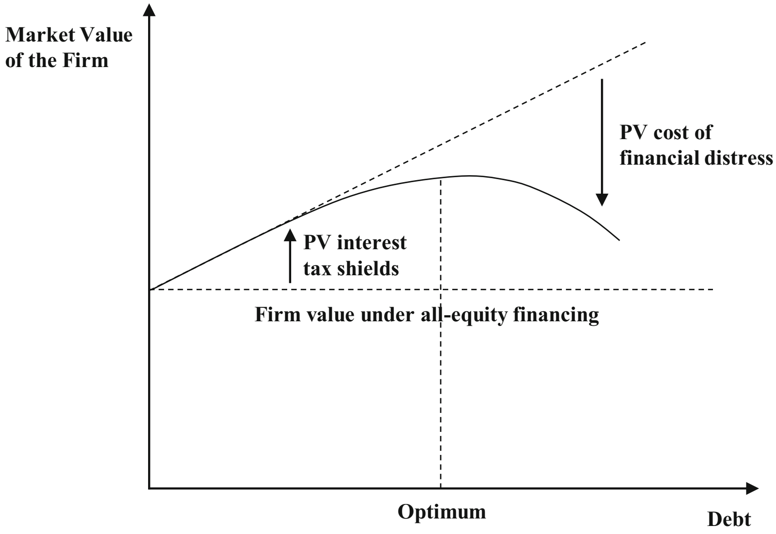

Fig. 7.3 The static trade-off theory predicts an optimal target over which the actual debt ration reverts. The optimum is derived from the interest tax shields and the costs of financial distress

7.3 Modern Theories of Capital Structure

Learning Outcomes

- Learn the static trade-off theory of capital structure and the pecking order hypothesis.
- Learn how to calculate the adjusted present value of an investment.
- Understand the role of taxes in the theory of capital structure.

7.3.1 Static Trade-Off Theory and the Pecking Order Hypothesis

The static trade-off theory was developed by Myers in 1984 and 2001, and focuses on the debt ratio of the firm. In particular, it states that there is an optimal target debt-to-value ratio and the firm's capital structure is gradually moving toward the optimum.

In order to determine the optimal level of debt to include in the capital structure, one must consider the balance between the tax shield advantage and the disadvantage coming from the bankruptcy costs (see Fig. 7.3).

The equilibrium corresponds to an optimum where the marginal benefit of an additional dollar of debt balances the marginal cost for it. For levels of debt above the optimal point, the bankruptcy costs will prevail. The theory can be illustrated by using a graph.

Some authors, like van der Wijst and Thurik (1993), argue that several factors affect the tax effect on the profitability of the firm. First of all, there are other non-debt tax shields, such as depreciation deduction, tax loss carry-forward, and investment tax credits, that can substitute efficiently the tax shield from leveraging.

Moreover, there is an offset effect of personal taxes on the interest of debt, when the tax on fixed income earnings is higher than the income from equity holdings, as described by Miller in 1977.

Third, different tax regimes can generate differences in investors' preferences. In countries with lower tax rates applying to capital gains than to dividends, investors prefer capital gains to stock ownership.

Tax shield interest generally creates incentive for corporate leveraging, and in case there are no costs offsetting the benefit of the tax shield, the debt will be used at its maximum level.

The reality is of course different. Standard borrowing of funds carries bankruptcy costs that are not avoidable. Therefore, the static theory claim that there is an optimal level of debt makes sense.

It is also useful to understand what these bankruptcy costs are and how they can impact on the choice of leveraging. They are all the costs that are added when the company goes bankrupt, and the failure is being processed, with the firm not changing its operating or external financing activities (Haley and Schall 1979).

The costs of bankruptcy can be divided in direct costs and indirect costs. The former category refers to all legal and administrative costs, plus the cost of negotiation with the stakeholders.

The indirect costs can arise from the imperfection of secondary markets or opportunity loss, such as decline in market share and distress sales. Investors react negatively to the hypothesis of financial distress, and the stock price goes down. Moreover, the benefits generated from tax advantage can be cancelled out by the increasing debt levels.

The pecking order hypothesis was first introduced by Myers and Majluf in 1984. The theory states that companies choose internal funds preferentially, up to some amount of internal funds available for investment.

If the investment needs exceed the threshold, the company raises the funds as debt, to fill the remaining part. Debt finance is then prioritized to equity finance, and equity finance will be used only when the investment exceeds beyond a further threshold, equal to the sum of internal funds, and the debt issued to fill the financing deficit.

Some companies may decide to not use all internal available funds in order to maintain a reserve of equity for future opportunities. In this case external resources will be needed when the investment exceeds the internal fund net of the reserves to be kept.

Due to the information-sensitive equity issuances, debt is issued to fill the financing gap along with insufficient internal funds if there is no significant financial distress. Literally, firms will never issue equity and the second (higher) threshold is infinite.

Firms have to issue equity in case the investment exceeds the debt capacity, which is the sustainable leverage. In case of a cost of equity lower than the value generated by positive NPV investments, the firm will raise funds through equity issuance.

The same holds in case the NPV of the total corporate assets is above the capital gain that can be realized on the newly issued share. The asymmetric information between inside management and investors generates variance of percentage changes in equity value. The firm will issue new equity in case the investors overestimate the variance rate.

The asymmetry of information between managers and shareholders is the basis of the pecking order theory. Current shareholders have different expectations compared to new investors and tend to act to protect themselves.

Therefore, equity is the last option for financing externally due to the fact that current shareholders are not willing to share the benefits of investment or cause the decline of share prices.

The advantage of internal funding in terms of costs is given by the absence of the typical costs of external financing, which are generated by the asymmetric information and the agency costs. Thus the availability of internal funds can heavily influence the major investment decisions of the company (Cleary 1999).

The information possessed by managers is always superior to that possessed by the investors. An issuance of debt or equity is a signal to the investors that internal funding is not enough to cover new investments.

If a decision of issuing equity was released, investors would feel the stock price is overvalued. The reaction of the market will be to sell the shares, and the stock price will drop, damaging the value of firm.

The adverse selection problem caused by the asymmetry of information has been described by Akerlof in 1970. If there are both good and bad opportunities for investment, investors cannot distinguish which ones are good or bad.

This is because investors have less accurate information so that they are possibly prone to take the worse opportunities while giving up the good ones according to the average level of products.

It follows that the good investment opportunities will be underestimated and not implemented, and the adverse selection cost will hit severely on the investors that will then be penalized by investing in bad projects.

Solutions to the problem have been proposed by Healy and Palepu (2001) among others. The authors state that optimal contracts between investors and firm management should be provided for disclosure of private information.

Secondarily, regulators should take care of forcing managers to disclose the private information they have. The final provision proposed by the authors is the introduction of information intermediaries, analysts, or agencies, to reduce the information gap.

The message from the pecking order hypothesis is that the equity capital is very sensitive to the information hitting the investors and carries a large adverse selection, compared to other securities. This is why for external financing, debt is usually preferred given the lower amount of adverse selection cost.

Small firms have commonly more asymmetric information than big ones, therefore facing more severe adverse selection problems. As a consequence, they are supposed to perform better in pecking order hypothesis.

But one should consider the lower credit capacity and higher riskiness of small businesses compared to the big ones. This may lead the former to opt for equity financing and perform worse in pecking order.

There are also moral hazard aspects connected to information asymmetry. As underlined by Holmstrom in 1979, the quality of financial investments is not perceived by the investors due to lack of information.

Managers are prone to cheat or fail to make the necessary efforts, because they often fail to act in the best interest of the investors. For instance, managers can expropriate investors' funds through both issuance of debt and equity.

7.3.2 The Adjusted Present Value

The traditional NPV analysis does not take into account the implications of debt financing when valuing a project. In order to capture these aspects, it is possible to run an adjusted present value (NPV) analysis.

APV is defined as the present value of a project if financed solely by equity plus the present value of financing benefits. It stands as an alternative valuation method similar to the NPV approach.

The difference is that it uses the cost of equity as the discount rate rather than WACC. And APV includes tax shields such as those provided by deductible interests. APV analysis is effective for highly leveraged transactions.

Recall the basic NPV capital budgeting equation defined as

$$\mathrm{NPV} = \sum_{t=1}^{N} \frac{C_t}{(1 + r_{\mathrm{WACC}})^t} + \frac{\mathrm{TV}_N}{(1 + r_{\mathrm{WACC}})^N} - C_0 \tag{7.3}$$

where:

C_t is the expected after-tax cash flow for year t.
TV_T is the expected after-tax terminal value.
C_0 is the initial investment.

The NPV is the difference between the present value of all cash flows, and recall that the related rule states that a project should be accepted when its NPV is positive, and rejected otherwise. The NPV decision rule is considered the superior framework for analyzing a capital budgeting expenditure.

First of all, the NPV equation must be expanded in order to innovate the analysis. Capital budgeting is concerned with the change in the firm's total cash flows assigned to the capital expenditure.

The incremental change in the total cash flows to the firm for a period t which results from the capital project is given by

$$C_t = NI_t + D_t + I_t(1 - \tau_C) \tag{7.4}$$

where:

NI is the net income at time t.
D_t is the depreciation at time t.
I_t is the interest expense at time t.
τ_C is the corporate tax rate.

Eq. (7.4) shows that the incremental cash flow is given by the sum of three main cash flows.

Recall that depreciation is not a cash flow, and the second term of the equation adds it back to the calculation, after it was subtracted from the Nit just for tax purposes in the previous mathematical passage.

Through simple math, it is possible to rework Eq. (7.4) to obtain

$$C_t = C_t^{\mathrm{OP}}(1 - \tau_C) + \tau_C D_t$$

where:

C_t^{OP} is the amount of operating cash flows.

It is now possible to expand the NPV model, by substituting Eq. (7.4) into (7.3), so that the NPV equation becomes

$$\mathrm{NPV} = \sum_{t=1}^{N} \frac{C_t^{\mathrm{OP}}(1 - \tau_C)}{(1 + r_{\mathrm{WACC}})^t} + \sum_{t=1}^{N} \frac{(1 - \tau_C D_t)}{(1 + r_{\mathrm{WACC}})^t} + \frac{\mathrm{TV}_N}{(1 + r_{\mathrm{WACC}})^N} - C_0$$

Recall that Modigliani and Miller derived a statement about the relationship between the value of a levered firm and the corresponding equivalent unlevered firm, as described by

$$V_L = V_U + \tau_C \mathrm{DBT}_t$$

where:

DBT_t is the debt at time t.

By separating the equity part and the debt part, the final version of the APV is given by

$$\mathrm{APV} = \sum_{t=1}^{N} \frac{C_t^{\mathrm{OP}}(1 - \tau_C)}{(1 + r_E)^t} + \sum_{t=1}^{N} \frac{\tau_C D_t}{(1 + r_D)^t} + \sum_{t=1}^{N} \frac{\tau_C I_t}{(1 + r_D)^t} + \frac{\mathrm{TV}_N}{(1 + r_E)^N} - C_0$$

where:

r_E is the cost of equity.
r_E is the cost of debt.

Capital budgeting through APV is a value-additivity approach that considers every source of value singularly. Cash flows are discounted at a relevant rate that is consistent with the risk inherent in that cash flow.

In particular note that the OCF_t and TV_T are discounted at the rate Ku. These cash flows in fact would reach the company from a capital project regardless of whether the firm was levered or unlevered.

The discount rate for the amount of tax shield savings, It, is the before-tax borrowing rate, i. There are also tax savings due to depreciation, τD_t, that should be discounted at the rate i as well, due to the relative low risk compared to operating cash flows.

The decision rule for APV is the same as for the NPV. The project should be accepted if APV ≥ 0 and rejected otherwise. A multinational company can use it to analyze one of its domestic capital expenditures.

The comparison of APV and cost of capital analysis shows that there are several important differences. In an APV valuation, the value of a levered firm is obtained by adding the net effect of debt to the unlevered firm value.

The effect of leverage is instead directly embedded in the cost of capital. The tax shield is in fact incorporated in the cost of debt, while the levered beta and pre-tax cost of debt incorporate the distress costs.

In theory the results from the two models should be equivalent, but there are differences. For example, the APV approach handles bankruptcy costs with more flexibility regarding treatment of the indirect costs.

The result from APV approach will therefore be more conservative in value estimation. Moreover, APV approach considers the tax benefit from a dollar debt value, while the cost of capital approach estimates the tax benefit from a debt ratio that may require the firm to borrow increasing amounts in the future.

7.3.3 Tax-Based Theories of Capital Structure

In order to introduce taxation in the theories of capital structure, it is necessary to make some assumptions to define the environment of a firm’s operations and the ideal market for business.

The first assumption is the absence of transaction costs to buy and sell securities on the market. Moreover, there is no bid-ask spread, so the buying price is the same as the selling price.

Another assumption is perfect competition on the capital market, with firms and investors being price takers. Bankruptcy costs are absent. Also recall that in the original MM framework, there are no corporate or personal taxes. Information is homogeneous on the market.

Consider two firms, which are identical in the distribution of cash flows but have different capital structure. It is possible to examine the implications of corporate and personal taxation for optimal capital structure.

The two firms are such that U is an all-equity firm, while firm L is leveraged, and they both generate a random cash flow, X˜, whose expectation is Xˆ in each period of operations.

Compared to the abovementioned original MM work, the assumption of absence of taxation is modified, and a unique corporate tax rate for all firms, τ_C, is introduced. Moreover, personal taxes on interest income, τ_D, and on equity (dividend and capital gains), τ_E, are also included in the model.

In order to complete the set of assumptions, there is an interest rate r_0 on tax-exempt bonds and a before-tax yield r on risk-free corporate bonds. By maintaining the assumptions of the original MM model and modifying the absence of corporate taxation with the introduction of the abovementioned taxation, the new model can be defined as

$$V_L = V_U + \left[1 - \frac{(1-\tau_C)(1-\tau_E)}{1-\tau_D}\right]D$$

In order to give a proof to the above equation, consider an investor holding a fraction α of the unlevered firm U. His per-period net future cash flow is

$$\alpha I(1-\tau_C)(1-\tau_E)$$

and the value of his portfolio is αV_U.

Consider an investment strategy consisting in selling the shares in firm U and using the proceedings to buy

– A fraction α of the equity of levered firm L
– A fraction $\beta = \alpha I(1 - \tau_D D)(1 - \tau_C)(1 - \tau_E)$ of the debt of firm L

In this way the investor uses his wealth to generate a leveraged position, which simulates the capital structure of the levered firm.

The strategy yields peculiar per-period payoffs. In particular, the payoff from equity is given by

$$\alpha I(1-\tau_D D)(1-\tau_C)(1-\tau_E)$$

and the net payoff from debt is

$$\beta r_D D(1-\tau_D)$$

Hence the total per-period net payoff of the investor is

$$
\begin{aligned}
&\alpha I(1-\tau_D D)(1-\tau_C)(1-\tau_E)+\beta r_D D(1-\tau_D) \\
&= \alpha I(1-\tau_D D)(1-\tau_C)(1-\tau_E)+\frac{\alpha(1-\tau_C)(1-\tau_E)}{(1-\tau_D)} r_D D(1-\tau_D) \\
&= \alpha I(1-\tau_C)(1-\tau_E)
\end{aligned}
$$

This result is equal to the payoff given by holding the fraction α of U's equity. It follows that the two investment strategies have the same payoff, and for no-arbitrage condition to hold, they should cost the same. Therefore

$$
\begin{aligned}
\alpha V_U &= \alpha(V_U - D) + \beta D \\
&= \alpha(V_U - D) + \left[1 - \frac{(1-\tau_C)(1-\tau_E)}{(1-\tau_D)}\right] D
\end{aligned}
$$

which corresponds to the MM third proposition, revisited for the introduction of taxation in the model. It is now possible to make several remarks on the proposition. First of all, if there are no taxes at all so that

$$\tau_C = \tau_E = \tau_D = 0$$

then

$$V_L = V_U$$

which is exactly the result of the MM proposition one.

If there are no personal taxes in the economy, then

$$\tau_E = \tau_D = 0$$

It follows that the value of the leveraged firm is given by

$$V_L = V_U + \tau_C D$$

which is exactly the result of M&M1 with corporate taxation. Therefore, the Miller's proposition can be viewed as a generalization of M&M to a world with personal taxes.

Moreover, it holds that

$$(1-\tau_C)(1-\tau_E) = (1-\tau_D) \quad \rightarrow \quad V_L = V_U$$

Recall that $(1-\tau_D)$ is the after-tax interest income on every dollar of debt, while

$$(1-\tau_C)(1-\tau_E)$$

is the after-tax income from dividends or and capital gains. When the after-tax income from debt and equity is equal to the one from debt, the condition

$(1-\tau_C)(1-\tau_E) = (1-\tau_D)$ holds. Therefore, the investors should be indifferent to the firm's capital structure meaning that the firm will have nothing to gain by using one type of securities rather than another.

A final consideration is that the Miller's proposition indirectly implies that the debt is preferred to equity if

$$(1 - \tau_D) > (1 - \tau_C)(1 - \tau_E)$$

This is a very strict relationship, which itself implies that, in case $\tau_D = \tau_E = \tau_C$ all firms would be

- All-debt if $V_L > V_U$
- All-equity financed if $V_L < V_U$

No company would therefore use a mix of equity and debt in the capital structure. In case of an all-debt firm, all the income goes to debtholders in the form of interest. Given exemption on taxability, the taxable corporate income would result to be zero. At a personal level, however, investors would have to pay a tax of τ_D per each dollar they receive, so their net payoff in this case would be X˜ $(1-\tau_D)$.

In case a firm was all-equity financed, the payoff of shareholders would consist in full dividends, if all income is distributed to them, or capital gains, if the income is retained as retained earnings.

Since there are no interest payments to deduct, the taxable income of the firm would be I, and the net income to distribute to the shareholders would be

$$I(1 - \tau_C)$$

The personal tax on that income would result in a payment of

$$I(1 - \tau_C)\tau_E$$

and the net payoff would be

$$I(1 - \tau_C)(1 - \tau_E)$$

Comparing this payoff with the payoff when the firm is all-debt, $I(1 - \tau_D)$, leads to the conclusion about the capital structure of the firm.

7.4 Summary

The capital structure of the company can get complex when different types of assets belong to it. For many years, scientists and practitioners have been interrogating themselves about the rationale behind choosing some particular structure.

From the classical theory, stating that debt would decrease the capital cost, being cheaper itself, the field of valuation moved to the Modigliani–Miller theorem, asserting that capital structure is meaningless to the value of the firm.

The irrelevance of capital structure is in fact at the center of the MM work, and it carries a lot of information about the riskiness of equity and debt and the interactions among the two.

One of the inspirations of the MM work is the Weighted Average Cost of Capital, as a measure of the cost of capital that takes care of the leverage of the firm. The authors take it as an inspiration for their arguments.

The WACC calculates the cost of capital as an average of cost of equity and cost of debt, weighted by the respective proportions of the two. When taxation is taken into consideration, the shield offered by debt lowers down the cost of capital of the levered firm, which become lower than if taxes were not there.

The payout policy of the firm has a central role, and the decision of the management regarding whether to pay out dividends or not impacts on the value of the firm. Also share repurchases have an effect on it.

Modigliani and Miller revisited their theorem based on the existence of the dividends updating their conclusions, in response to the criticisms received by the other scientists about such a limitation of their model.

Other theories of capital structure are based on information and take into account the asymmetry of information on the market to justify different choices of the management for the capital structure.

Theories like the trade-off theory, the pecking order hypothesis, the adjusted present value, and the theories based on the presence of taxes are all meant to relax the assumptions of the standard MM theorem, so as to make the model structure fit the reality.

Problems

1. Tecom LTD is an all equity firm with a current market value of €750,000,000 and will be worth €600,000,000 or €900,000,000 in 1 year. The risk-free interest rate is 3%. Suppose Tecom LTD issues zero-coupon, 1-year debt with a face value of €850,000 and uses the proceeds to pay a special dividend to shareholders. Assuming perfect capital markets, use the binomial model to answer the following:
   (a) What are the payoffs of the firm's debt in 1 year?
   (b) What is the value today of the debt today?
   (c) What is the yield on the debt?
2. Given the company and the number in Exercise 1:
   (a) According to the Modigliani–Miller theory, what is the value of the equity before the dividend is paid?
   (b) What is the value of equity just after the dividend is paid?
3. Media Corp. has planned free cash flow in the coming year of €18,000,000, expected to grow at a rate of 2% per year perpetually, afterward. Media Corp. has a cost of equity of 16%, a cost of debt of 7%, and corporate tax rate of 38%. The debt to equity ratio is 0.65. What is the value of Flagstaff as an all equity firm?
4. Rina Industries is an all-equity firm with 65,000,000 shares outstanding and €180,000,000 in cash. The firm expects future free cash flows of €68,000,000 per year. Money can be used to expand the business and increase the expected

future free cash flows by 12%. The cost of capital is 10%, and capital markets are perfect. An alternative strategy is to use the €200,000,000 to repurchase shares instead of funding the expansion. If you were advising the board:
   (a) What course of action would you recommend, expansion or repurchase?
   (b) Which provides the higher stock price?
5. According to Modigliani and Miller, what is the significance of a company's capital structure? How did they come to this conclusion?
6. According to the trade-off theory:
   (a) What is traded off against what in the trade-off theory?
   (b) When is optimal capital structure reached? Be precise in your answer.
7. For each of the four characteristics below, does the trade-off theory predict that it will lead to more or less debt in optimal capital structure, other things equal?
   (a) Selling durables that need future maintenance and repair
   (b) Having very volatile earnings
8. Firms can change their capital structures with stock repurchases and with equity offerings.
   (a) What does the trade-off theory of optimal capital structure predict about the effect on the value of the firm of stock repurchases, will the value go up or down?
   (b) What does the trade-off theory of optimal capital structure predict about the effect on the value of the firm of equity offerings, will the value go up or down?
9. The trade-off theory and the pecking order theory both explain firms' capital structures as a function of firms' characteristics. The table below lists a number of empirical proxy variables that are often used to test these theories. Complete the table below by writing "+," "–," or "0" in the columns behind each variable, depending on whether the theory predicts it is associated with more (+) or less (−) debt in capital structure or 0 if the theory does not predict anything regarding this variable.

Proxy variable	Trade-off theory	Pecking order theory
Depreciation/total costs		
Return on equity		
Standard deviation of stock returns		
Fixed-to-total assets		
Market-to-book value		
R&D expenses-to-total costs		
Size (total assets)		

10. Two companies A and B have the same assets that produce the same perpetual cash flow of €10 million. Both companies have 16,000,000 shares outstanding. Company A has outstanding debt with a value of €18,000,000 and current price of shares €8, for an annual return of 11.5%. Company B has outstanding debt

with a value of €80,000,000. All debt is risk-free and the risk-free interest rate is 5%. Assume a Modigliani–Miller world without taxes.

(a) Calculate the value of the assets of company A. Use an alternative calculation to check your results.

(b) Calculate the price and return of the shares of company B. Check your results.

11. Arko LTD has generated a considerable amount of cash and it now wants to pay out €15 million of it to its shareholders. Its balance sheet is depicted below. Arko LTD has 15,000,000 shares outstanding. Assume no taxes and transaction costs.

Balance sheet of Arko LTD (€M)			
Cash	20	Debt	25
Other assets	80	Equity	75
Total	100	Total	100

(a) Calculate the value per share after Arko LTD has paid out €7,000,000 dividends to its shareholders and demonstrate that this does not affect the wealth of the shareholders.

(b) Calculate the value per share after Arko LTD has used €7,000,000 to buy back its shares and demonstrate that this does not affect the wealth of the shareholders.

(c) Describe under which circumstances the management of Arko LTD would prefer buying back shares instead of paying cash dividends.

Appendix: Risk-Adjusted Return on Capital

The risk-adjusted return on capital (RAROC) is a risk-adjusted performance measurement tool, which has become important in assessing the profitability of business units.

Generally, risk adjustments compare return with capital employed in a way to incorporate an adjustment for the risk involved in the business, therefore taking in account the fact that the metrics is affected by uncertainty. RAROC is the ratio of adjusted income over economic capital:

$$\begin{aligned} \text{RAROC} &= \frac{R - C - E_L}{E_C} \\ &= \frac{A_{\text{NI}}}{E_C} \end{aligned}$$

where:

R is the amount of revenues.

C is the amount of costs.

E_L is the expected loss.

E_C is the economic capital.

A_{NI} is the adjusted net income.

For a bank issuing a loan, the numerator of RAROC measure for that loan will look like

$A_{\mathrm{NI}} = I - E_L - C_O$

where:

I is the financial income.

C_O is the amount of operating costs.

Assuming τ is the corporate tax rate, a step further consists in multiplying the amount obtained by $(1 - \tau)$, in order to get the post-tax RAROC measure.

A further degree of complication can be added by multiplying the economic capital by a compounding factor obtained from the risk-free rate. The amount obtained is added to the numerator of RAROC equation.

RAROC can be related to CAPM analysis in order to capture the relationship with the hurdle rate. Recall CAPM equation to be

$R_i = R_f + \beta_i(R_m - R_f)$and

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{\rho_{im}\sigma_i\sigma_m}{\sigma_m^2} = \frac{\rho_{im}\sigma_i}{\sigma_m}$$

The CAPM equation becomes

$R_i = R_f + \frac{\rho_{im}\sigma_i}{\sigma_m}\left(R_m - R_f\right)$ from which

$R_i - R_f = \frac{\rho_{im}\sigma_i}{\sigma_m}\left(R_m - R_f\right)$ and

$\frac{R_i - R_f}{\rho_{im}\sigma_i} = \frac{R_m - R_f}{\sigma_m}$

The equation sets an important equivalence for the asset i in the portfolio. The left-hand side is the RAROC of the asset, while the right-hand side is the hurdle rate on the asset. The two are equal.

Case Study: Capital Structure

Payoux Ltd

The Case

A company shows the following financial information:

- FCFF for next year = 25,000,000
- Tax rate = 30%
- Debt = 100,000,000
- Cost of debt = 5%
- Unlevered cost of capital = 10%

Payoux is a company active in the industry of recycling. Established in 1992, it is still one of the oldest and biggest companies in the field, and employees have a good share of the people living in the nearby village.

Management has always been concerned about managing the company properly, due to the heavy social impact that distress may have on the local population, as well as for the contagion to the partner companies.

The company is quoted on the regulated exchange, and the share price has been stable to the current price of $50 in the last months. Expectations are bullish on the stock due to recent expansion and increase in the amount invested in modern recycling methods.

One of the main concerns of the managers is the riskiness of the capital of the company and how this can impact on the perception of the investors. It is therefore important in their opinion to run an analysis of the capital structure and current cost of capital.

On top of that, the company has current extra cash of $2,000,000 to be fully distributed to the shareholders. The management has to decide which payout policy to apply in the current year, dividends, or share repurchases.

The company is planned to generate, starting from the following year, an average stable amount of extra cash to be distributed to shareholders, in the order of $3,000,000 per year. The outstanding shares are 1,000,000.

The price of a stock is currently $50 and supposed to reach a price of $55 1 year from now. There is another stock on the market that currently costs $100 and supposed to reach a price of $108 1 year from now, after paying a dividend of $2 right before the end of the 1-year period. Taxes on dividends are 25%, and taxes on capital gains are 12%. Assume you want to invest in (buy) one of the two stocks and sell it back after the 1-year period.

Questions

1. What is the debt-to-equity ratio of the firm?
2. What is the WACC (MM second case) of the firm?
3. How should the capital structure change to support that increase?
4. Show that according to the MM theory of dividends (irrelevance), the choice between dividends and shares repurchase is irrelevant, at the current cost of capital.
5. Which stock would you invest in, if you believe in the irrelevance theory of dividends? Show calculations.
6. Which stock would you invest in, if you believe in the tax-preference theory of dividends? Show calculations.

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8 Company Valuation

Company valuation is the core of many corporate finance courses, and it represents one of the most challenging tasks to perform in relation to the analysis of the company financials.

The standard valuation relies on a good mix of science and art, in that it involves careful measures of critical input variables, mixed with a large part of interpretation and intuition.

There are several institutions specialized in valuing companies, and none of them ever reached unanimous consensus about the value of the same company. This is an example of how complicated the task can be.

Company valuation resembles many different stages and inputs, and it relies on the appropriate estimation of key parameters, like growth potential, right cash flow discounting, and more.

The result is an educated guess on what the theoretical value of a company should be based on the available information, the balance sheet, and the robust theoretical background developed in recent years.

A good financial manager is supposed to understand the value of the company he manages, thus proper knowledge of the methodologies for corporate valuation is at the basis of a complete education in corporate finance.

After studying this chapter, you will be able to answer the following questions, among others:

- How can the CAPM be implemented in practice, for returns estimation?
- How can its parameters of CAPM be estimated?
- What are the different valuation methodologies available for corporate valuation, and how do they differ from each other?
- What is the main feature of the Facebook IPO, and how did it affect financial markets?
- What are the reasons behind the mistakes in valuing Facebook?

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The first section of the chapter is about the estimation of CAPM parameters, so as to apply it in practice for cost of equity estimation. Also the market return and beta estimation are described. The second section is about the different valuation methodologies available to the analyst. The third section focuses on the real case of Facebook's IPO and its interesting features as an example of strange valuation.

8.1 CAPM Parameters Estimation

Learning Outcomes

- Learn how to estimate the risk-free rate input for the CAPM calculation
- Learn how to estimate the risk premium input for the CAPM calculation
- Learn how to estimate the beta input for the CAPM calculation

8.1.1 The Risk-Free Rate

The measurement of risk-free rate should be consistent with the cash flows, meaning that if, for example, cash flows are measured in euro currency, the reference risk-free rate should be the EU reference rate or the lowest yield among all the bonds issued by sovereigns in the euro area.

One implication of that choice is that the actual location of the company does not determine the choice of the risk-free rate. The choice is driven by the currency in which the project of firm cash flows is estimated.

For example, a multinational company like Nestle can be valued using cash flows estimated in Swiss Francs, discounted back at an expected return estimated using a Swiss long-term government bond rate, or it can be valued in British pounds, with both the cash flows and the risk-free rate being British pound rates.

The purchasing power parity should assure that the same company or project, valued at different currencies, should result in consistent valuations. The only differences should be explained only by differences in expected inflation, which of course affects both the cash flows and the discount rate (Stulz 2000).

It follows that a low discount rate due to high inflation will be offset by a decline in expected nominal growth rates for cash flows, and the value will remain unchanged.

The values obtained in different currencies will differ consistently when the differences in the interest rates across different countries are not due to differences in expected inflation.

Any value will be higher when the currency with lower interest rates is used and lower when the opposite occurs. The risk, however, is that the interest rates will have to rise at some point to correct for this divergence, at which point the values will also converge.

When inflation is high and unstable, the valuation is done in terms of real values, with an estimation of cash flows directly linked to the real growth rates, excluding the part coming from inflation.

Consistency implies that the discount rates must also be expressed in real terms, starting from a real risk-free rate. While government bills and bonds offer returns that are risk free in nominal terms, they are not risk free in real terms, since expected inflation can be volatile.

Subtracting the inflation from nominal rates gives a good estimate of the real risk-free rate. This operation was hard to implement in the past, but recently the introduction of inflation-indexed treasuries has filled the gap.

The difference between a standard treasury security and an inflation-indexed treasury security is that the latter does not offer a guaranteed nominal return, but instead provides a guaranteed real return.

Thus, an inflation-indexed treasury that offers a 3% real return will yield approximately 7% in nominal terms if inflation is 4% and only 5% in nominal terms if inflation is only 2%.

Unfortunately, most countries do not offer treasury inflation-indexed securities, making it very difficult to account for the effects of inflation on the nominal rates. The real risk free rates in these markets can be estimated according to two arguments:

- Free flow of capital between countries with highest real returns would homogenize the real risk-free rate across markets. Therefore, the real risk-free rate estimated in countries where inflation-indexed treasury securities are issued can be used as the real risk-free rate in any market.
- According to another argument, frictions and constraints in capital flows across markets make the expected real return on an economy approximate the expected real growth rate (in the long term). Thus, the real risk-free rate for a mature economy like that of the United States should be much lower than the real risk-free rate for an economy with greater growth potential, such as Estonia.

The assumption that governments cannot default is not exact for all countries. Many emerging economies cannot be defined as default free; therefore, their rates cannot be assumed to be risk free (Godfrey and Espinosa 1996).

These governments are considered at risk to default on just local debt, and the fact that they also borrow not locally just amplifies the risk. Therefore, there are scenarios where obtaining a local risk-free rate, especially for the long term, becomes difficult.

The estimation of a risk-free rate for those economies must be therefore done under specific compromises.

The trick is to look at the safest companies in that market, which can be considered safer than the government itself. These companies are default free, and it is possible to estimate the risk-free rate as marginally lower than the corporate borrowing rate.

8.1.2 The Risk Premium

The factors that influence the equity risk premium to be inputted in the CAPM are of primary importance. In this section, we look at the role played by equity risk premiums in corporate financial analysis, valuation, and portfolio management and then consider the determinants of equity risk premiums.

The risk premium results from the expectations that investors make about how much risk can be seen in an economy and the price to be attached to that risk. This judgment is a determinant of the expected return on a risky investment and the value estimated for the investment (Booth 1999).

The natural consequence of the above is that investors allocate wealth across different asset classes in different ways. Also the choice of what specific assets or securities to invest in for each class is then affected.

The estimation of equity risk premium comes after careful analysis of the factors that determine it. After all, equity risk premiums should reflect not only the risk that investors see in equity investments but also the price they put on that risk.

The most important factor is the risk aversion of investors. More risk-averse investors in fact will ask for higher premiums, while for declining risk aversion, the asked premium will go down.

The equity risk premium is determined by the collective risk aversion of the market. The latter will result in changes in the premium, and it is opportune to focus on the factors that are more likely to change with time.

Investor age, for example, is an important factor, and studies have confirmed evidence that individuals become more risk averse as they start aging. As a consequence, the markets populated by older investors should have higher risk premiums.

Bakshi and Chen (1994), for instance, examined risk premiums in the United States and noted an increase in risk premiums as investors aged. Liu and Spiegel calculated the *M*/*O* ratio (middle-age investors over old-age ones). Their results show that the price/earning (*P*/*E*) ratios are closely and positively related to the *M*/*O* ratio for the US equity market from 1954 to 2010 (Chan et al. 1992).

Another important factor is the preference for current consumption, in that the expected equity risk premium should be expected to increase as preference for future consumption increases.

Markets characterized by investors that are net savers are then supposed to show lower equity risk premiums compared to markets where individuals are net consumers. Consequently, equity risk premiums should increase as savings rates decrease in an economy.

The direction of the relationship between risk aversion and risk premium is quite simple to establish. However, it is not straightforward to determine the relationship between the two.

The class of equity risk is linked to the general considerations about the predictability of the overall economy. When the inflation is easily predictable, for example, the equity risk premium should be lower. The same should apply when there is predictability of interest rates and economic growth.

A relevant branch of research examines the relationship between equity risk premium and inflation. The results are quite controversial, and some studies find high correlation, while others find little or no correlation.

Brandt and Wang (2003) argue that news about inflation dominates news about real economic growth and consumption in determining risk aversion and risk premiums.

They present evidence that equity risk premiums tend to increase if inflation is higher than anticipated and decrease when it is lower than expected.

Basically the conclusion is that the expectations on inflation determine the equity risk premium and not the level of the inflation itself.

A measure of the risk implied in the economy, when investing in equities, is given by the volatility in the earnings and cash flows of the individual firms operating in the economy.

The above changes are transferred to the markets in terms of information. In the last 20 years, the role of information available to the investors and the quality of it have changed substantially.

The relationship between information and equity risk premium is complex. The precision of the information should be inversely related to the risk premium, with more precise information corresponding to lower equity risk premiums.

Precision here must be defined in terms of what the information tells us about future earnings and cash flows. Providing more information about last period's earnings may create more uncertainty about future earnings.

The differences in information stand as the reason why, for example, larger risk premiums are demanded by investors in some emerging markets rather than others. Markets vary widely in terms of transparency and information disclosure requirements. Markets, in which companies provide little information about important aspects of the firm, should have higher risk premiums than markets where information on firms is reliable and easily accessible to investors.

There is another risk to be considered on top of the underlying real economy and the bad quality of information on the markets. There is also an additional risk related to the liquidity.

If investors have to accept large discounts on estimated value or pay high transaction costs to liquidate equity positions, they will pay less for equities today (and thus demand a large risk premium).

The general thought is that the net effect of illiquidity on equity markets should be very small given that the market is equally split in buyers and sellers. But the argument can be contrasted based on at least two main reasons.

First of all, not all the stocks in the market are widely traded, with liquidity being very variable across different stocks. Therefore, the cost of trading a widely held, large market cap stock is very small, but the cost of trading an over-the-counter stock will be much higher.

Another aspect is related to the cost of illiquidity, which can be variable over time, with significant effects given by even small changes. In particular, the cost of illiquidity seems to increase when economies slow down and during periods of crisis.

There is also a behavioral component, making experts believe that equity risk premiums are guided by changes in the rationality of investors. There are two main strands to this analysis, namely:

First of all, the money illusion is a phenomenon by which the equity prices in the 70s rapidly declined, followed by an increase in inflation rates. According to Modigliani and Cohn (1979), the low equity values of that period were determined by investors being inconsistent about inflation expectations.

The explanation they give is that investors used historical growth rates in earnings, reflecting past inflation, to forecast future earnings. The simultaneous use of current interest rates, which reflect the expectations on future inflation, biased the estimation of discount rates.

When inflation increases, this will lead to a mismatch, with high discount rates and low cash flows resulting in asset valuations that are too low (and risk premiums that are too high).

The second aspect is the narrow framing, which deviates from the standard Portfolio Theory. The latter in fact assumes that investors assess the risk of an investment in the context of the risk it adds to their overall portfolio and demand a premium for this risk.

Experts in behavioral economics state that when investors are offered new gambles, they isolate these gambles, separately from other risks that they face in their portfolio, leading them to overestimate the risk of the gamble.

8.1.3 The Beta

Recall that the beta of an investment measures the risk attached to an exposure to the market volatility and not the specific asset risk. The market portfolio of all investable assets has a beta of exactly one.

When the beta is less (more) than one, the investment has lower (higher) volatility than the market, and the asset price movements are not highly (highly) correlated to the market.

An example of the first is a treasury bill: the price does not go up or down a lot, so it has a low beta. An example of the second is gold. The price of gold does go up and down a lot but not in the same direction or at the same time as the market.

Negative betas are also observable for the investment that does not follow the market but moves to the other side. There are several types of investments like derivatives and equity put options that can show large negative betas.

The risk measured by beta is not diversifiable, and it does not measure the risk of an investment held on a stand-alone basis, but the amount of risk the investment adds to an already diversified portfolio.

The use of beta in finance is mainly divided in two parts. The first part is the ranking of assets and portfolio compared to the systematic risk of the market and is mainly performed by practitioners.

Another important field of use is the testing of CAPM, with cross-sectional regressions intended for testing the mean-variance efficiency of the market portfolio and the linear relationship between expected returns and betas (Kandel and Stambaugh 1995).

When estimating beta, there is also a problem of separation between risk aversion, which is supposed to entail the asymmetric treatment of deviation from the regression, and the statistical loss function that is symmetric due to equal treatment of observations.

Recall that the betas measuring risk in financial models are characterized by two main features. First of all, they measure the risk added to a well-diversified portfolio, rather than the total risk associated to an asset. Therefore, there is the chance that an investment is very risky from an individual point of view while being safe in terms of idiosyncratic risk.

Furthermore, all betas measure the relative risk of an asset and thus are standardized around one. In the CAPM, the market capitalization-weighted average beta across all investments should be equal to one, and the property should be consistent in any multifactor model.

Diversification can be done at both equity and other asset level, plus there is a geographical dimension, regarding diversification across domestic assets or global assets.

In the CAPM, for instance, the diversified portfolio includes all asset classes and is globally diversified, when transaction costs are not considered. In case of transaction costs and barriers to global investment, some asset classes and global diversification could be excluded from market portfolio.

The beta of an asset can be estimated through a regression of the asset returns on some index that represents the market portfolio, over a reasonable time period as in Fig. 8.1.

The regression equation for a stock i is given by

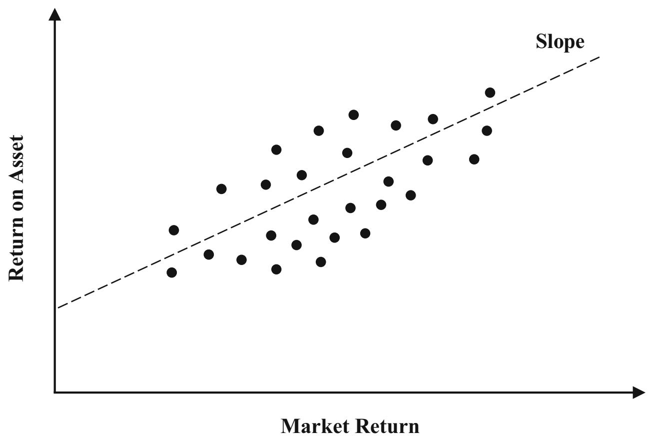

Fig. 8.1 Graph of a linear regression

$$r_i = \alpha_i + \beta_i\left(r_{m,t} - r_{f,t}\right) + \varepsilon_{i,t}$$

where:
α_i is the intercept parameter of the regression.
β_i is the estimated beta (slope) of the regression.
$r_{m,\ t}$ is the value of the market return for the observation at time t.
$r_{f,\ t}$ is the value of the risk-free rate for the observation at time t.
$\varepsilon_{i,\ t}$ is the error term for observation at time t.

The beta is the slope of the regression, measuring the risk added on by the investment regressed to the index used as proxy of market portfolio. In addition, it also fulfills the requirement that it be standardized, since the weighted average of the slope coefficients estimated for all of the securities in the index will be one.

There are measurement issues involved in the right estimation of beta:

The choice of a market index, for example, is controversial because there are no indices that are so close to the market portfolio. There are equity market indices and fixed income market indices, which measure the returns on subsets of securities in each market.

All these indices are not complete, and they include only a subsection of the market. An example of the market is the S&P 500, which is used for estimating beta for US companies. But it includes only 500 of the thousands of equities that are traded in the US market.

The choice of the market index can make a big difference in beta estimation, given that different indices include different types of equity assets and none of these indices include other asset classes, such as fixed income or real assets.

Indices that include more securities should then provide a better estimate of the beta, especially if they are market weighted. Moreover, the index should reflect the extent to which the marginal investor in that market is diversified.

Given the weighting issue, the S&P 500 becomes the best index to use, because, even if it contains less stocks than other indices, it comes with the advantage to be market weighted, and it includes the 500 largest firms.

Another important factor in beta estimation is the choice of the time period to consider. There is not much information in risk and return models about how long the estimation time should be.

There is a trade-off involved in choosing a time period for beta estimation, given that the further one goes back in time, the more observations are available, but at the cost of including information that is probably too old, compared to the actual status of the firm.

The company in fact might have changed its characteristics, in terms of business mix and leverage, over that period. Recall that the main objective in estimating beta is not to obtain the best beta over the last period but to obtain the best beta we can for the future.

For firms that have maintained a stable business, without experiencing major innovations, going back in time is not a problem. The opposite holds for a company that experienced a change in terms of business mix and leverage.

The final factor affecting beta estimation is the choice of the return interval, used to measure returns historically. Recall that measurement can be done at daily, weekly, monthly frequencies, and so on, with intraday data representing the limit.

A shorter interval in the regression implies a richer database, for even time period. Anyway the downside is that assets do not trade continuously, and this can affect the beta estimation when an asset is not traded. In particular, non-trading on an asset during a return period can reduce the measured correlation with the market index and consequently the beta estimate.

The problem of missed trading can be reduced by choosing a longer time period, at the cost of fewer observations. In particular quarterly and annual returns result in few observations, but monthly returns seem to be the best solution in that they provide a reasonable amount of data without the problem of non-trading.

When estimating betas based on daily or weekly data, the risk is to have a bias in the measurement, with illiquid firms reporting lower betas than they should be and liquid firms reporting higher betas than is justified. Another method for beta estimation involves calculation of short-term interval returns, followed by an adjustment of the betas for the extent of non-trading.

Different choices of time period, return interval, and market index produce different estimations for the beta, so that different services often end up with different estimates of beta for the same firm.

Some services have standard adjustment methods, based on simple techniques. For example, Bloomberg estimates the adjusted beta through the formula

Adjusted beta = regression beta (0.67) + 1.00 (0.33)

This effectively pushes all regression beta estimates closer to one. The weights remain the same for all companies and are not a function of the precision of the beta estimate.

8.2 Valuation of Private Firms

Learning Outcomes

- Learn how to calculate the cost of capital of a private firm
- Learn how to estimate the beta of a private firm
- Explain the concept of terminal value

8.2.1 The Cost of Capital

Recall that the valuation of a private company is a different task compared to valuing a public company. The focus is on choosing the right model and deciding whether to focus on valuing the firm or the equity.

Moreover, the valuation must take into account whether the company is at a mature stage, therefore requiring a steady state model. Alternatively, for a growing company, the choice is between a two-stage or a three-stage model.

Another challenge is the estimation of the discount rate for the cash flows, either in the form of cost of equity, cost of debt, or overall cost of capital. The choice must be consistent on what type of cash flows have to be discounted.

The estimation of the cost of equity for a private firm is challenged by the limitations of classical models, like CAPM, that rely on available market data. For private firms and divisions, the data are not available, because these firms are not traded.

It is therefore necessary to develop methods for the estimation of the model parameters, in order to overcome the limitations imposed by the lack of data.

Estimation of the beta, for example, can be done in several steps. First of all, one should collect a group of publicly traded comparable firms, preferably in the same line of business, but more generally, affected by the same economic forces that affect the firm being valued.

It is possible to test for the comparability of selected firms by estimating the correlation between the financials of the firm under valuation and the comparables. A high correlation confirms the comparable firms can be used for the analysis.

The second step is to estimate the average beta for the publicly traded comparable firms. After that, the average market value of the debt-to-equity ratio for the comparable firms must be estimated. This leads to the calculation of the unlevered beta of the firm.

$$\beta_U = \frac{\beta_L}{1 + (1 - \tau_C)\left(\frac{D}{E}\right)}$$

where:

β_U is the unlevered beta.

β_L is the levered beta.

τ_C is the corporate tax rate.

D is the market value of the debt of the firm.

E is the market value of the equity of the firm.

The last step involves the estimation of the debt-to-equity ratio for the private firm. The main limitation is the availability of only the book values for both. In order to overcome the issue, the average debt-to-equity in the industry can be used.

Assuming that the private firm will move to the industry average debt ratio, the beta for the private firm will then also converge on the industry average beta. This might not happen immediately but over the long term.

$$\widetilde{\beta}_L = \frac{\beta_U}{1 + (1 - \tau_C)\left(\frac{D_{\mathrm{AVG}}}{E}\right)}$$

where:

β_{PVT} is the beta of the private firm.

D_{AVG} is the industry average debt.

Another approach involves estimating the theoretical optimal debt-to-equity ratio for the private firm, based on the analysis of the financial statement. The determined ratio can then be used for the beta calculation. One should consider to then use the same ratio also in the estimation of cash flows.

$$\widetilde{\beta}_L = \frac{\beta_U}{1 + (1 - \tau_C)\left(\frac{D_{\text{OPT}}}{E}\right)}$$

where:

D_{OPT} is the optimal debt.

The final step of this method is to use the beta obtained through the above steps to estimate the cost of equity of the private firm.

An alternative method consists of estimating an accounting beta, again with a process divided in several steps. First of all, the historical accounting earnings of the private firm must be collected for a reasonably long time.

Collecting the accounting earnings for the index of reference, as a proxy of the market, for the same time period is the following step. The changes in earnings for the private company must then be regressed against the changes in the market returns. The beta is then obtained as the slope of the regression line.

Much research has been done by researchers to relate the betas of publicly traded firms to observable business variables. Fundamental betas are the result of the efforts of authors like Beaver et al. (1970).

They examined the relationship between betas and seven different business variables, namely, the dividend payout, asset growth, leverage, liquidity, asset size, earnings variability, and the accounting beta.

Also Rosenberg and Guy (1976) have performed a similar analysis. The betas have been regressed on the variables in order to find the appropriate coefficients to relate the beta for the private firm to its own business variables.

By measuring each of these variables for a private firm, it is then possible to estimate the beta for the firm. While this approach is simple, it is only as good as the underlying regression.

Another type of beta to use for valuing the cost of equity of private firms is the bottom-up beta. It can be estimated by starting with the businesses that a firm is in and estimating the fundamental risk or beta of each of them. The weighted average of these risks is then calculated and adjusted for leverage.

The four main steps involved in the estimation of bottom-up beta are:

- Break the company down into the businesses that it operates in. The business should not be defined too narrowly in order for the process to run smoothly.
- Estimate the risk (beta) of being in each business. This beta is called an asset beta or an unlevered beta.
- Take a weighted average of the unlevered betas of the businesses, weighted by how much value the company gets from each business.
- Adjust the beta for the company's financial leverage (debt-to-equity ratio).

The adjustment of the bottom-up beta to the leverage of the firm is a controversial point since such an adjustment should be normally made on market values of equity and debt, which is only possible for public firms.

While many analysts use the book value debt-to-equity ratio to substitute for the market ratio for private firms, there are several alternatives that can be implemented.

One option is to assume that the market leverage of the private firm resembles the average leverage of the industry sector. In this case the levered beta can be defined as an industry beta.

Another method is to use the private firm's target leverage ratio, or the optimal debt ratio, to estimate the beta. The difficulties of the method are that the leverage ratio is disclosed upon discretion of the management, while the optimal debt ratio must be estimated.

The beta can then be adjusted for operating leverage, by simply considering the proportion of the fixed costs over overall costs. If this proportion is greater than is typical in the industry, the beta used for the private firm should be higher than the average for the industry.

A further adjustment can be made for diversification effects. One should recall in fact that the beta represents the risk added by an investment to a diversified portfolio. In case of the private firms, the owner is often the only investor and thus can be viewed as the marginal investor.

As a consequence, the entrepreneur is not well diversified but concentrates most of his wealth in the private firm. It can be stated as a consequence that the betas calculated for a private firm will generally underestimate the risk exposure.

As in the case of operational leverage, it is possible to make a simple adjustment in order to account for the non-diversifiable risk when calculating beta. The method is based on the standard deviation.

Define the standard deviation in the private firm's equity value by σ_i and the standard deviation in the market index by σ_m. If the correlation between the stock and the index is defined to be ρ_{im}, the market beta can be written as:

$$\beta_m = \rho_{im} \frac{\sigma_i}{\sigma_m}$$

where:

σ_i is the volatility of the returns of stock i.

σ_m is the volatility of the market returns.

ρ_{im} is the correlation coefficient between market returns and returns of stock i.

The total risk to which the firm is exposed is given by dividing the market beta by σ_{jm}. This would yield the following:

$$\frac{\beta_m}{\rho_{im}} = \frac{\sigma_i}{\sigma_m} \tag{8.1}$$

Equation (8.1) represents the relative standard deviation of the firm, scaled against the standard deviation of the market. It is possible to indicate such a measure as the total beta, which accounts for non-diversifiable risk.

$$\beta_{\mathrm{TOT}} = \frac{\beta_m}{\rho_{im}}$$

The total beta is dependent on the correlation between the market and the firm and will be higher than the market beta. In general, for lower values of the correlation, the total beta will be higher.

For a private firm, the total beta calculation may seem difficult, due to the absence of market prices, which hinder the calculation of a market beta and the correlation coefficient.

However, one should recall that it is possible to estimate the market beta of the sector by looking at publicly traded firms in the business. The correlation coefficient is obtained by looking at the same sample and using it to estimate a total beta for a private firm.

The reason for making the company valuation is another important factor in assessing whether the total beta adjustment is needed or not. In case of valuation for sale, for example, the need for adjustment depends on the potential buyer.

In case of an IPO, there should be no adjustment for non-diversification given that the market investors, who are the buyers in this case, are assumed to be potentially diversified in their investments.

If the valuation is for sale to another individual or private business, the extent of the adjustment is dependent on the diversification of the buyer. The more diversified the buyer, the higher the correlation with the market and the smaller the total beta adjustment.

The estimation of the cost of debt for a private firm presents a basic problem, similar to that encountered in estimating the cost of equity. As for equity in fact, private firms do not access public debt market, and they are therefore not rated.

The biggest part of the liabilities in the balance sheet is made of bank debt. Using the interest rate paid on the bank, loans may be misleading and not reflect the real rate at which the firm can borrow.

One possible method is to assume that the private firm can borrow funds at a rate equivalent to that paid by the comparable firms in the industry. In this case the cost of debt can be expressed as

$$\tilde{r}_D \cong r_{\mathrm{AVG}}$$

where:

r_{AVG} is the cost of debt for similar firms in the industry.

Another solution is to estimate an appropriate credit rating for the company, based upon financial ratios, and use the interest rate estimated bond rating. This leads to a formulation of the type

$$\tilde{r}_D \cong r_{\mathrm{RAT}}$$

where:

r_{RAT} is the interest rate based upon estimated bond rating for private firm.

The third method involves considering the actual long-term debt of the private firm and using the cost of that debt as a proxy of the estimated cost of debt. This is especially valid if the debt is recent, therefore reflecting the actual creditworthiness of the firm.

$$\tilde{r}_D \cong \frac{I}{D}$$

where:

I is the interest expense.

Once the cost of equity and cost of debt have been somehow estimated, it is then possible to estimate the overall cost of capital. However, there are issues also at this stage.

The debt ratios for private firms are in fact stated in book value terms, rather than market value. Furthermore, the debt ratio for a private firm that plans to go public might change as a consequence of that action.

One possible solution is to assume that the firm debt ratio will converge to the industry average ratio in a reasonable term. In this case the following equality holds:

$$\frac{\tilde{D}}{\tilde{V}} \cong \frac{D_{\mathrm{AVG}}}{V}$$

where:

$\tilde{D}$ is the debt of the private firm.

$\tilde{V}$ is the total value of the private firm.

D_{AVG} is the industry average value of debt.

Otherwise it is possible to assume that the company will move toward its optimal debt ratio, so that

$$\frac{\tilde{D}}{\tilde{V}} \cong \frac{D_{\mathrm{OPT}}}{V}$$

where:

D_{OPT} is optimal value of debt.

It is very important that the debt ratio assumptions used to calculate the beta, the debt rating, and the cost of capital weights are consistent. When assuming that the firm will move to the industry average debt ratio, one must therefore calculate the beta using the industry average debt/equity ratio, estimate the bond rating by looking at similar firms in the industry, and calculate the cost of capital using the same debt ratio.

8.2.2 Beta Estimation

Recall that beta measures the systematic risk of a security compared to the market, and how the return on equity of a company changes, according to changes of the return of the market.

Beta is the slope coefficient obtained through regression analysis of the stock return against the market return. The following regression equation is employed to estimate the beta of the company:

$$\Delta S_i = \alpha_i + \beta_i \Delta S_M + e_i$$

where:

ΔS_i is the change in price of stock i.

ΔS_M is the change in the value of the market.

α_i is the intercept value of the regression.

β_i is the beta of the stock i.

e_i is the error term.

The problem with the above regression is that it can be conducted only for public companies, which are listed on stock exchange and for which historical data about equity returns are available (Damodaran 2002).

But what about for private companies? Due to lack of market data on stock prices of private companies, it is not possible to estimate stock beta. Therefore, other methods are required to estimate beta of the private company.

One straightforward method is to derive the private company beta directly from that of comparable public companies. The objective of the method is to calculate the average beta of a group of public companies that operate in the same business as the private company.

The estimated beta from comparable firms is a proxy for the industry average levered beta. After that, the beta needs to be unlevered, by applying the average debt-to-equity ratio for these comparable companies. The final step is re-levering beta, using private company's target debt/equity ratio.

Example 8.1 Consider estimating the beta of an illustrative company with a target debt-to-equity ratio of 0.5, by using the information about four comparable companies:

Comparable companies	Beta	Debt	Equity	*D/E*
Company 1	1.60	7840.00	16,267.00	0.48
Company 2	1.65	10,565.00	37,850.00	0.28
Company 3	1.71	523.23	1653.47	0.32
Company 4	1.69	1627.84	4079.74	0.40
Averages				
Weighted average beta	1.64			
Weighted average *D/E*	0.34			

The equity-weighted average beta of the four companies is 1.64. This is close to the arithmetic average of about 1.66. The next step is to unlever the beta by using the average debt-to-equity ratio for these companies, which is 0.34. It follows that

$$\beta_U = \frac{\beta_L}{1 + (1 - \tau)\frac{D_{\text{AVG}}}{E_{\text{AVG}}}} = \frac{1.64}{1 + (1 - 0.35)0.34} = 1.343$$

where:
β_L is the levered beta.
$\frac{D_{\text{AVG}}}{E_{\text{AVG}}}$ is the average debt-to-equity ratio of comparable firms.
τ is the tax rate.
Thus, we get the unlevered beta of 1.343 approximately.

In the final step, we need to re-lever the equity using the target *D*/*E* ratio of the private company, which equals 0.5:

$$\beta_L = \beta_U\left[1 + (1 - \tau)\frac{D}{E}\right] = 1.343(1 + 1 - 0.35)0.5 = 1.78$$

The method of the average beta of comparable firms has the drawback consisting in the fact that it neglects the differences in size between companies. It happens often that the public comparable companies considered are much larger than the private firm.

Another possible method is the earnings beta approach that becomes useful in cases when it is difficult to obtain reliable comparable betas. It happens quite often in fact that public companies operate in many segments, making it difficult to identify which companies are really comparable to the private firm under valuation.

The earnings beta approach uses the company's historical earnings change and regresses them against the market returns. An appropriate market index can be used as a proxy for market.

The beta obtained as above needs to be adjusted in order to reflect the expected future performance of the firm. Recall that beta has a mean-reverting feature and tends to get to one in the long term. The adjusted beta can be expressed as

$$\beta_{\text{Adj}} = \alpha(1 - \alpha)\beta_H$$

where:
β_{Adj} is the adjusted beta.
α is a smoothing factor.
β_H is the historical beta.

The smoothing factor can be derived through complex statistical analysis based on historical data, but as a rule of thumb, the value of 0.33 or (1/3) is used as a proxy.

There are drawbacks in the earnings beta approach, depending on the fact that private companies do not exhibit longtime series of earnings, therefore making the regression analysis less reliable.

Moreover, the accounting earnings are subject to smoothing and accounting policy changes. Therefore, these may not be appropriate for statistical analysis, unless necessary adjustments have been made.

The use of CAPM for valuation of private companies can be problematic, due to the lack of a direct method to estimate the beta of a private firm. The above-presented methods represent a good compromise for implementing the model.

8.2.3 Growth and Terminal Value

The terminal value in the valuation of discounted cash flows represents the estimated value of the potential company's cash flow to be realized after the final discrete projection period.

Corresponding to the forecast of the future cash flows over such a terminal period, an analyst also chooses a long-term growth rate to incorporate the value of the cash flows into a perpetuity.

The calculation of the terminal value is therefore based on the strong assumption that future cash flows over a specified time horizon will grow at a constant long-term growth (LTG) rate forever.

The most commonly used method to calculate the terminal value in the standard DCF model is the Gordon growth model (GGM), which can be described by the equation

$$\mathrm{PV} = \frac{\mathrm{NCF}_0(1+g)}{r_e - g} \tag{8.2}$$

where:

PV is the present value.

NCF_0 is the net cash flows in the final discrete projection period.

g is the selected long-term growth rate.

r_e is the selected cost of capital.

The first step in terminal value calculation is to use the growth model to estimate the normalized long-term series of cash flows (terminal period NCF) during the estimated projection period.

The cash flow stream should reflect the assumption of stable, normalized returns for the company. This is a very important passage in order to ensure a fair valuation of the growth potential of the business.

The following step entails the capitalization of the terminal NCF at an appropriate risk and growth-adjusted capitalization rate, which is commonly indicated as the direct capitalization rate.

The Gordon growth model assumes that the terminal value is estimated based on the assumption of an increase (or decrease) of the NCF, at a constant annual rate, in perpetuity.

The good proxy for the capitalization rate is the company's WACC, which incorporates the risk embedded in the business cash flows, net of the selected long-term growth rate (which incorporates the expected growth of the company cash flow).

The capitalization rate calculated as above is then used to discount the projected terminal period's NCF, in order to determine the terminal value, to be added to the discounted previous cash flows (Pratt and Grabowski 2010).

Because the terminal value is calculated as of the end of the discrete projection period, the last procedure in the application of the GGM is to compute the present value of the terminal value.

In that final discounting phase, the long-term growth rate is not used, but the same present value discount rate applied to the final cash flow of discrete period NCF calculation should be used instead.

The assumption of a permanent positive or negative growth rate for a business may appear too strong and unrealistic. This is particularly the case when an analyst decides to use a growth rate above the projected inflation rate in the economy.

This implies in fact that the company will grow to infinite, and it will do it on an infinite time span. However, the majority of the terminal value is generated by the cash flow that occurs within the first few periods beyond the discrete projection periods.

By selecting the LTG rate for the Gordon model, an analyst has a view of the cash flow growth of the company for a time interval of at least 10–20 years, after the terminal period.

Recall that the GGM is a collapsed version of the summation of the present value of each individual period in the terminal period discounted to the beginning of the terminal period at the direct capitalization rate.

As per the equation as stated above, the terminal value is generated (for each period successive to the last discretely discounted cash flow) based on the spread between the WACC and the LTG.

In other words, given the same WACC, a lower LTG rate causes a higher proportion of terminal value to be generated in the near term, while a higher LTG rate causes the opposite result.

A higher LTG in fact corresponds to a steep increase in the cash flow projections in the future. Following this approach, the future periods tend to correspond to higher present values.

A higher long-term growth rate corresponds then to higher proportion of value generated in later periods. This drives uncertainty such that a high LTG amplifies it through relatively increased proportion of the terminal value.

The issues related to the right choice of the LTG rate are many. First of all, there is necessity to match it with the inputs considered when measuring the related cash flows to discount.

Also important is to consider appropriate factors in the selection of the right rate, while excluding the inappropriate ones from the analysis, in a process that requires careful selection.

The purpose of this section is also to give a summary with explanation of the quantitative and qualitative factors that should be considered when selecting the appropriate LTG rate for application to the Gordon model.

Sometimes the analysis behind the choice of the LTG rate mainly relies on factors like historical and projected growth, inflation, or any other similar metric that can be directly measured.

The issue with such an approach is that other important factors are ignored, which relate to the overall business life of the company and to its overall growth perspectives. Understanding what those factors are is important because the period covered by the LTG rate variable within the GGM may start so far into the future that there could be problems.

First of all, it is very difficult to precisely project the LTG rate. Secondarily it may not be practical or possible to identify and select the specific company initiatives that may contribute to the overall growth of the business.

Qualitative factors are important in determining the appropriate permanent growth factor of the company and may include items like the projected business performance, the assets of the company, and managerial strategy.

According to *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset* by Aswath Damodaran, a company's LTG rate should be "determined by a number of subjective factors—the quality of management, the strength of a firm's marketing, its capacity to form partnerships with other firms, and the management's strategic vision, among many others."

The first way to measure LTG is from the company's existing assets. Such an approach focuses on both tangibles and intangibles and results in being the most commonly addressed factor when thinking about long-term growth.

Among the assets that analysts normally include in the assessment of the expected growth are:

- The overall industry growth
- The company's market share
- Inflation
- The growth of the existing assets prior to the terminal period

Industry growth is important and sometimes expected to be higher or lower in the discrete projection period rather than in the terminal period. In all cases, the latter is the one to be assumed as the factor for LTG calculation.

The company's market share is also very important, and sometimes, if the company shows a positive trend of market penetration, this factor has a positive impact on the estimation of the growth.

Existing assets in the company are sometimes the basis for the estimation of LTG rate. But in most cases, and for most companies, this is not enough. Some business produce new assets due to innovation. It follows that if the analyst only considers growth from existing assets in the LTG rate, he or she may understate the company's LTG rate.

It is a complicated task to identify the growth from new internally developed assets and separate it from that coming from existing assets. This is because the projection of cash flows should be then based on a product not developed yet. The analysis also implies that the idea or product generating the increase in assets will be commercially viable.

Another important category of growth is the growth by acquisition, by which it is possible to create new company assets or grow the already existing ones, including the intangibles and goodwill.

The analysis of LTG from the perspective of growth by acquisitions involves establishing how likely the company is to make acquisitions during the terminal period and what contribution these will have on the projected growth.

Many factors affect the above considerations. First of all, it is important to consider what is the history of the company in terms of acquisitions. It is also important to understand the acquisition policy of the company.

Another important factor is the level of acquisition activity in the subject company industry. The analysis should rely on the merger and acquisition databases to analyze the number of transactions that occurred in the subject company industry in the years preceding the valuation date.

Last but not least, it is very important to assess the prospective financial ability of the company to make acquisitions. This can be done by analyzing the past financial statements as well as the financial statements projected for the discrete projection period.

One should recall that the most relevant period for the analysis is the terminal period, so that the potential of the company to make acquisitions in perspective constitutes the core of the analysis.

For example, if there is a major capital expenditure to be financed through debt, during the discrete projection period, it may be difficult for the company to complete acquisitions during the first part of the terminal period.

There are also factors that should be excluded from the LTG analysis, in that they are not meaningful to assess the growth perspective of the company. Factors like speculation, hypothetical situations, or opportunities maybe existed as of the analysis date but are not relevant in that probably not exploited.

The other big area of analysis includes quantitative factors, represented by empirical data that are useful to support the choice of the LTG rate. These data may include both company-specific information and projected economic growth, both real and nominal.

Sometimes qualitative data and quantitative data may lead to different values of the LTG. In this case it is the duty of the analyst to understand the differences and try to reconcile the different values.

The corporate financial statements are a primary source of information, both in the form of historical financial information and in the form of management-prepared projections about the financials of the business.

If the business was operating under similar business conditions, historical financial information is useful because it provides snapshots of the economic results of the business.

The projections of the management are also very important in that they reflect the point of view of someone who has generally a deep understanding of the economic drivers of the business and give reasonable predictions of the business for a number of years into the future.

The analysis of historical financial information for the estimation of the LTG relies on the assumption that the economic factors driving a company in the near past will continue to affect the company in the near future.

The belief is that the recent trend of cash flows in one direction or the other may have a significant impact on the future direction of those, unless the trend has been determined by major shocks on operations.

One important factor to consider when analyzing the company's historical financial information is the age of the company. This is because a company in the business for a longer time may exhibit a much more stable economic condition. On the other hand, a longer series of time periods provides a higher amount of data to analyze.

The trend of the recent cash flows assumes a significant role when the company reports stable economic returns over a reasonably high number of periods. In this case in fact no major changes should be expected, and the weight of the trend in historical financial information should be considered relevant.

The analysis can be simplified by looking at a sample of three hypothetical companies. The different types illustrated below correspond to different impact of their circumstances on the usefulness of their historical financial data in the selection of the LTG rate.

- Company X has been recently founded. It shows consistent profits in the early years, and it is expected to continue like that. However, volatility of cash flows is high, and the main sources of income are two big customers not bounded by any long-term agreement.
- Company Y is young as well, with a path of increasing cash flows in time, without being dependent on any particular customers for the cash flow pattern to continue in the future.
- Company Z is a very old company that already reached its steady state and shows stable cash flows. The company grows at a rate consistent with the overall industry, with no projected major change in the business structure in the near future.

Company A does not have links to stable economic factors and has volatile cash flows, with unstable customer relationships as well. For such a company, it is clear that the historical data are less meaningful than for other companies. For such a company, it would be more useful to rely on management projections, industry projections and expectations, and other factors.

The case of company B is different, and historical data may be more useful, even if the company B is as young as company A. This is because it appears that company B's economic drivers are more predictable.

As a matter of fact, it is obvious that for a company as old as company C, historical data have the most use. With a demonstrated past growth and an actual good average reduced growth, it shows to be very interesting for the analysis.

The specific factors of the company at hand will determine what information will carry the most weight and what quantitative methods the analyst may use to provide an indication of a supportable LTG rate.

One important question is: how to estimate the growth in earnings? There are several possible approaches, and it is very important to analyze all of them in order to get a deep understanding of how growth impacts on value.

A good starting point for the analysis is generally the historical growth in earnings per share. Moreover, it is important to have knowledge of what the figures are for other businesses, as measured by other analysts.

Ultimately, all growth in earnings can be traced to two fundamentals—how much the firm is investing in new projects and what returns these projects are making for the firm.

Among the various methods of EPS growth estimation, there are arithmetic and geometric averages and simple regression models. In all cases one must put attention to the period used in estimation and the management of negative earnings, both having a serious impact on the estimation.

The work of the analysts about growth rate estimation is driven by several propositions. First of all, one must take into account that there is far less private information and far more public information in most analyst forecasts than is generally claimed.

Another proposition states that private information mostly comes from the company itself. A third proposition tells us that there is danger when all analysts barely agree with each other on estimations (lemmingitis) and when they agree to little (in which case the information that they have is so noisy as to be useless).

The inputs involved in the estimation of expected long-term growth in EPS are reinvestment rate R_R and return on equity *ROE* which are calculated as

$$r_R = \frac{I_R}{I_C} = R_R$$

where:

r_R is the reinvestment rate.

I_R are the retained earnings.

I_C are the current earnings.

And the return on investment r_I is

$$r_I = \text{ROE} = \frac{I_N}{E_B}$$

where:

I_N is the net income.

E_B is the book value of the equity.

One must be aware that small changes in ROE translate into large changes in the expected growth rate. Moreover, the lower the current ROE, the greater the effect of changes in ROE will be.

In the long term, the company will hardly sustain growth in earnings due to increase in the ROE. After some level, the improvement in ROE will be almost meaningless to the growth in earnings (Pratt 2008).

If the change is over multiple periods, the second component should be spread out over each period.

How to value the company then? First of all, let's focus on public companies. A public company is assumed to have infinite life, so that its value P can be calculated as the present value of its cash flows over an infinite time span, as

$$P = \sum_{t=1}^{\infty} \frac{C_t}{\left(1 + \tilde{r}\right)^t}$$

where:

$\tilde{r}$ is the discount rate applied to the cash flows.

Given that it is impossible to estimate cash flows forever, the common method is to estimate a series of cash flows for a limited growth period and then add up a terminal value, in order to account for the following infinite time.

$$P_0 = \sum_{t=1}^{T} \frac{C_t}{\left(1 + \tilde{r}\right)^t} + \frac{\mathrm{TV}}{\left(1 + \tilde{r}\right)^T}$$

where:

TV is the terminal value.

The simple form of stable growth equation for estimating the terminal value represents the firm's cash flows as growing at a constant rate forever, so that the present value of those cash flows is

$$P_0 = \frac{C_1}{\tilde{r} - g}$$

where:

g is the expected growth rate.

The constant growth rate as so calculated is a stable rate that can never be higher than the growth rate of the economy in which the company operates. All companies have an initial growth which can be high for some periods and will approach the stable growth at a more mature stage of their life.

The overall economy growth in some country of reference resembles the average of the growth rates of the many companies that populate it. It is therefore important to know the general state of the economy in order to guess a stable growth for the firm.

If one assumes that the economy is composed of high-growth and stable growth firms, the growth rate of the latter will probably be lower than the growth rate of the economy.

In the assumption that the company is going to disappear in the future, an implicit conclusion is that the growth rate can be negative, so that the terminal value will be lower.

The chosen growth rate should follow the nature of cash flows and discount rates, so that if those are expressed in nominal terms, also the long-term growth should be nominal and expressed in the currency in which the valuation is denominated. One simple proxy for the nominal growth rate of the economy is the risk-free rate.

It is important to underline that the terminal value is not much linked to stable growth, as it is to the excess returns in the stable growth stage. If one assumes that the ROC will equate the cost of capital in stable growth, the growth rate will not impact on the terminal value.

On the other hand, if the assumption is that the firm will earn positive (negative) excess returns in perpetuity, the terminal value will increase (decrease) as the stable growth rate increases.

The first period of discrete cash flows discounting and high growth can be characterized by several behaviors. In particular, there are four different assumptions that can be made.

The first scenario is that there is no high growth, and the firm is already in stable growth. The second scenario assumes high growth for a period, after which the growth rate will drop to stable growth.

The third scenario is similar to the second but involves high growth followed by a decline that will gradually lead to stable growth in a three-stage process. Finally, there is the case where each period will be characterized by different growth and margins.

At the same time, assumptions about excess returns must be made as well. In general, the excess returns will be large and positive in the high-growth period and decrease as you approach stable growth (the rate of decrease is often titled the fade factor).

There are several factors determining the growth pattern of a company. They range from company size to barriers to entry, passing through current growth rate. First of all, the bigger the firm, the lower the growth rate, due to reaching limits.

Secondly, past growth is not always a reliable indicator of what growth will be in the future. Long-term growth will be always lower than current growth. Barriers to entry help companies maintain large market share and therefore enhance higher returns and corresponding higher growth.

There are peculiar characteristics that a firm in stable growth should have. In particular, the risk of the company should be comparable to a stable growth firm in terms of beta and rating.

The beta should be around one, with a rating of BBB or higher on the debt issue by the firm. Also the debt ratio of the firm might increase to reflect the larger and more stable earnings of these firms. Also, the reinvestment rate of the firm should reflect the expected growth rate and the firm's ROC.

Concluding, always be aware that the terminal value will always be a large proportion of the total value. The point reflects the reality that the majority of the returns from holding a stock for a finite period comes from price appreciation.

The proportion of terminal value on the overall value increases as the growth increases. In some cases, the present value of the terminal value is greater than 100% of the current value of the stock.

8.3 A Real Case: The IPO of Facebook™

Learning Outcomes

- Define the road to the firm's IPO
- Understand the firm's fundamentals
- Comment on subsequent developments toward a fair valuation

8.3.1 The Road to the IPO

The history of Facebook is as interesting as controversial, in terms of its relationship with the markets. For many years in fact, the company resisted several attempts to be acquired and also rejected going public for years.

Starting from early 2006, Mark Zuckerberg turned down several acquisition offers, from the $75,000,000 offer from Viacom in 2006 to the valuation given by independent advisors in 2011 at $50 billion.

Facebook's valuation steadily increased in the days leading up to the IPO price. Zuckerberg wanted to wait to conduct an initial public offering, saying in 2010 that the managers were definitely in no rush.

The official path to the IPO began in February of 2012 when the company filed the documents to the SEC, as per guidelines and regulation of IPOs. The preliminary prospectus announced that the company had 845 million active monthly users and that its website featured 2.7 billion daily likes and comments.

The report to the SEC clearly showed that the impressive figures about memberships and income-related assets were slowing down, with deceleration projected to continue in the following years.

As for other companies before, the management of Facebook wanted to protect the ownership and control rights of the early investors, by instituting a dual-class stock structure, in 2009.

When the decision about going public was made, Facebook's founder Zuckerberg was puzzled about the IPO and the consequences of it. The fear of the founder was that the mission of the company would have to be compromised in the name of profit. Moreover, one should not forget about the burden of releasing a company's financial details to the public.

However, the company was at such a growth stage that involved the need for a consistent injection of capitals. Facebook needed money, and private companies face restrictions on how much stock they can issue for cash.

An attempt had been made in 2011, with the help of Goldman Sachs, to raise equity capital by creating a special investment product to sell the private shares of the company to selected Goldman Sachs's customers.

However, the SEC started investigating the plan immediately, putting pressure on the management of Facebook. The founder then decided that going public was the only solution to the capital need.

In late 2011, The Wall Street Journal reported that Facebook was anticipating an IPO valuation of $100 billion, nearly four times more than Google's market cap when it went public in 2004.

The company filed the forms to the SEC for starting the IPO process in early February 2012. The form contained everything an investor needs to know before buying shares at an IPO, including basic financial information and the business model.

The S−1 form filed to the SEC contained information regarding the core business of the company, which at the time was primarily focused in the display advertising business. Net profits were reported at $1 billion in 2011 from total revenue of $3.7 billion.

The company also listed the investment banks involved in the underwriting of the IPO. The pool of banks involved in the operation, being led by Morgan Stanley, JPMorgan Chase & Co., and Goldman Sachs Group Inc., included 33 investment banks.

The next step in the path to the IPO was, like for many other companies going public, the road show, the promotional tour where executives drum up support for their IPO before large investors.

The first meeting with investors took place in May 2012, just 11 days before the IPO day. The top management of Facebook met the investor at the Sheraton Hotel in midtown Manhattan.

There was a massive presence of journalists, as expected, and a long line of potential investors eager to understand the potential of the IPO. Inside the meeting, Facebook played a video introducing the business model to special clients of its underwriting banks.

That was the time when the road to IPO started being hindered by unwanted news. While the road show was taking place, Morgan Stanley found out Facebook had cut revenue projections, resulting in a nearly unprecedented last-minute correction.

The new internal forecasts affected the road show, which is usually functioning as a way to boost the expectations on the IPO. In the case of Facebook, that stage was heavily affected by the bad news.

Bad news about the company's new internal forecasts focused on revenue growing slower than expected, due to high-tech users flocking to smartphones faster than the company could serve mobile ads.

The last-minute cut in revenues by Facebook resulted in an unprecedented downward revision, characterized by poor timing. As a matter of fact, the public disclosure of such information would have turned the IPO down.

For an IPO of that magnitude in fact, even if corrections in the revenue were quite small, statistically, in IPO showbiz statistics run second to momentum, and nothing kills momentum like a poorly timed downward revision.

Obviously something had to be disclosed, but the issue was about what to disclose. The law requires companies to share all information that would likely influence an investor's decision to buy stock, and some of the underwriters were still advising clients based on figures that Facebook later reported to be wrong.

With only a few days left before the IPO, Morgan Stanley and the other banks decided to save the company from being ashamed of their figures, through a procedure that turned out to be quite controversial and harmful to the banks afterward.

First of all, Facebook would amend its public birth certificate filed to the SEC, the S−1 form, to include information about mobile usage cutting into revenue. Second, the company would disclose more specific information about the downgraded projections.

This amendment of the official form to the SEC resulted in just changing 3 pages out of the 170 pages included in the form. The only way to gather information about the ongoing challenge, to serve ads to mobile users, was to have knowledge of those three pages.

On page 14 and 17, the company said that its users were growing faster than ads due to the increased use of mobile phones and product decisions that allowed fewer ads per page.

Another amendment appeared on page 57, stating that the mobile trend discussed elsewhere in the document had continued in the second quarter, due to users shifting from computers to phones. But no financial consequences of such a persistent trend was ever mentioned anywhere in the updated pages.

That was all the information that was given out, and no mention was made about the projected lower revenue estimates. Even the most sophisticated retail investors, with specialized software to read the forms deposited at the SEC, had no clue about the consequences on revenue of the issues reported in the amendment.

Facebook was already projecting to trade at high multiples given its earning figures was slashing its annual projections. The underwriters were not involved into any type of information disclosure themselves.

There is a rule in information disclosure by analysts, which hinders them from publishing or emailing any type of information about a new public company in the first 40 days after the IPO.

Such a rule protects investors from being hit by the findings of analysts, which may bias their personal judgment about the investment to undertake. On the other hand, this information may then flow privately to institutional clients without a paper trail.

It is clear how such a game of telephone calls and private disclosure became so important in the peculiar situation of Facebook. Analysts had the chance to inform only selected investors about the last-minute change in the revenue projections.

After the company's surprising 11th-hour amendment, the unenviable job of explaining Facebook's revisions to the research analysts fell to Cipora Herman, Facebook's vice president of finance.

Bloomberg information system reports on May 11 that the interest of the institutional investors on the IPO was not as strong as expected. Another report from Reuters on the other hand stated that demand was much higher than the planned offer of stocks, with one large unnamed institutional investor was calling around syndicate desks trying to get more shares.

The market in those days was therefore characterized by large institutional investors being fully informed about Facebook's last-minute troubles and retail investors being unaware of them.

This is why the retail investors clamored for Facebook shares, while some large investors were planning a massive short selling, therefore essentially betting against the stock's buyers.

Telephone calls reached the managers of top analysts, from hedge funds and other big investors declaring they would have short sold the stocks of Facebook, due to the revelations.

But despite the growing consensus among some large investors that Facebook was overpriced, on May 15, 3 days before Facebook's market debut, the underwriting banks increased the IPO range from \$28–\$35 to \$35–\$38, citing heavy demand. A day later, they increased supply to more than 420 million shares.

The new share and price allocation placed Facebook's valuation at the iconic \$100 billion mark.

8.3.2 The Firm's Fundamentals

This section summarizes the findings of the case study about Facebook given to students during the Corporate Finance course given in the years 2013 and 2014. It is based on the financials of the company at the end of 2011.

Students were given the chance to play with the financial statement of Facebook and analyze the balance sheet (see Table 8.1), income statement (see Table 8.2), statement of cash flows (see Table 8.3), and major ratios (see Table 8.4). The following paragraphs summarize the results obtained in terms of fair pricing.

Based on the financials of the company, it is possible to calculate what the fair price of an IPO of Facebook should have been. In particular, it is also possible to grasp the idea of what went wrong in the actual process of the firm going public.

Several valuation methods are available for company valuation, as summarized in Table 8.5:

Students were asked to use free cash flow to firm (FCFF) valuation method. FCF valuation is most suitable when:

- The company is not dividend paying.
- The company is dividend paying but dividends significantly differ from FCFE.

Table 8.1 The balance sheet of Facebook™ as of year 2011

Balance sheet ($M)	2011	2010	2009	2008	2007
Assets					
Cash	1512	1785	633	297	305
Short-term investments	2396	0	–	–	–
Receivables	547	373	–	–	–
Inventories	0	0	–	–	–
Other current assets	149	88	–	–	–
Total current assets	*4604*	*2246*	*633*	*297*	*305*
Net property, plant, equipment	1475	574	–	–	–
Intangibles	162	96	–	–	–
Other long-term assets	90	74	477	208	143
Total long-term assets	*1727*	*744*	*477*	*208*	*143*
Total assets	*6331*	*2990*	*1110*	*505*	*448*
Liabilities and equity					
Accounts payable	63	29	–	–	–
Short-term borrowing	279	106	–	–	–
Other short-term liabilities	557	254	–	–	–
Total current liabilities	*899*	*389*	*0*	*0*	*0*
Long-term borrowings	398	367	–	–	–
Other long-term liabilities	135	72	241	170	174
Total long-term liabilities	*533*	*439*	*241*	*170*	*174*
Total liabilities	*1432*	*828*	*241*	*170*	*174*
Preferred stocks	615	615	–	–	–
Share capital	2684	947	868	335	273
Retained earnings	1600	600	0	0	0
Total equity	*4899*	*2162*	*868*	*335*	*273*
Total liabilities and equity	6331	2990	1109	505	447

- The company's FCF's align with company's profitability within a reasonable time horizon.
- The investor has a control perspective.

FCF valuation is very popular with analysts.

A firm potentially has an infinite life. Therefore, valuation must be based on a two-stage procedure where there are N future years (usually 5–10) of growth forecast of cash flows, followed by the final (discounted) terminal value.

$$V_0 = \sum_{i=1}^{N} \frac{\mathrm{FCF}_i}{(1 + r_{\mathrm{WACC}})^i} + \frac{\mathrm{TV}}{(1 + r_{\mathrm{WACC}})^N}$$

The stable growth rate cannot exceed the growth rate of the economy, but it can be set lower. If the economy is assumed to be composed by high-growth and stable growth firms, the latter group will normally grow slower than the economy.

Table 8.2 The income statement of Facebook™ as of year 2011

Income statement ($M)	2011	2010	2009	2008	2007
Operating revenue	3711	1974	777	272	153
Cost of revenue	(537)	(354)	(145)	(124)	(41)
Gross profit	*3174*	*1620*	*632*	*148*	*112*
Selling/general/admin expense	(707)	(305)	(205)	(47)	(113)
Research and development	(388)	(144)	(87)	(156)	(113)
EBITDA	*2079*	*1171*	*340*	*−55*	*−114*
Depreciation and amortization	(323)	(139)	(78)	0	0
Other income, net	19	2	(2)	0	(10)
Operating income	*1775*	*1034*	*260*	*(55)*	*(124)*
Share of results of associated companies	0	0	0	0	0
EBIT	*1775*	*1034*	*260*	*(55)*	*(124)*
Interest income (expense)	(42)	(22)	(10)	(1)	(11)
Pretax income	*1733*	*1012*	*250*	*−56*	*−135*
Taxes	(695)	(402)	(25)	0	(3)
Net income	*1038*	*610*	*225*	*(56)*	*(138)*
ROE	*24.23%*	*39.43%*	*25.92%*	*0.00%*	*0.00%*

Table 8.3 The statement of cash flows of Facebook™ as of year 2011

Cash flows ($M)	2011	2010	2009
Net income	1038	610	225
Depreciation and amortization	323	139	78
Accounts receivable	(174)	(209)	(112)
Accounts payable	102	120	(7)
Inventory	5	0	0
Other adjustments	221	23	(60)
Cash from operating activities	*1515*	*683*	*124*
Capital expenditures	(606)	(293)	(33)
Acquisitions and other investing activity	(2400)	(22)	(32)
Cash from investing activities	*(3006)*	*(315)*	*(65)*
Dividend paid	0	0	0
Sale or purchase of shares	1030	506	209
Increase in short-term borrowing	603	115	82
Increase in long-term borrowing	(431)	160	(48)
Cash from financing activities	*1202*	*781*	*243*
Foreign exchange adjustment	3	(3)	0
Change in cash and cash equivalents	*(289)*	*1149*	*302*
Changes in working capital	5	(70)	(179)
Free cash flows	*802*	*408*	*101*

When nominal cash flows and discount rates are used, the growth rate should be nominal in the currency in which the valuation is denominated. One simple proxy for the nominal growth rate of the economy is the risk-free rate.

Table 8.4 Major ratios of Facebook™ as of year 2011

Indices	2011	2010	2009	2008	2007
Debt/equity	0.29	0.38	0.28	0.51	0.64
Debt/assets	0.23	0.28	0.22	0.34	0.39
Equity/assets	0.77	0.72	0.78	0.66	0.61

Table 8.5 Valuation methods for the firm and equity

Valuation method			
	Dividend discount	*FCF to equity*	*FCF to firm*
Cash flow	Dividend	FCFE	FCFF
Expected growth	In dividends	In FCFE and equity income	In FCFF and operating income
Discount rate	Cost of equity	Cost of equity	Cost of capital
Steady state	Constant *g* for dividends	Constant g for FCFE	Constant g for FCFF

The historical growth in earnings per share is usually a good starting point for growth estimation. Analysts estimate growth in earnings per share for many firms. It is useful to know what their estimates are. Ultimately, all growth in earnings can be traced to how much the firm is investing in new projects and what returns these projects are making for the firm.

Growth estimation can be performed by analyzing the EPS of the firm. There are two possible methods for it. The first method assumes that the ROE is expected to stay constant and defines the growth rate as

$$g_{\text{EPS}} = r_R \times \text{ROE}$$

The expected growth rate in earnings for a company cannot exceed its return on equity in the long term. When the ROE is expected to change, equation becomes

$$g_{\text{EPS}} = (r_R \times \text{ROE}_{t+1}) + \frac{(\text{ROE}_{t+1} - \text{ROE}_t)}{\text{ROE}_t}$$

Please recall that small changes in ROE translate into large changes in the expected growth rate and that the lower the current ROE, the greater the effect on growth of changes in the ROE.

The stable growth can be estimated by taking into account simple rules. First of all no firm can, in the long term, sustain growth in earnings per share from improvement in ROE.

In stable growth, firms should have the characteristics of other stable growth firms. In particular, the risk of the firm, as measured by beta and ratings, should reflect that of a stable growth firm.

The reinvestment rate of the firm should reflect the expected growth rate and the firm's return on capital

$$r_R = \frac{g_{\mathrm{OC}}}{\mathrm{ROC}}$$

When a firm's cash flows grow at a "constant" rate forever, the present value of those cash flows can be defined as a growing perpetuity, as seen in previous chapters.

The stable growth rate cannot exceed the growth rate of the economy but can be negative. The terminal value will be lower, and you are assuming that your firm will disappear over time. One simple proxy for the nominal growth rate of the economy is the risk-free rate.

While analysts routinely assume very long high-growth periods (with substantial excess returns during the periods), the evidence suggests that they are much too optimistic. A study of revenue growth at firms that make IPOs in the years after the IPO shows that typically the revenue growth rate of a new public company outpaces its industry average for only about 5 years.

In the case study, the students were recommended to use an approach of the log-descend type, where the revenues for each comparable company have to be logged, and after that the trend of increase of the log price from 1 year to each other should be recorded (see Table 8.6).

The last trend represents a fair proxy of the beginning of the stable growth period for all companies, and it is then wise to take an average of them, so to obtain a risk-free type of rate.

8.3.3 After the IPO

The trading of Facebook stocks on the regulated exchange became on May 18, 2012, as planned. However, technical problems delayed the start of trading from 11.00 am to half hour later.

In general, during the first day of trading, many technical problems were experienced, and some orders did not go through due to glitches of various kind. Moreover, investors got confused as to whether or not their orders were successful.

During the first hours of trading, the stock price rose to a maximum of $45, but the initial rally of trade was quite unstable. Due to the tendency of the stock price to stay below the IPO price for most of the day, many underwriters bought back shares to support the price.

If it were not for the technical problems mentioned above, the market price would have fallen below the IPO level already on the first trading day. The closing price of the first day ended up being $38.23, only 23 cents above the IPO price and down $3.82 from the opening bell value.

After a disappointing first day, the trading went on and, it was supported by a huge figure in terms of volume, standing at 460 million shares exchanged in the first day of trading.

Table 8.6 An example of growth estimation through historical log growth of revenues

	Google			Linkedin		Yahoo					
	Rev.	Log	Trend	Rev.	Log	Trend	Rev.	Log	Trend	Avg. trend	Risk-free
2010	29,321	10.29	2.13%	215	5.37	12.18%	6324	8.75	−0.24%	4.69%	3.56%
2009	23,651	10.07	0.82%	120	4.79	37.67%	6460	8.77	−1.23%	12.42%	
2008	21,796	9.99	2.81%	32	3.48	52.39%	7209	8.88	0.38%	18.53%	
2007	16,594	9.72	4.83%	10	2.28	41.84%	6969	8.85	0.93%	15.86%	
2006	10,605	9.27	6.27%	5	1.61		6426	8.77	2.34%	4.30%	
2005	6139	8.72	8.12%				5258	8.57	4.72%	6.42%	
2004	3189	8.07	10.66%				3574	8.18	10.66%	10.66%	
2003	1466	7.29	19.79%				1625	7.39	7.78%		
2002	440	6.09	36.47%				953	6.86			
2001	86	4.46									
2000											

The IPO ended up raising $16 billion, making it the third largest operation of that kind in US history, behind only General Motors and Visa Inc. The stock price corresponded to a market capitalization higher than most US corporations.

At the time, giants like Amazon and Disney ended up being worth less than the freshly public company. Also the wealth of the founder got considerably high, with Zuckerberg's stocks worth $19 billion.

In the subsequent days, Facebook's share turned out to be bullish on 9 of the 19 days after the IPO, with only 4 bearish days. On May 21 the closing price was as low as $34, and the price dropped to $31 the day after that.

The stock increased modestly in coming days, and Facebook closed its first full week of trading at $31.91. The stock returned to losses for most of its second full week and had lost over a quarter of its starting value by the end of May.

By June 6 investors had lost $40 billion in capitalization. The third trading week of Facebook ended up in a low $27.10 even worse than a week previous. The company finished its fourth full week with a modest gain, reaching a closing price of $30.01.

The volatility was so high that analysts started delivering controversial predictions of stock price range. Some of them in fact recommended buying in early June, while others recommended selling, in the same period.

The difficult path of Facebook's shares right after the IPO impacted on the stock market as a whole. The high-tech sector suffered from a decrease in prices and the exchanges as a whole suffered from the bearish trend.

The impact of the Facebook IPO was immediate on the market, affecting other technology companies as well. Investment firms faced considerable losses, as a consequence of the technical troubles.

Bloomberg estimated that retail investors may have lost approximately $630 million on Facebook stock since its debut. UBS alone may have lost as much as $350 million.

The Nasdaq stock exchange offered $40 million to investment firms plagued by offering-day computer glitches.

But the investors' losses were too high to be compensated by such an amount, and the other NY exchange, the NYSE, commented severely on the decision of increasing the $3 million reimbursement limit usually observed in these cases.

IBM was hired by Nasdaq to take care of the technical issues and overcome them. The IPO impacted both Facebook investors and the company itself. It was said to provide healthy rewards for venture capitalists who finally saw the fruits of their labor.

The wrong IPO definitely lowered the interest of the investors toward Facebook. It was also the case of all the stakeholders, including employee, who changed their view of the investment.

The situation was very risky for the company, also in view of raising further capital for future investments and expenditures. On top of that, the IPO could jeopardize profits for underwriters who face investors skeptical of the technology industry.

From a legal point of view, more than 40 lawsuits were filed regarding the Facebook IPO in the month that followed the actual operation, making the situation even worse for the shareholders and the underwriters.

The latter ones, namely, Morgan Stanley (MS), JP Morgan (JPM), and Goldman Sachs, (GS) all cut their earning forecasts for the company in the middle of the IPO road show.

It is believed that adjustments to earning estimates were communicated to the underwriters by a Facebook financial officer, who in turn used the information to cash out on their positions while leaving the general public with overpriced shares.

8.4 Summary

Company valuation is a very important field of specialization involving companies and investment banks. Based on the financials of a firm, it is possible to give an estimate of the market value of the equity.

The task can be accomplished systematically for publicly traded firms. The estimation of parameters like risk-free rate premium and beta that serve as input for CAPM valuation can in fact be estimated through historical data.

The valuation of private firms is different, in that no historical data are available, and it is usually performed at the time of initial public offering to assess the right asking price of the operation.

The cost of capital is normally based on book values rather than market values, and the same goes for the other financial aspects of the firm. Beta estimation is not possible through regression as for the public firm, and it involves an analysis of comparable firms.

Another major aspect to be careful about when dealing with a private firm is the growth rates and the terminal value. Making the right assumptions about both the discrete cash flow period growth and the terminal (permanent) growth is important to not overestimate or underestimate the firm.

The chapter presents a case study regarding the IPO of Facebook, one of the most controversial financial operations of the last years. The firm in fact went public after a publicized escalation to the level of big corporation.

There were many expectations about the company going public, and the analysts were speculating for months about the value the shares would have recorded at the beginning of the trading.

Facebook still remains as an example of an overrated IPO, and the days following the first trading hours confirmed that. By studying such a case, it is possible to understand how sometimes the bias in the information can generate disasters on the market.

Problems

1. Assuming we are valuing a going concern, which of the following types of income streams would be most appropriate for valuing the company?
   (a) Earnings before interest and taxes
   (b) Free cash flows
   (c) Operating income after taxes
   (d) Price to earnings ratio
2. The following estimates have been made for the year 2006:
   Operating income (EBIT): €6000
   Depreciation: €500
   Cash taxes to be paid: €950
   Income from nonoperating assets: €60
   No capital investments or changes to working capital are expected. Based on this information, the projected free cash flows for 2006 are:
   (a) €5610
   (b) €4550
   (c) €4490
   (d) €6550
3. Marshall Company is considering acquiring Lincoln associates for €600,000. Lincoln has total outstanding liabilities valued at €200,000. The total purchase price for Marshall to acquire Lincoln is:
   (a) €200,000
   (b) €400,000
   (c) €600,000
   (d) €800,000
4. The valuation process will often analyze several value drivers in order to understand where value comes from. Which of the following value drivers would be least important to the valuation?
   (a) Return on invested capital
   (b) Earnings per share
   (c) Cash flow return on investment
   (d) Economic value added
5. You have been asked to calculate a terminal value for a valuation forecast. The normalized free cash flow within the forecast is €11,400. A nominal growth rate of 3% will be applied along with a weighted average cost of capital of 15%. Using the dividend growth model, the terminal value that should be added to the forecast is:
   (a) €78,280
   (b) €86,200
   (c) 95,000
   (d) 97,850
6. Information from a valuation model for Gemini Corporation is summarized below:
   Total present value of forecasted free cash flows: €150,000

Terminal value added: €450,000
Total present value of nonoperating assets: €20,000
Total present value of outstanding debts: €120,000
If Gemini has 20,000 shares of outstanding stock, the value per share of Gemini is:
(a) €15.00
(b) €25.00
(c) €30.00
(d) €35.00

Case Study: Company Valuation

Cashgrow Ltd

The Case

Cashgrow Ltd is a well-established company operating in the field of renewable energies. They recently topped up to a record $1.85bn in sales, after launching a new power generator with increased efficiency and environmental friendly.

The generator has been the last of a series of successful projects undertaken by the company, which comes from 5 years of continuous growth in sales, gross income, and net income.

For this reason, and given the need for the company to step up and increase the scale of its operations, the management of Cashgrow decided that the company will go public in the coming year.

Going public means the company will go through an IPO, with related consequences in terms of market capitalization, share appreciation, and cash inflows. The IPO will coincide with the most important capital restructuring for the company since its birth.

Jihad is one of the senior financial analysts working in the team of the CFO office. He has been selected to lead a team of analysts who will carry on a preliminary valuation of the company Equity, so to have an idea of what to expect from the IPO.

After consulting his team, Jihad tells the management that they are going to apply a discounted cash flow (DCF) approach to the valuation of Cashgrow. That should give a good estimation of the market value of the company.

The valuation team gathers the most important financial statement items, as well as the main variables involved in the analysis. The entries from both the balance sheet and the income statement are summarized in Table 8.7 below.

Since Cashgrow is not public yet, the valuation must be run by using comparable firms and the sector as a source of financial information. In particular, there are three companies that are comparable to Cashgrow. The financial variables of those companies, and the market, that are pertinent to the analysis are summarized in Table 8.8.

Table 8.7 Accounting items of Cashgrow in the last 5 years

Item	2017	2016	2015	2014	2013
Total assets	5,575,000	5,050,000	4,350,000	3,975,000	3,560,000
Total debt	2,075,000	2,050,000	1,650,000	1,450,000	1,260,000
Total equity	3,500,000	3,000,000	2,700,000	2,525,000	2,300,000
Sales	1,850,000	1,550,000	1,300,000	1,150,000	1,000,000
Cost of goods sold	820,000	725,000	675,000	620,000	550,000
Depreciation	105,000	115,000	121,000	90,000	85,000
Other expenses	90,000	82,000	67,000	79,000	42,000
Taxes	230,000	190,000	157,000	95,000	82,000
Changes in NWC	(5500)	6200	(3500)	4500	5200
Capital expenditure	12,000	15,300	13,100	18,150	9500

Table 8.8 Financial variables of the companies and the market

Variable	Company A	Company B	Company C	Market
Volatility of stock returns	14.35%	16.25%	12.76%	11.45%
Correlation with the market	35.43%	42.25%	37.12%	100%
Long-term risk-free rate	3.25%	3.25%	3.25%	3.25%
Cost of debt	5.35%	5.55%	5.15%	
Average tax rate	33.25%	33.75%	32.75%	
Average (market) debt-to-equity ratio	31.75%	32.50%	30.55%	

Given that the company has substantial debt in the book, Jihad specifies to the managers that the team will be valuing the company based on a two-stage free cash flow to firm (FCFF) variant of the DCF model.

Questions

1. If you were Jihad, how would you proceed for the calculation of the company's value? What are the main steps in the two-stage FCFF valuation model?
2. How can the variables in Tables 17.1 and 17.2 be used to calculate the parameters needed in the analysis?
3. What choices would you make for the growth of the cash flows over time? How can you justify these choices?
4. What model would you use to calculate the cost of equity of the firm?
5. What discount rate would you apply to the projected cash flows, to run the valuation? What model would you use to calculate it?

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Financial and Real Options

9

Financial derivatives constitute a large proportion of trading in world financial markets, due to their unbeatable features. Derivatives are popular in fact for their flexibility and combinability.

Derivatives have large application for risk hedging, but they can also be traded for speculative purpose, making them at the same time an important tool for wealth preservation and also a highly risky form of investment.

When it comes to corporate finance, the focus is on options and their features. Real options in fact can be used to modify the profitability profile of projects and investments available to the company.

In most cases, adding some option to projects that in theory present a negative NPV, therefore likely to be rejected, can change their profile, making it worthwhile to invest in them.

It is therefore very important to analyze the properties of real options as a tool for corporate management, and, in order to do so, it is first important to understand the rationale and functioning of financial options.

The purpose of the chapter is in fact to give a robust insight on financial options, their pricing and possible use, and to then proceed to the analysis of real options, as a tool for improved project valuation.

After studying this chapter, you will be able to answer the following questions, among others:

- What are financial derivatives, and what is their use in financial investment?
- How can financial options be described, and what is their payoff profile?
- How is it possible to valuate financial options? What types of model are available for option pricing?
- What are real options, and what is their use in corporate finance?
- How can real options be priced? What types of model are available? How can real options be incorporated in projects?

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The first section of the chapter is an introduction to financial derivatives and their use for hedging or speculation purposes, with insights on option payoff structure. The second section deals with the pricing of financial options, by using analytical or numerical models (trees). The final section focuses on real options, their pricing by using either analytical or numerical approach, and their use in corporate finance as options embedded in some project.

9.1 Basic Concepts

Learning Outcomes

- Learn what financial derivatives are and how they can be used.
- Explain the structure and payoff profile of financial options.
- Define various strategies and combinations of options.

9.1.1 Financial Derivatives

The value of a financial derivative is linked to a specific underlying instrument or indicator or commodity. Through derivatives it is possible to trade specific financial risks and hedge positions in the underlying asset.

The value of the transactions in financial derivatives should be separate from the value of the underlying transactions, and no principal amount is advanced to be repaid and no investment income accrues. They are used for a number of purposes including risk management, hedging, arbitrage between markets, and speculation.

Financial derivatives allow to transfer some specific risk to a counterparty which is interested in bearing it. The entities receiving the risk are better suited to manage the risk involved in the transaction.

It is possible to trade the risk by embedding it directly in the contract, like in the case of options. Another way is to create a new contract embedding the risk features so as to match those of the existing contract.

Basically it is possible to offset the risk associated with a derivative by simply creating a new derivative with reversed characteristics. Buying the new derivative is the functional equivalent of selling the first derivative, as the result is the elimination of risk (Chance and Brooks 2008).

Derivatives contracts usually involve cash settlement and before maturity in some cases. This is consequent to using them to trade risk independently of ownership of an underlying item. But in some cases, the position in financial derivatives involves dealing with the underlying asset too, like in the case of foreign currency derivatives.

The future value of the underlying asset is not predictable with certainty, making the value of the financial derivative an estimated anticipated guess. The underlying assets of a derivative range from commodities, financial instruments, interest rates, exchange rates, other derivatives, spreads, and indices.

Due to their peculiar features, some items cannot be identified as financial derivatives. This is the case of a fixed price contract, unless standardized and tradable. Also timing delays do not represent a financial derivative.

Insurance cannot be considered as a form of financial derivative. It is a form of financial intermediation in which funds are collected from policyholders and invested in assets (technical reserves) to meet future event driven claims.

Contingencies, such as guarantees and letters of credit, are also not financial derivatives. In fact, they are not instruments that facilitate the trading of specific financial risks.

When the derivative features are embedded in another standard instrument, from which it cannot be separated, it is also impossible to talk about financial derivative. This is because both the derivative part and the underlying asset involve the same counterparties.

The whole instrument must then be classified according to the feature of the primary security, even if the value will be different from comparable straight security, because of the embedded derivative.

Examples are bonds that are convertible into shares and securities that carry the option of repaying the principal in a different currency from that of issuance.

Many institution nowadays trade in derivatives, including commercial banks, investment banks, central banks, fund managers, insurance companies, and other nonfinancial corporations.

In order to understand the use of derivatives, one should recall that market participants can be divided into hedgers and speculators. Hedgers seek for protection against adverse changes in the values of their assets or liabilities, while speculators seek for huge profits in a short time.

The hedging purpose of derivatives is fulfilled by taking a position such that, in case the value of the assets fall, the value of the derivative increases by a corresponding amount. The aim of a speculator is instead to anticipate changes in market prices or rates or credit events by entering a derivative contract.

The definition explains how riskier the activity of speculators compared to that of hedgers is. It therefore requires close regulatory monitoring, even if in practice it is not easy to separate the two categories of investors.

From a corporate finance perspective, companies can use financial derivatives to achieve better financing terms. For example, it is much easier to get a loan for a company that managed to hedge part of its risk.

Fund managers sometimes use derivatives to achieve specific asset allocation of their portfolios. For example, passive fund managers of specific index-tracking funds may need to use derivatives to replicate exposures to some not so liquid financial assets.

The earliest trading of derivatives dates back to the twelfth century in Venice, Italy. The primitive form of credit derivative at that time took the form of loans to fund a ship expedition with some insurance on the ship not returning.

After the development of derivatives on commodities in the sixteenth century, the market was still hindered by the high cost of transportations, limiting the range of action of traders.

In the early 1970s, financial derivatives came back to popularity among investors, especially in response to some major changes in the global financial markets. In 1971 there was the collapse of the Bretton Woods system of fixed exchange rates.

With the collapse of the system, and the turning back to floating currencies, the demand for protection against exchange risk increased a lot.

Other changes in financial policy mostly apply to the United States, which has been the most vibrant market for derivatives in the last decades. From the monetary policy adoptions of the FED in late 1970s to the establishment of target rates for fed funds in the early 1990s, the demand for derivatives to hedge against adverse movements in interest rates raised.

Also, the many emerging market financial crises in the 1990s, which were often accompanied by a sharp rise in corporate bankruptcy, greatly increased the demand of global investors for hedging against credit risk.

Theoretical steps in financial theory also contributed to making financial derivatives one of the most popular types of investment. The derivation of analytical pricing models like Black-Scholes-Merton model for options pricing, provided a new framework for portfolio managers to manage risks.

Finally, one should not deny the crucial role of the technology developments, which, starting from the late 1990s, allowed traders and asset managers to develop new types of derivatives, as tools for managing more complex types of risk.

Derivatives are classified into four main categories, namely, forwards, futures, options, and swaps. The analysis of all these classes allows for a better understanding of the dynamics underlying derivatives markets (Kolb and Overdahl 2003).

A forward has similar characteristics of futures and is an agreement to buy or sell a specified quantity of an asset at a specified price with delivery at a specified date in the future.

The main difference is then in the way the transactions take place. Investor trading futures experience gains and losses on a daily basis, while forwards involve cash settlement at delivery date.

Moreover, futures contracts have standardized features, while the forwards are tailored to meet the needs of the counterparties. Also important is the fact that futures contracts are settled through a clearing house, while forwards are settled between the counterparties instead.

Finally, because of being exchange-traded, futures are regulated, whereas forwards, which are mostly over-the-counter (OTC) contracts, are loosely regulated (at least until the global financial crisis).

There are two types of option, call and put. The call option gives the buyer the right (but not the obligation) to buy a specified quantity of the underlying asset, at a specified (strike) price, at expiration day, or before it. The put option on the other hand gives the right to sell the underlying asset to the counterparty, again at the strike price, and before or coincident with maturity.

European options are those that can be exercised only at maturity, and not before, while American options can be exercised at any time before maturity. The difference makes the two types of option very distinctive.

In options transaction, the purchaser pays the seller an amount for the right to buy or sell, which is called the option premium. As opposed to forwards and futures, options do not require the purchaser to buy or sell the underlying asset under all circumstances.

If an option is not exercised at expiration, the owner gets a zero payoff from it, therefore losing just the premium paid to buy the option. In case the option is exercised the seller of the option is liable of the costs of changes in value of the underlying benefiting the purchaser.

Financial swaps are contracts where the counterparties agree to exchange a series of cash flows, for a determined period of time. The periodic payments can be charged on fixed or floating interest rates, depending on contract terms. There is an agreed nominal amount on what the calculation of the payments is based.

Equity derivatives represent a large share of the derivatives market and can be written on single stocks or indices. Way back in 1982, trading of futures based on S&P’s composite index of 500 stocks began on the Chicago Mercantile Exchange (CME).

One year later, futures on the S&P 500 index also started trading on the CME and are still very popular nowadays. The investors can still buy futures on stock indices in that market or others in the world.

Another class of commonly traded equity derivatives is the so-called equity swaps. They are contracts where two counterparties agree to exchange the total return on a stock in exchange of a floating rate of interest.

Equity swaps can be used to hedge the positions in equity without giving up ownership of his share. At the same time, the party receiving equity return enjoys exposure without actually taking ownerships of shares.

The class of interest derivatives includes the interest rate swaps (IRS), which come in the form of an agreement between a bank and a counterparty where the former agrees to pay a floating rate to the latter in exchange of fixed interest rate payments.

The IRS is useful for interest risk management, and it is therefore widely used by banks. A decline of interest rates in fact would reduce the income from loans while keeping the interest payment on the deposits unchanged.

By entering an interest rate swap contract and receiving fixed rate receipts from a counterparty, banks would be less exposed to the interest rate risk. Meanwhile, interest rate futures contract allows a buyer to lock in a future investment rate.

The market of commodity derivatives has been the earliest, in the twentieth century, but it was subject to the problems connected to storing, delivering, and seasonality. In the 1970s the new market for commodity derivatives raised prominently.

At that time in fact monopolistic dominance of few commodity producers had been overcome, toward an improved competition. This allowed the price movements to better reflect the market supply and demand conditions.

The increased competition increased trading and volatility as a consequence. Following that pattern, the commodity traders increased their demand for derivatives in order to hedge the associated risks.

For example, forwards contracts on Brent and other grades of crude became popular in the 1970s following the emergence of the Organization of Petroleum Exporting Countries.

Deregulations of the energy sector in the United States since the 1980s also stimulated the trading of natural gas and electrical power futures on the New York Mercantile Exchange (NYMEX) in the 1990s.

In the last decades, the world has been subject to an increasing financial and trade integration. As a consequence, the demand for protection against exchange rate movements rose.

A forward exchange contract is an agreement to buy or sell a certain amount of foreign currency at a predetermined exchange rate, with a certain maturity.

Example 9.1 A European aircraft building company is selling material to a US air carrier, for a payment of $10,000,000, to be cashed in 12 months from current time. The current spot exchange rate is €0.80 per dollar. That corresponds to a current theoretical amount of $8,000,000 for the deal. But the payment is expected to come after 1 year, and if the euro appreciates by 10% against dollar is expected, the amount in euro expected by the supplier becomes €7,200,000. The solution can be a hedging strategy to be implemented by purchasing dollars forward at a rate of €0.76 per dollar. This will correspond to a certain amount of €7,600,000, which is an intermediate value between the current spot rate and the worst scenario of a 10% appreciation of the euro. The exchange rate is locked at current time in order to avoid surprises in 12 months.

Dollar and euro are both vehicle currencies, and they are widely used for payment on foreign markets. Some other currencies are thinly traded currencies or currencies of those countries with restrictions on capital account transactions, the profit or loss resulting from the forwards transaction can be settled in an international currency. The forward contracts in that type of currencies are called non-deliverable forward contracts, and they are traded offshore (Gardner 1996).

Cross-currency swaps involve two parties exchanging payments of principal (based on the spot rate at inception) and interest in different currencies. A liquid cross-currency swap market is an important for local currency bond market developments, because such instruments allow foreign borrowers in local bond markets to swap back their proceeds to their own currencies while hedging against the interest rate risk.

Credit derivatives are contracts where a party sells credit protection to another party. This happens through a promise of payment from the credit protection buyer to the credit protection seller, in case a particular reference entity incurs a credit event.

A credit event in general refers to an incident that affects the cash flows of a financial instrument (the reference obligation). There is no precise definition, but in practice, it could be filing for bankruptcy, failing to pay, debt repudiation or moratorium.

An important and fast-growing type of credit derivatives is the credit default swap (CDS), a type of contract that protects the buyer against the loss generated on the face value by the default of the issuer of the reference bond.

The contract involves payment of a periodic premium from the buyer to the seller of the CDS. The premium reflects the default probability of the reference entity and the relative expected loss on the nominal value, as assessed by the CDS buyer.

In case the reference entity incurs a credit event, the buyer has the right to receive compensation from the seller. Normally a CDS contract is written on just one reference entity, therefore being named single-name CDS.

In some cases, a basket of single-name CDS are also considered, as an index, on entities with similar ratings. Such a grouping gives investors the opportunity to take on exposures to specific segments of the CDS index default loss distribution (Amato and Gyltenberg 2005).

Financial derivatives are traded on regulated exchanges as well as in OTC markets, with substantial differences in the way the trading takes place. While in exchange-traded markets, derivatives contracts are standardized, in OTC markets they are not.

The contracts traded on a regulated exchange have specific delivery and settlement terms, with negotiations conducted on the credit floor and electronic trading system. Exchange-traded derivative trades are publicly reported and cleared in a clearing house.

The clearing house stands as a body taking care of the trade settlement if the seller of an asset defaults. All positions on the market are marked in order to ensure sufficient funds for the solvency of the clearing house, by means of collected margins.

In the OTC market on the other hand, the derivative trades are bilateral, without any filter or buffer interposed. All contract terms such as delivery quality, quantity, location, date, and prices are negotiable between the two parties.

The transactions are planned and executed via telephone or the Internet, and prices are not publicly disclosed. The two markets complement each other and together fulfill all the needs of investors (Nystedt 2004).

Derivatives exchanged in a regulated market have better price transparency than those traded on OTC. Moreover, the clearing house ensures the risk is smaller, with all trades on exchanges being settled daily with the clearing house.

The advantage of OTC markets on the other hand is the higher flexibility that makes OTC trading very good for low-order flows and investors with special needs. This is why the OTC market works well for the placement of new financial products.

There are several steps in treating financial derivatives statistically, governing how they are recorded in the balance of payments. First of all, the exchange of claims underlying a derivative contract creates true asset and liability positions.

Moreover, the transactions in secondary (OTC) market must be registered as financial transactions, and any payment or settlement must be registered as transactions in financial derivatives, so as to generate no income.

9.1.2 Option Structure and Payout

An option is a financial derivative that gives the right to buy or sell, a reference security (underlying) at a pre-specified price. The two major classes of options are call options and put options (see Figs. 9.1 and 9.2).

The buyer of a call option has the right to buy some asset at a predetermined price, defined as the strike price. The contract is settled at maturity. A put option on the other hand gives the right to sell a particular asset at a pre-specified price.

The option gives the owner rights but no obligations, and the holder of a long position in a call option, for example, gains from the price of the underlying asset being above the strike price of the call at maturity (Hull 2005).

In case the strike goes below the strike price, it is not optimal to exercise the option given that the difference with the strike price is theoretically negative. Therefore, the buyer will let it expire. The payoff from a long position in a call option is

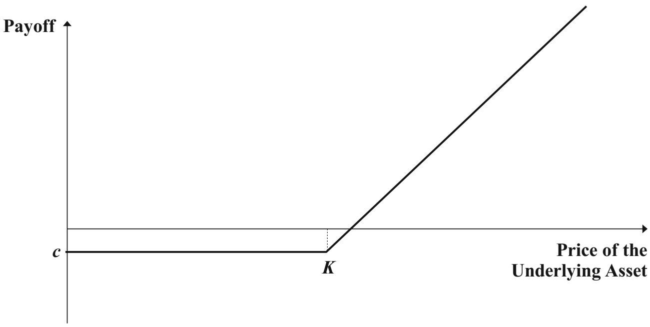

Fig. 9.1 The payoff function for a long position in a call is flat, beginning at an underlying asset price of zero and continuing to the strike price, with a loss equal to the call premium (c). As the price of the underlying asset rises beyond the strike price, the call comes into the money, and payoff begins to rise with the price of the underlying asset

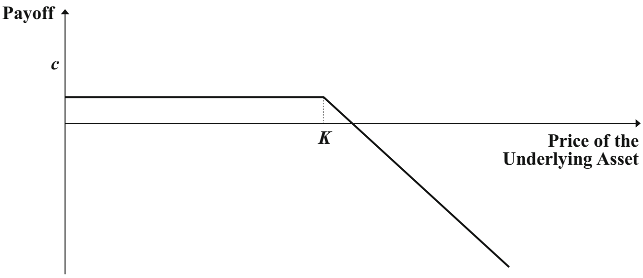

Fig. 9.2 The payoff function for a short position in a call is flat, beginning at an underlying asset price of zero and continuing to the strike price, with a profit equal to the call premium. As the price of the underlying asset rises beyond the strike price, the call comes out of the money, and the loss begins to rise one-for-one with the price of the underlying asset

$$c = \max(S_T - K, 0)$$

where:

S_T is the price at maturity T of the underlying.

K is the strike price specified in the contract.

The simplest way to trade call options is through getting long positions on them. When a stock has a bullish trend, the trader can buy call options in order to get the underlying stock for the strike price at maturity, and cash it on the market for a high price.

Some investors instead benefit from short positions in call options. When the outlook on the underlying is bearish, the seller of the options believes it is possible to benefit from the counterparty not exercising the right, and the gain on the premium cashed to sell.

For the buyer the situation is opposite, and the gain comes from the price of the underlying being below the strike at maturity. As opposed to a call option, when the price is above the strike, it is not optimal to exercise the option.

The payoff of the long position in the put option is given as in Fig. 9.3.

$$p = \max(K - S_T, 0)$$

Holding a long position in a put also has a protective purpose, in that it hedges against a bearish trend of the underlying asset, if held. If the stock is bearish, the trader can buy put options to cover the lower asset price and balance it.

There is also the possibility to benefit from short positions in put options, in case the underlying asset shows a bullish trend (see Fig. 9.4). The seller of the option in this case benefits from the expiring of the option without exercise and the gain on the premium cashed to sell.

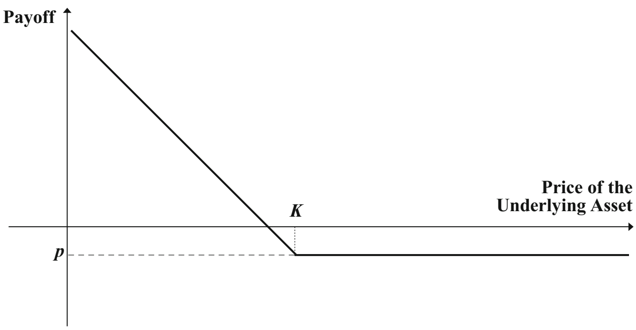

Fig. 9.3 The payoff function for a long position in a put option, as oppose as a call option, shows declining gains for possible future stock price lower than the strike and a flat premium loss afterward. As the price of the underlying asset stays below the strike price, the put option comes into the money, and payoff declines afterward

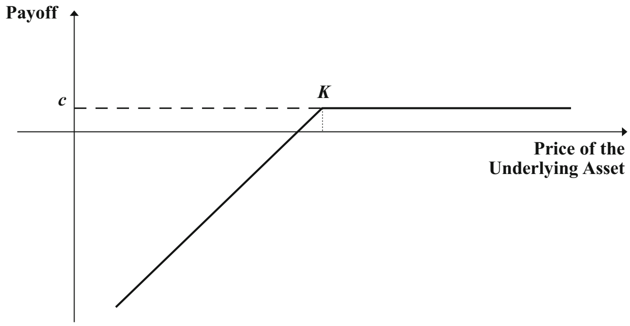

Fig. 9.4 The payoff to writing a put option is exactly the opposite of the payoff to buying one. The writer receives the put option premium so long as the price of the underlying asset remains above the strike price of the put. Once the underlying asset's price falls below the strike price, the payoff to option writer falls one-for-one as the price of the underlying asset falls

Like many financial derivatives and some other assets, most financial options are not traded in a regulated exchange but only in over-the-counter (OTC) markets, with different regulation.

Whereas exchange-traded options are standardized contracts, OTC options are usually tailored to a particular risk. If an investor has a need to hedge a position in currency or some other underlying asset, for a specific maturity, but the available exchange-traded options cover only part of it.

9.1.3 Strategies and Combinations

Besides being an important type of stand-alone investment, for speculation or hedging, financial options can also be combined in order to form several payoff profiles that can accomplish multiple hedging tasks (Neftci 2000).

The simpler form of combination is called covered call (or buy-write strategy), and it consists in purchasing an asset, and simultaneously selling a call option on that same asset, in order to cover your position. The volume of assets owned should be equivalent to the number of assets underlying the call option.

Such a strategy is indicated when the outlook of the stock is stable, with low volatility forecasted, and the option is then used to generate additional profit through cashing the option premium from sale. It also offers protection against a fall in the asset price (see Fig. 9.5).

A married put strategy involves buying or owning an asset, and the simultaneous purchase of a put option written on an equivalent amount of assets. The strategy is used to protect against bullish trend of the asset price to hedge short-term losses. The strategy essentially works like an insurance policy and establishes a floor should the asset's price plunge dramatically (see Fig. 9.6).

In a bull call spread strategy, an investor will simultaneously buy call options at a specific strike price and sell the same number of call options with the same expiration

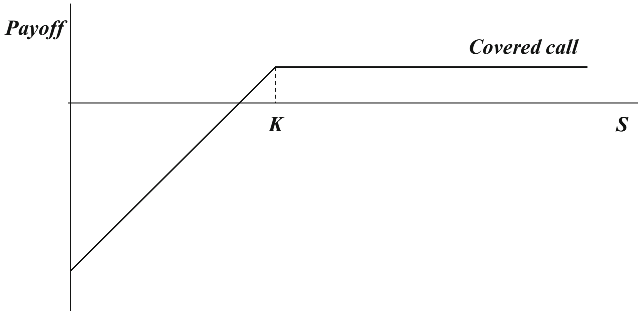

Fig. 9.5 Payoff diagram of a covered call strategy

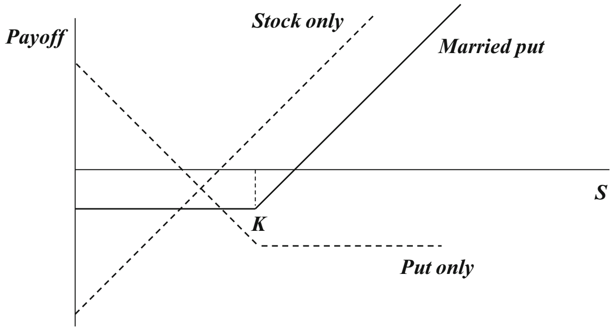

Fig. 9.6 Payoff diagram of a married put strategy

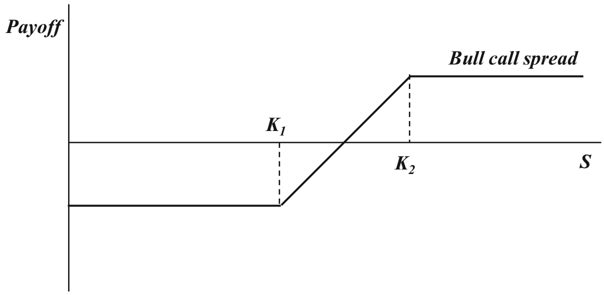

Fig. 9.7 Payoff diagram of a bull call spread strategy

month and underlying asset but with a higher strike price. A bullish investor will use such a strategy when a moderate rise in the price of the underlying asset is expected in the short term (see Fig. 9.7).

Another type of vertical spread is the bear put spread, when an investor simultaneously purchases put options at a specific strike price and sells the same number of puts at a lower strike price. As for the bull call spread, the put options share the same

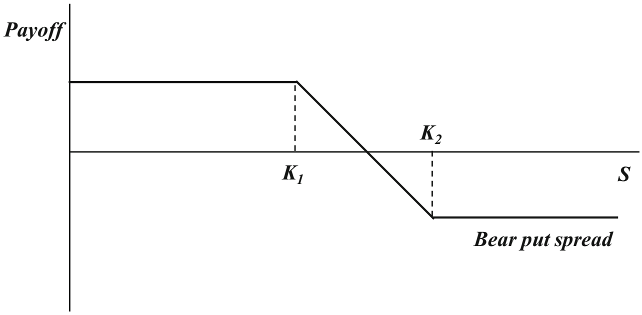

Fig. 9.8 Payoff diagram of a bear put spread strategy

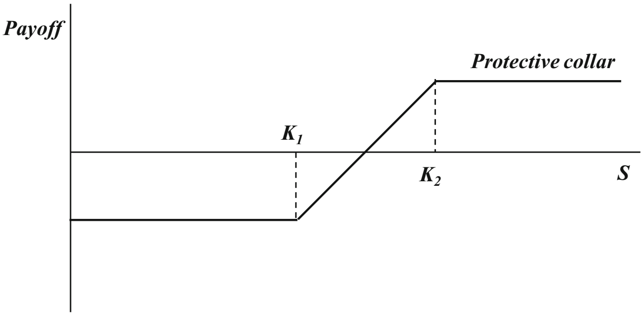

Fig. 9.9 Payoff diagram of a protective collar strategy

underlying asset and have the same expiration date. If the trader is bearish, the strategy is used to hedge from an expected decline in the underlying asset's price (see Fig. 9.8).

A protective collar combination is obtained by getting a long position in an out-of-the-money put option and a simultaneous short position in an out-of-the-money call option. Both options have the same underlying asset.

Such a strategy is often used by investors after a long position in a stock has experienced substantial gains. In this way, investors can lock in profit without selling their shares (see Fig. 9.9).

A long straddle strategy happens when an investor buys a call and a put option at the same time. The options must have the same strike price, underlying asset and expiration date.

The resulting strategy covers the investor in case the belief is that the price of the underlying will move significantly, but there is no certainty about which direction the move will take. This strategy allows the investor to maintain unlimited gains, while the loss is limited to the cost of both options contracts (see Fig. 9.10).

The investor in a long strangle option strategy takes a long position in both a call option and a put option with the same maturity and underlying asset, but with different strike prices.

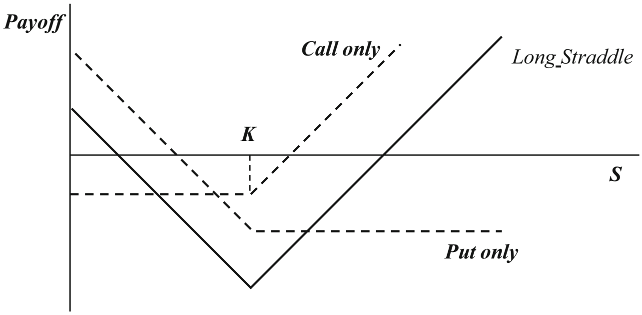

Fig. 9.10 Payoff diagram of a long straddle strategy

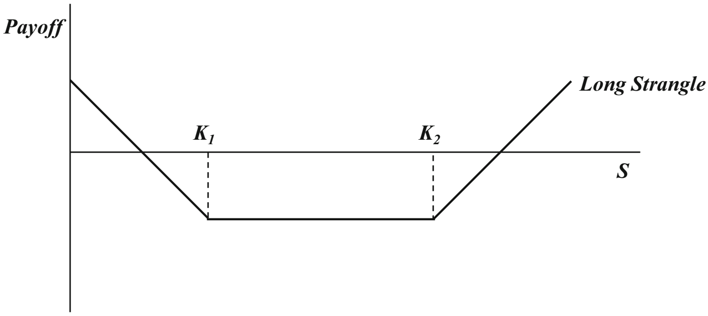

Fig. 9.11 Payoff diagram of a long strangle strategy

Normally, the put strike price is below the call strike price, with both options being out of the money. Investors opting for this strategy believe the underlying asset's price will experience a large movement but is unsure of which direction the move will take. Losses are limited to the costs of both options; strangles will typically be less expensive than straddles because the options are purchased out of the money (see Fig. 9.11).

A butterfly spread is a strategy that involves both a bull spread strategy and a bear spread strategy. So it is basically a combination of strategies involving more than two options.

For example, one type of butterfly spread involves purchasing one call (put) option at the lowest (highest) strike price, while selling two call (put) options at a higher (lower) strike price, and then one last call (put) option at an even higher (lower) strike price (see Fig. 9.12).

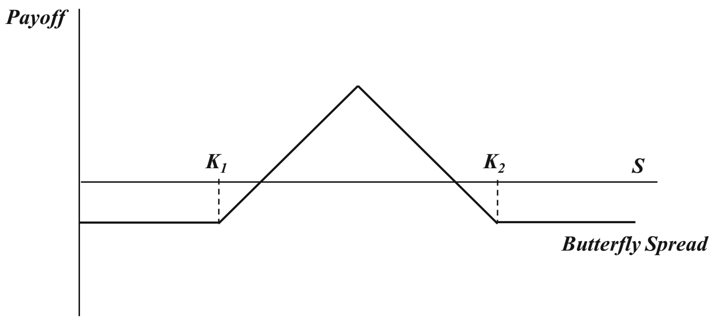

Fig. 9.12 Payoff diagram of a butterfly spread strategy

9.2 Option Valuation

Learning Outcomes

- Learn how to use binomial trees for option pricing.
- Understand the rationale behind the Black-Scholes-Merton model for option pricing.
- Perform pricing of financial options and compute the market price of risk.

9.2.1 Binomial Trees

A popular model for option pricing, based on a numerical approach, was proposed by Cox, Ross, and Rubinstein in 1979. The binomial tree model assumes that stock price movements are composed of a large number of small binomial movements (Cox et al. 1979).

The resulting model translates in discrete steps the features of continuous time models for option pricing, in a simplified structure as in Fig. 9.13.

To show the model, assume a stock has initial price S at time t with the possibility to move up to $S_h = Sh$ or down to $S_l = Sl$, depending on the value of a multiplying factor taking possible values $h > 1$ or $l = \frac{1}{h} < 1$, respectively, at the future time $T = t + \Delta t$.

Denote with p the probability of the price going up. The expected price at time $t = 1$ as from the tree is therefore given by a weighted average expressed as

$$E(S_T) = pSh + (1 - p)Sl$$

Such an expected value can then be compared with the one extracted from the risk-neutral valuation algorithm

$$E(S_T) = Se^{r\Delta t}$$

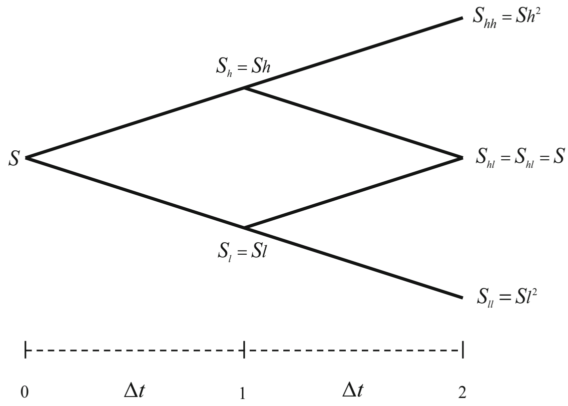

Fig. 9.13 An example of binomial tree for option valuation. At each step, the price of the underlying asset can go up by a factor h or down by a factor l

meaning that the expected price in the future is given by compounding the price today by a factor influenced by the risk-free rate and the time horizon of measurement. Calculating the expected price from the stock at $t = 1$ and making use of risk-neutral valuation,

$$\begin{aligned}
&E(S_T) = pSh + (1-p)Sl = Se^{r\Delta t} \\
&\Rightarrow S_0 = e^{-r\Delta t}E(S_t) \\
&\qquad = e^{-r\Delta t}E(Sh + Sl) \\
&\qquad = e^{-r\Delta t}(pSh + (1-p)Sl) \\
&\Rightarrow e^{r\Delta t} = (ph + (1-p)l) \\
&\qquad = p(1-l) + l \\
&\Rightarrow p = \frac{e^{r\Delta t} - l}{h - l}
\end{aligned}$$

The resulting value of p is called risk-neutral probability and identifies the probability of an upper movement of the underlying asset, as derived from the assumption of risk neutrality of the investors (Capinski and Zastawniak 2003).

A similar argument gives the formulation of u and d in terms of the volatility. Consider the variance of the stock price as from the tree, as difference between the expectation of the square and the square of expected return.

In the case of the binomial tree, the return can be expressed as the probability-weighted average of u and d. Therefore, the variance has the form

$$ph^2 + (1-p)l^2 - [ph + (1-p)l]^2 \tag{9.1}$$

It is then possible to equate this variance with its expression $\sigma^2\Delta t$:

$$\begin{aligned}
\sigma^2\Delta t &= ph^2 + (1-p)l^2 - [ph + (1-p)l]^2 \\
&= \ldots \\
&= p(1-p)(h-l)^2
\end{aligned}$$

After some mathematical steps, it gives as a result

$$
\begin{aligned}
h &= \frac{\sigma^2 \Delta t + 1 + e^{2r\Delta t}}{e^{r\Delta t}} - \frac{1}{h} \\
&= \frac{\sigma^2 \Delta t}{e^{r\Delta t}} + \frac{1}{e^{r\Delta t}} + e^{r\Delta t} - \frac{1}{h}
\end{aligned}
$$

The next step is to approximate the continuous compounding factor to a linear simple factor, as defined by

$$e^{r\Delta t} \approx (1 + r\Delta t)$$

It follows that the following approximations hold

$$
\begin{aligned}
&\frac{1}{e^{r\Delta t}} + e^{r\Delta t} \approx 2 \\
&\frac{\sigma^2 \Delta t}{e^{r\Delta t}} \approx \sigma^2 \Delta t
\end{aligned}
$$

Therefore

$$h^2 = \left(\sigma^2 \Delta t + 2\right) - 1$$

The solution to the second-order equation gives the root

$$h \approx 1 \pm \sigma\sqrt{\Delta t} \approx e^{\sigma\sqrt{\Delta t}}$$

By repeating the process for the case of l and d, the resulting formulas are

$$h \approx e^{\sigma\sqrt{\Delta t}}, l \approx e^{-\sigma\sqrt{\Delta t}}$$

This allows to price an option on the binomial trees knowing only the volatility of the stock, the risk-free rate, and the time interval considered (see Fig. 9.14).

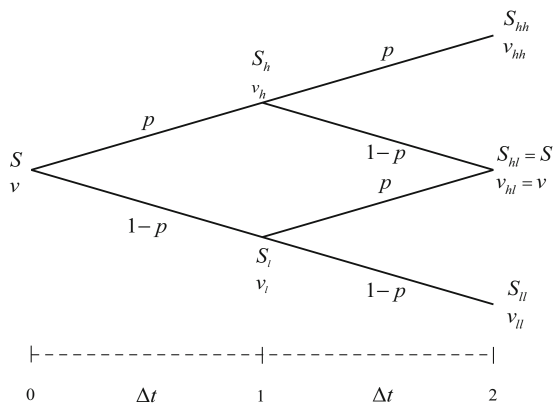

Fig. 9.14 The same binomial tree can be used to also show the corresponding values of the option, at each node, given the risk-neutral probability p of an upper movement

The resulting risk-neutral probabilities are assigned to each up and down branch, for up and down movements of the asset. It is then possible to price the options on the binomial tree, according to the assumption of risk neutrality.

In order to get the price of a claim on a binomial tree, one must work backward on the tree starting from the end of it, corresponding to the maturity. The payoffs of the options can be calculated, corresponding to the stock values at each terminal node.

Each couple of nodes gives the price of the claim on the common node to which they are connected, which is observed by making one step backward. The price is obtained by applying the relevant risk-neutral probabilities at each node.

To illustrate binomial tree, assume a simplified model in one time step, ranging from time $t = 0$ to time $t = 1$. According to the value of the stock moving either up to S_h or down to S_l at time $t = 1$, the value of the option associated to those values moves to v_u or v_d, respectively. Given the probability of an upper movement, the value of the option at time $t = 0$ is given by

$$v = e^{-r\Delta t}[pv_h + (1 - p)v_l]$$

The simple model can then be extended in order to comprise two time steps. The complication is subtle but can be overcome by seeing each node at time 1 as the starting node of another one step tree, terminating at time 2. Therefore, the pricing of the option at time 1 is

$$v_h = e^{-r\Delta t}[pv_{hh} + (1 - p)v_{hl}] \tag{9.1}$$

and

$$v_l = e^{-r\Delta t}[pv_{hl} + (1 - p)v_{ll}] \tag{9.2}$$

After that, working backward the price of the option at time zero is given again by 4.tt. An alternative is to substitute for (9.1) and (9.2) in (9.3) and do some math to get

$$v = e^{-rT}\left[p^2 v_{hh} + 2p(1 - p)v_{hl} + (1 - p)^2 v_{ll}\right] \tag{9.3}$$

Such an equation resembles in one mathematical step the separate formulas for a two-step valuation, notably simplifying the math. The price of the option is directly obtained as a function of the expected payoffs at time 2, without going through the intermediate time step.

Recall the payoffs at time 2 for the call and put options at each node are given by

$$c = \max(S_T - K, 0)$$
$$p = \max(K - S_T, 0)$$

Example 9.2 Consider 3-month call option with a strike $K = 21$ on a stock priced €20 today and volatility 19.06%. Assume the current continuously compounded interest rate r (at 3 month maturity) is 12%. The high and low factors can be computed as

$$h \approx e^{0.1906\sqrt{0.25}} \approx 1.1, l \approx \frac{1}{h} \approx e^{-\sigma\sqrt{\Delta t}} \approx e^{-0.1906\sqrt{0.25}} \approx 0.9$$

It follows that the risk-neutral probabilities are

$$p = \frac{\frac{20}{e^{-0.12\times0.25}} - 18}{22 - 18} = 0.65, 1 - p = 0.35$$

We can then calculate the value of the call option as

$$c = e^{-0.12\times0.25}(1 \times 0.65 + 0 \times 0.35) = 0.63$$

9.2.2 Black-Scholes-Merton Model

The fair pricing of financial derivatives is one of the core issues in modern financial economics. In the 1970s Fisher Black, Myron Scholes, and Robert Merton developed a model (BSM) for option pricing, yielding them a Nobel Prize.

It is the most popular options pricing model in the world, and it is used to measure the theoretical price of European put and call options. It was originally developed without taking into consideration the effects of dividends paid during the life of the option.

Anyway, once derived through strict assumptions, the model, can be adapted to account for the dividends paid on the underlying stock, whose ex-dividend value must at first be determined.

In order to analyze the price of a claim, using any model, it is necessary to describe the behavior of the underlying stock, which can assume any positive value. The BSM model applies this concept through extremely complex mathematics, in order to derive pricing formulas for options written on an underlying following a random process (Black and Scholes 1973).

There are several assumptions simplifying the reality so to make it possible to develop the model. Most of these are just needed for model derivation, but they do not limit the efficiency of the pricing kernel that can be adapted to frameworks where some of the assumptions are relaxed.

The following list resembles the main assumptions:

- During a stock's life, no dividend is paid ever.
- All options considered are European.
- Markets are efficient, normal, and arbitrage-free.
- Transaction costs and commissions are absent.
- The risk-free rate is the reference rate for lending and borrowing.
- Prices are normally distributed, and returns are log-normally distributed.

The behavior of underlying asset prices follows a geometric Brownian motion, with a variance rate proportional to the square root of the price. This is stated formally as

$$\frac{dS}{S} = \mu dt + \sigma dW \tag{9.4}$$

where:
S_t is the price of the underlying stock.
μ is the expected return on S.
σ is the standard deviation of the stock returns.
W is Wiener process (Brownian motion).

Consider a derivative v written on an underlying stock S. A popular tool in mathematics is the Itô's Lemma, which allows to derive the process of a claim from the process of the underlying stock. In this case, if the stock process is in the form of a geometric Brownian motion, the process of the claim is given by

$$dv = \left(\frac{\partial v}{\partial S}\mu S + \frac{\partial v}{\partial t} + \frac{1}{2}\frac{\partial^2 v}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial v}{\partial S}\sigma S dW \tag{9.5}$$

The first step for the derivation of the BSM equations is to form a portfolio of one share of stock and an amount $\delta = \frac{\partial v}{\partial S}$ of the option. The value of such a portfolio is

$$V = S - \frac{v}{\delta}$$

It is then possible to express the potential change in value of such a portfolio as

$$\Delta V = \Delta S - \frac{\Delta v}{\delta} \tag{9.6}$$

Plugging (9.4) and (9.5) into (9.6), the change in value of the portfolio can be expressed as

$$\begin{aligned}
\Delta V &= \mu S \Delta t + \sigma S \Delta W - \left[\left(\frac{\partial v}{\partial S}\mu S + \frac{\partial v}{\partial t} + \frac{1}{2}\frac{\partial^2 v}{\partial S^2}\sigma^2 S^2\right)\frac{\Delta t}{\delta} + \frac{\partial v}{\partial S}\frac{\sigma}{\delta}S\Delta W\right] \\
&= \ldots \\
&= \left(-\frac{\partial v}{\partial t} - \frac{1}{2}\frac{\partial^2 v}{\partial S^2}\sigma^2 S^2\right)\frac{\Delta t}{\delta}
\end{aligned}$$

The condition on the portfolio being riskless, combined with the need for the market to be free of arbitrage, implies that the portfolio must instantaneously earn the risk-free rate r, and the change in portfolio value in the time interval is equivalent to the current value of the portfolio multiplied by the risk-free rate and the time interval itself.

$$\left(-\frac{\partial v}{\partial t} - \frac{1}{2}\frac{\partial^2 v}{\partial S^2}\sigma^2 S^2\right)\frac{\Delta t}{\delta} = r\left(S - \frac{v}{\delta}\right)\Delta t$$

Multiplying both sides by $\frac{\delta}{\Delta t}$ and rearranging terms yields the BSM PDE as

$$\frac{\partial v}{\partial t}+\frac{\partial v}{\partial S}rS+\frac{1}{2}\frac{\partial^2 v}{\partial S^2}\sigma^2S^2=rv$$

The solution of the BSM PDE is a complicated task (Baxter and Rennie 2006). What is interesting for our purposes is to analyze the specific solution for the call and put options.

The generic solution for the boundary of an option is

$$c=e^{-r\Delta t}\widehat{E}[\max(S_T-K,0)]$$

where:
$\widehat{E}$ denotes expectation under risk-neutral measure.
S_T is the price of the stock at maturity of the option T.
K is the strike price of the option.
Recall the process for the stock is

$$dS=\mu Sdt+\sigma Sdz$$

Also recall that S is lognormal by input of the mean, and the variance of the stock into the general solution yields the specific solution for the call option written on a stock S, as

$$\begin{aligned}c&=e^{-r\Delta t}\widehat{E}[\max(S_T-K,0)]\\&=e^{-r\Delta t}\left[Se^{r\Delta t}N(d_1)-KN(d_2)\right]\\&=SN(d_1)-Ke^{-r\Delta t}N(d_2)\end{aligned}$$

where

$$\begin{aligned}d_1&=\frac{\ln\left[\frac{E(S_T)}{K}\right]+\frac{\sigma^2+\Delta t}{2}}{\sigma\sqrt{\Delta t}}\\&=\frac{\ln\left[\frac{S}{K}\right]+\left(r+\frac{\sigma^2}{2}\right)\Delta t}{\sigma\sqrt{\Delta t}}\\d_2&=\frac{\ln\left[\frac{E(S_T)}{K}\right]-\frac{\sigma^2+\Delta t}{2}}{\sigma\sqrt{\Delta t}}\\&=\frac{\ln\left[\frac{S}{K}\right]+\left(r-\frac{\sigma^2}{2}\right)\Delta t}{\sigma\sqrt{\Delta t}}=d_1-\sigma\sqrt{\Delta t}\end{aligned}$$

and $N(.)$ is the cumulative standardized normal distribution defined as

$$N(x)=\int\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx$$

The same reasoning leads to the particular solution for the price of a put option

$$p = Ke^{-r\Delta t}N(-d_2) - SN(-d_1)$$

Example 9.3 Assume valuating a call option on a stock with price €99, strike price €95, time to expiration 1 year, risk-free rate 5%, and volatility 25%. These values make it possible to calculate the Black-Scholes theoretical option value, and the first task is to calculate values for d_1 and d_2

$$d_1 = \frac{\ln\left(\frac{99}{95}\right) + \left(0.05 + \frac{0.25^2}{2}\right)1}{0.25\sqrt{1}} = 0.49$$

$$d_2 = 0.49 - 0.25 = 0.24$$

$$N(0.49) = 0.6879, N(0.24) = 0.5948$$

It is then possible to calculate the price of the option

$$c = 99(0.6879) - 95e^{-(0.05)(1)}(0.5948) = €14.35$$

9.2.3 The Greeks

Looking at the components of the PDE, it can be observed that it is made of a linear combination of sensitivities (ratios of derivatives) of the option price to the various factors affecting it.

Some of these sensitivities, called Greeks, are directly observable in the PDE, while others can be derived. Recall the PDE comes in the form

$$\frac{\partial v}{\partial t} + \frac{\partial v}{\partial S}rS + \frac{1}{2}\frac{\partial^2 v}{\partial S^2}\sigma^2 S^2 = rv$$
$$\rightarrow \Theta + \delta rS + \frac{1}{2}\Gamma\sigma^2 S^2 = rv$$

It is directly observable what is the sensitivity of the option price to changes in the price of the underlying asset, in first (delta) and second (gamma) order, as well as the sensitivity to the time to maturity (theta).

It is then also possible to calculate other sensitivities, not explicated in the PDE, namely, the sensitivity to the volatility of the underlying asset (vega) and in the level of the interest rate (rho) (Shreve 2005).

These are the most important Greeks, but many more can be defined just considering the sensitivity to each parameter at higher orders. The equation for delta δ of a call option in the BSM framework can be derived by differentiating the equation for the call price with respect to the price S of the underlying asset, as

$$\delta_c = N(d_1) \tag{9.7}$$

and recalling

$$c = SN(d_1) - Ke^{-r\Delta t}N(d_2) \tag{9.8}$$

where

$$d_1 = \frac{\ln\left[\frac{S}{K}\right] + \left(r + \frac{\sigma^2}{2}\right)\Delta t}{\sigma\sqrt{\Delta t}}$$
$$d_2 = d_1 - \sigma\sqrt{\Delta t}$$

As a first step, chain rule is applied in order to express the first-order derivatives of d_1 and d_2 with respect to S as

$$\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S} = \frac{1}{S\sigma\sqrt{\Delta t}}$$

The first-order derivative of (9.7) with respect to S is then given by

$$\delta_c = \frac{\partial c}{\partial S} = N(d_1) + \frac{1}{S\sigma\sqrt{\Delta t}}\left[SN'(d_1) - e^{-r\Delta t}KN'(d_2)\right] \tag{9.9}$$

where:
$N'(.)$ is the standard normal probability density.
The solution comes from the fact that the two terms in square brackets on the right-hand side of (9.9) are equal; therefore they cancel out, as defined by

$$\begin{aligned}
e^{-r\Delta t}KN'(d_2) &= e^{-r\Delta t}KN'\left(d_1 - \sigma\sqrt{\Delta t}\right)\\
&= e^{-r\Delta t}K\frac{1}{\sqrt{2\pi}}e^{\frac{-\left(d_1-\sigma\sqrt{\Delta t}\right)^2}{2}}\\
&= e^{-r\Delta t}K\frac{1}{\sqrt{2\pi}}e^{\frac{-d_1^2}{2}}e^{\frac{\left(2d_1\sigma\sqrt{\Delta t}-\sigma^2\Delta t\right)}{2}}\\
&= e^{-r\Delta t}KN'(d_1)e^{\frac{\left\{2\ln\left[\frac{S}{K}\right]+2r\Delta t+\sigma^2\Delta t-\sigma^2\Delta t\right\}}{2}}\\
&= e^{-r\Delta t}KN'(d_1)\times\frac{S}{K}e^{r\Delta t} = SN'(d_1)
\end{aligned}$$

The same procedure applied to the price of a European put option yields the delta of the latter, δ_p, as

$$\delta_p = -N(-d_1)$$

The application of the chain rule, as for the delta, combined with the above results gives the gamma of a call option as defined by

$$\Gamma_c = \frac{\partial \delta_c}{\partial S} = \frac{\partial \delta_c}{\partial d_1}\frac{\partial d_1}{\partial S} = \frac{N'(d_1)}{S\sigma\sqrt{\Delta t}}$$

As for the delta, it is then possible to apply the same reasoning to a put option, which results in being the same as for the call option (Avellaneda and Laurence 2000).

As mentioned above, the sensitivity of the option price to the variation of the time to maturity is called theta. Using a similar argument as for the above chain rules, the theta is given by

$$\begin{aligned}
\Theta_c &= \frac{\partial c}{\partial t} \\
&= e^{-r\Delta t} K N'(d_1) \frac{\partial d_1}{\partial t} - rKe^{-r\Delta t} N(d_2) \\
&\quad - e^{-r\Delta t} K N'(d_2) \frac{\partial d_2}{\partial t} \\
&= e^{-r\Delta t} K N'(d_2) \frac{\partial d_2}{\partial t} - \frac{\sigma e^{-r\Delta t} K N'(d_2)}{2\sqrt{\Delta t}} \\
&\quad - rKe^{-r\Delta t} N(d_2) - e^{-r\Delta t} K N'(d_2) \frac{\partial d_2}{\partial t} \\
&= -Ke^{-r\Delta t} \left[rN(d_2) + \frac{\sigma N'(d_2)}{2\sqrt{\Delta t}} \right]
\end{aligned}$$

For a put option, theta can be defined as

$$\Theta_p = \frac{\partial p}{\partial t} = Ke^{-r\Delta t} \left[rN(-d_2) - \frac{\sigma N'(d_2)}{2\sqrt{\Delta t}} \right]$$

The rho of a call option is defined as

$$\begin{aligned}
Rho_c &= \frac{\partial c}{\partial r} \\
&= SN'(d_1) \frac{\partial d_1}{\partial r} + Ke^{-r\Delta t} N(d_2) \Delta t - e^{-r\Delta t} K N'(d_2) \frac{\partial d_2}{\partial r} \\
&= Ke^{-r\Delta t} N(d_2) \Delta t + \frac{\partial d_1}{\partial r} \left[SN'(d_1) - e^{-r\Delta t} K N'(d_2) \right] \\
&= Ke^{-r\Delta t} N(d_2) \Delta t
\end{aligned}$$

The rho of a put option is

$$Rho_c = -Ke^{-r\Delta t} N(-d_2) \Delta t$$

The last sensitivity of interest is the one to the volatility of the underlying asset, vega. By observing that through chain rule, it results that

$$\frac{\partial d_1}{\partial \sigma} = \frac{\partial d_2}{\partial \sigma} + \sqrt{\Delta t}$$

The vega of a call option is defined as

$$\mathcal{V}_c = SN'(-d_1)\sqrt{\Delta t}$$

The vega of a put option is

$$\mathcal{V}_p = Ke^{-r\Delta t}N'(-d_1)\sqrt{\Delta t}$$

Greeks can be considered the risk sensitivities, risk measures, or hedge parameters and are vital tools in risk management. They allow to treat component risks in isolation and rebalance the portfolio accordingly to achieve a desired exposure.

9.3 Real Options

Learning Outcomes

- Understand the various types of real options.
- Learn about the most common valuation methods.
- Calculate the value of a real option on an investment.

9.3.1 Types of Real Options

The traditional approaches that discount cash flows, like the NPV rule, do not allow for management flexibility and revision of decisions in response of unexpected changes in the environment. They just assume an expected scenario of cash flows and consider managers to be passively committed to a certain static operating strategy.

In the real world, a business must face uncertainty, change, and competitive interactions, with new information hitting the market every day. Once the uncertainty about the market conditions is resolved by the incoming information, the management may want to adapt their strategy to the new state of the world. This managerial operating flexibility is like financial options and is known as strategic options or real options.

Real options are a useful method to value investment decisions, and their use has been evolving since the 1980s. It involves using the methodology of financial options to value investment opportunities.

Traditional methods like NPV and decision tree analysis (DTA) lack flexibility and do not allow to adapt the investment valuation to sudden changes in the overall conditions.

Corporate management is now represented with the option to act on it if initial assumptions about the project change or more information about the development of the project is available later on.

There can be changes in several factors like commodity prices, property rights, and new products, and these changes drive the management to take actions like expanding, deferring, or abandoning the current project, due to changes in the conditions.

All of the abovementioned options add value to a project and can be assimilated to equivalent financial options written on some underlying asset. Instead of the underlying asset in the option being a stock, the underlying of a real option is an investment.

Sometimes the NPV analysis may give a negative outcome on some project, so as to make it unappealing and rejected. Real options can add up on the project in order to make the NPV positive.

Proper choice of an embedded real option can change the NPV of the project from negative to positive and then justify that a project should be undertaken then confirm that the NPV is likely to undervalue the investment itself.

The traditional NPV and DTA methods fail to assess the risk and uncertainty associated to new investment opportunities. Real options can capture these factors, and the management can build up the corporate portfolio of projects at the desired risk level.

Once real options are chosen for valuation, the firm can go for a single real option on a project or build up a portfolio of projects with different underlying asset or projects with different exercise prices.

Real options are strictly connected to financial options, especially in terms of valuation. The right of exercising a real option may raise from several factors, like a patent, ownership of some asset.

More generally, the real option rights arise from the resources which are available inside a company, such as technological knowledge, the position the company has on the market, or the scale of the company.

As pointed out by Dixit and Pindyk (1994), the availability of options on some investments gives power to a corporation, adding competitive advantage over other companies in the business, through the possibility to add flexibility to their investment decisions.

Table 9.1 Differences between financial and real options

Feature	Financial options	Real options
Option price	Decided by the market	Not fixed, paid to change the nature of an investment
Exercise price	Defined in the option contract	Paid to trade the underlying real asset
Expiration time	Defined in the contract	Known in some cases, not so in others
Control on the option value	None	Defined by the quality of management
Value as a function of option life	Larger for longer maturity of the option	Competition may revert the time effect
Value as a function of volatility	Increases	Increases

Also the rationale of exercising a real option on some investment is similar to that of a financial option. The option should be exercised when such an action gives a favorable income to the option holder.

Projects can then become valuable in case, for example, an extension option is exercised. Moreover, unfruitful projects can be abandoned so to limit the losses and gain a salvage value on the abandoned investment.

Real options can be seen as a call or put option on an underlying asset or project. The connection between important parameters in a financial call option and a real option is listed in Table 9.1.

Besides the many common aspects between financial options and real options, there are also some differences between them. Financial options can be always exercised by the owner, depending on the convenience of doing so.

With real options the underlying asset is a project or an outcome of a market condition, which causes a competitor in the same industry to be able to affect the option by exercising a similar option before you do.

Competitors can have an active role in making the option worthless, thus affecting its outcome as well. Good timing is crucial with real options, and management can exclude the influence of the competition which holds a similar option, by exercising at the right time.

The issue with real options is that, as opposed to financial options, the value of the underlying asset is not so straightforward to determine. Consider the option to expand which can only be applicable for a certain company or it can also be a compound option that needs to be sold as a package.

A real option can however be sold as well between companies in the same industry. The other company may benefit from buying a real option, for example, a patent or a development of a product.

The use of a real option for investment analysis is particularly suitable in some specific situations. First of all, real option analysis is useful when there is a contingent investment decision.

Another case is when there is a large uncertainty about the profitability of an investment, and it is wise to wait for more information, in order to avoid the binding of an irreversible investment.

Real options are good also when the potential of the project is more concentrated in its future potential rather than the actual cash flows. They should also be used when the uncertainty calls for a high degree of flexibility.

Finally, there might be the case when there will be project updates and midcourse strategy corrections. Both authors also categorize strategic investments and put them into the perspective of real options.

The strategic investment of a company can be classified into six main types:

- Irreversible investments are those that cannot be reversed after they are put in place, unless accepting a high loss on their value. The real option attached to such an investment drastically cuts the loss, being therefore very valuable.
- Flexibility investments add up flexibility through real options, in a way that is not implementable through traditional methodologies.

- The insurance investments reduce the generic exposure to risk.
- Modular investments associate modules to options, in order to have the possibility of interchanging the modules. Therefore, a modular product can be viewed as a portfolio of options to upgrade.
- Platform investment is such that create potential future developments, as in the case of research and development. The value of the investment relies in the innovations released, and it is often undervalued by standard tools.
- Learning investments are meant to obtain information that would be not available otherwise and are mostly represented by the explorations prior to work undertaking.

The peculiar nature of the above investments defines the types of option that are involved in the investment valuation. The Trigeorgis groups real options into the following classes:

- The option to defer allows to change the time point of an investment, which is not determined and can be optimized later. It also includes the cases where the time point of an investment is uncertain but has to be optimized. Those options can also arise from changes in the term structure of interest rates over time even if the future cash flow is deterministic.
- A variation of the option to defer is the staging option, giving the right to phase the project into several stages, with progressive amounts invested according to the new information generated by the project at each stage. Unlike the option to defer, this strategy requires at least a minimal initial investment. A time-to-build option allows to interrupt an investment at a certain step, given that the investment is progressive, on a step-by-step basis. This is usually the case when the market turns unfavorable.
- The option to exit is the only way to avoid a negative NPV attached to a project, in that it gives the right to abandon the project at any time, and obtain a liquidation amount on top of that. The option to abandon a project allows the investor to sell the project before it is completed. The amount earned by the sale is called salvage value; it is then included in the project valuation, with an impact on the NPV.
- The operating option allows investors to organize their operations to benefit from changes that would occur in business conditions. There is also the option to alter the operating scale of an investment, which allows to expand the investment in case of favorable conditions, and to compress it in a bad environment. It is the case of the introduction of a new product of the market, where the response of the market determines the future of the investment.
- Growth option gives the chance to access additional projects on top of some investment that may result in negative NPV. This is especially the case of investments in research that initially give no benefit, but they rather provide positive NPV once additional investments are made, with positive and large enough NPV to cover the loss generated by the original, indispensable, project. Growth options are relevant for projects that have potential to generate profits in the future. Therefore, they are used in R&D sector mostly, where the journey

from the idea to the product may show other important applications to invest in, leading to positive NPV for the whole investment.

The aim of this section is do deepen the analysis of some of the most important of the above types of options. It is, for example, very important to correctly asses the option to delay (defer).

When an investment has initial negative NPV, it is very important to understand whether delaying the project will make the investment profitable so as to give additional value to the business.

In his work, Damodaran (2002) shows that a delaying option can be compared to a call option value where the underlying asset is the value of the project to be realized, calculated as the discounted value of the projected cash flows.

The exercise price corresponds to the cost of the project, assumed to stay constant overtime in real terms, and not subject to uncertainty (evolution is deterministic and not stochastic).

The time to maturity is determined as the length of benefiting period for the user of the project, or the amount of time the project can be used by the investor exclusively, before competitors reply to it.

The risk-free rate is represented by the expected return on a risk-free asset with the same maturity as the project. Good proxies can be the usual government securities or benchmark rates like LIBOR.

The volatility of the underlying asset can be determined by observing the volatility for the cash flows of similar projects or by looking at hard data to calculate the volatility of the financial cash flows in the same industry. On top of that, Monte Carlo simulation can also be used. It is also possible to compare the loss of financial flows for each year of delay, to a dividend.

All the above elements are the typical inputs for analytical option pricing models like the Black-Scholes-Merton model. This is the model that is also used to price real options, based on the identification of the parameters as shown above.

By replacing in the model the elements that characterize the project (the difficult stage is to identify and estimate these elements), the model yields the value of the option attached to the project, making it possible to recalculate the NPV.

The binomial model, based on the Cox-Ross-Rubinstein approach can also be used, but it involves more calculations because a value for the option must be calculated for each node of the tree.

Moreover, it is a discrete time model, therefore allowing to consider a finite number of values for the underlying asset. The call option value determined by the Black-Scholes model is always smaller but close to the value determined using the binomial model (using Black-Scholes there is no risk of overvaluation).

The exit (abandonment) option is assimilated to a put option, where the value of the underlying asset is represented by the current value of all financial flows expected to be received by the project, and the volatility is estimated by the same categories of methods.

The period of time during which the exit decision can be made represents the maturity of the associated option. The strike price is given by the liquidation value of the project, as obtained when liquidating the assets upon exit.

The evaluation model is applied to the situation that this value remains unchanged over time (real assets don't lose their value). In reality, the investment is eroded over time (both physically and morally), and the loss of value may be surprised by the dividend rate (Luehrman 1998).

The current value at the time of exiting the project can be lower than the value estimated. Reasons for that can be the lack of demand for that type of product, a market not adequate to meet the demand and supply, or sudden non-forecasted losses.

Sometimes there are charges for abandoning a project, plus nothing can be earned on the assets, so that the liquidation value becomes negative. In this case, the project will be abandoned only if the losses generated by continuing the operations are larger than the costs of liquidation proceedings.

The value of the exit option increases when the structure of the project is made so flexible that abandonment is not complicated, so to allow greater ease adoption of the liquidation decision. Examples can be the adoption of flexible factors like temporary work force, short rental for equipment, and other factors.

An expansion option is evaluated at the beginning of the project, when the first investment is made. It is implicit in the option evaluation in fact that such an option can only exist if the investor made the first step to start the project at least.

The items to be introduced in the evaluation model are similar to the ones of the option to delay. Characteristic expansion option depends on two projects (and not one), so entries must be defined accordingly.

The underlying asset in this type of option corresponds to the expansion project, whose value is given by discounting the potential cash flows of the extended project, if extended today.

The cost for project expansion represent the exercise price of the expansion option, and the time to maturity is not externally determined, like for the deferral option, but it depends on the internal constraints set by the investor.

The risk-free rate in this case is also represented by the rate of return expected on an investment without risk. The maturity of such an asset must be equal to the time established for the option.

Also the calculation of the volatility follows the standard methodologies as presented for the other options above. The dividend rate in this case is represented by the loss of cash flows until the decision, if expansion is not accomplished as soon as it becomes possible (which is also a delay cost).

There are some downturns in calculating the price of the expansion option, like the fact that both the underlying asset price and the strike price are set from the beginning, even if related to an expansion project that will be done in the future. Moreover, estimating the time to maturity of the option is not an easy task, because it is determined subjectively by the investor.

There are several features that make real options so different from financial options. First of all, real options are owned by more investors. Many competitors could be interested in a particular market or area.

In order to ensure the project is exclusive and not affected by competition, the competitive advantages of the company over the competitors must be identified. This allows to create barriers to the entry of new players in the market.

Real options also differ from financial options because they are not negotiable. The owner of a real option can either exercise the option by then investing in the project, or let the option expire, without making any investment decision. In fact, an organized market for selling these rights through negotiation does not exist.

Also, real options give the investor who made the decision a right of preemptive investment. The company that decides on starting an investment project before its competitors benefits of preemption, while in conditions of no competition, it would have been optimal to defer a decision and wait for an accumulation of new information.

Financial options, even if they can be combined, are commonly traded singularly; the real options are mostly composed. It is very common in fact to have interdependency between several projects, and consequently the real options associated to these projects are often more complex than financial ones, because their exercise may depend on the exercise of other options.

On the other hand, there are also similarities, especially in valuation. It is in fact possible to expand the use of models for financial option pricing, in order to achieve a fair valuation.

It follows that basically the factors influencing the value of a real option are the same as for the financial options. These factors are the underlying asset price, exercise price, time to maturity, risk-free rate, and volatility of the underlying asset.

Real options can be differentiated from financial options according to different parameters. All of them are important in defining the unique nature of real options as instruments useful to be embedded into real business investments.

For example, real options are normally owned by multiple investors, and the possibility to perform some kind of investment or enter a specific market can be always considered by more than one competitor (Luehrman 1995).

In order to eliminate competition and to ensure the exclusivity of a project, a company must identify the competitive advantages it has over competitors in order to form some kind of entry barriers.

Moreover, real options cannot be negotiated, and there is no organized market for trading and exchanging them. The owner of a real option can either exercise it or let it expire but cannot get rid of it by selling to someone else.

As opposed to financial options, real options carry a preemptive right on the investment. The company that decides starting investment project before its competitors benefits of preemption while in other conditions (no competition) would probably have preferred to defer a decision and wait for accumulation of new information.

In reality real options can be more complex than as described. This is due to the fact that investment projects are often interdependent. Therefore, real options are often more complex than financial ones, because their exercise may depend on the exercise of other options.

9.3.2 Valuation Methods

The valuation models for real options are adapted from those for financial options. Most cases of real options have American style exercise at any point in the project's life and are impacted by multiple underlying variables.

Standard model for financial options are normally limited in terms of dimensionality and early exercise. The analyst must take these aspects into consideration when deciding which model to use. The model must also be flexible enough to allow for the relevant decision rule to be coded appropriately at each decision point.

The standard Black-Scholes-Merton framework can be utilized for real options valuation, but only for European styled options or perpetual American options. Similar to the condition of constant volatility for financial options, the application of Black-Scholes-Merton to real options assumes constant costs.

As an example consider a business man negotiating to open a new hotel downtown. The contract with the hotel chain obliges the investor to open the hotel either at current time or in 1 year from the time of contracting (Luenberger 1998).

Opening the hotel involves an initial investment of €7,000,000. The expected cash flow associated with an immediate opening is about €750,000 initially, with growth potential of 3% per year in the future.

Such an investment is considered feasible when the cost of capital is 13%. It follows that the value of the hotel if opened today is given by the value in perpetuity of growing cash flows, as per

$$V = \frac{750,000}{0.13 - 0.03} = €7,500,000$$

The amount of

$$\text{NPV} = 7,500,000 - 7,000,000 = €500,000$$

represents the net present value from opening the hotel at the current time and the minimum value of the contract.

The option to delay embedded in the contract allows also to choose to wait and open the hotel in 1 year from now. In this case the decision is not difficult in 1 year, given that the value of the hotel at that time will be known.

However, there are uncertainties related to the cash flows not earned in the waiting period. Recall that the option to delay an investment must be valued as a European call option.

In this case the option has maturity 1 year and a strike price equal to the initial investment in the hotel, €7,000,000. Assuming a risk-free rate of 3% and an estimated volatility (from observable comparable hotels) of 31%, the value of opening later is equivalent to that of the corresponding financial option.

Such an option can be evaluated using the Black-Scholes-Merton model, by considering the missed cash flow in the first year as a dividend embedded in the value. It is therefore necessary to compute the ex-dividend value as

$$S^* = S - PV(CF) = \frac{750,000}{(0.13 - 0.03)} - \frac{750,000}{(1 + 0.13)} = 7,500,000 - 663,716$$
$$= €6,836,284$$

We compute the present value of cash flow using the project´s cost of capital of 12%. Next, we need to compute the present value of the cost to open the hotel in 1 year:

$$PV(K) = \frac{7,000,000}{1.03} = 6,796,116$$

Now we can compute the value of the call option to open the hotel:

$$d_1 = \frac{\ln\left[\frac{S^*}{PV(K)}\right]}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2} = \frac{\ln\left[\frac{6,836,284}{6,796,116}\right]}{0.31} + \frac{0.31}{2} = 0.1740$$

$$d_2 = d_1 - \sigma\sqrt{t} = 0.1740 - 0.3100 = -0.1360$$

The value of the call option on the hotel is

$$C = S^*N(d_1) - PV(K)N(d_2) = (6,836,284 \times 0.5691) - (6,796,116 \times 0.4459)$$
$$= €860,141$$

The resulting €860,141 exceeds the NPV of €500,000 from opening the hotel today. Thus we are better off waiting to invest, and the value of the contract is €860,141.

In case there are several future growth opportunities; one can think at a series of real call options on the project. Given that generally the call options that are out of the money are riskier than those that are in the money, the growth component of firm value is riskier than the ongoing assets of the firm. Future growth options cannot be valued through the BSM model, given they are often exercised before maturity, therefore being assimilated to American options.

In case of American-style options, it is possible to employ numerical methods like binomial tress. These are more widely used given that most real options are American styled.

Many investment options allow for the possibility of revaluating the decision to invest at a later point in time. Future decisions and uncertainty resolution can be represented on a decision tree.

The decision tree contains two kinds of nodes: decision nodes marked with square boxes (pay and get versus do nothing) and information nodes, in which uncertainty is involved that is out of the control of the decision-maker (rain or sunshine).

Once a binomial tree is designed for a real option, the model allows for great flexibility in terms of the right to exercise. At each node it would be possible to encode different rules.

Example 9.4 A pharmaceutical company is investing in developing a new drug for some type of disease. The drug is at the test and development stage. The first phase will last 1 year at a cost of €200,000, and the chance of tests being successful is 55%. The full-scale production will then cost €550,000 and will occur over 5 years.

The income statement for the 6-year (1+5) full-scale production time looks like that. The calculation shows what is the NPV of the production phase calculated discounting cash flows at the cost of capital of 12% by subtracting the €750,000 investment from the PV.

If success	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Cash flow	−€550,000	+500,000	+500,000	+500,000	+500,000	+500,000

$$\mathrm{NPV}_{PH.2} = -550,000 + \sum_{t=1}^{6} \frac{500,000}{(1.12)^t} = €522,246.10$$

If failure	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Cash flow	−€550,000	+200,000	+200,000	+200,000	+200,000	+200,000

$$\mathrm{NPV}_{PH.2} = -550,000 + \sum_{t=1}^{6} \frac{200,000}{(1.12)^t} = -€121,101.56$$

The company has to make two choices: to test or not to test and to invest or not to invest (see Fig. 9.15).

Moving backward to phase 1, it is possible to calculate the associate expected payoff as

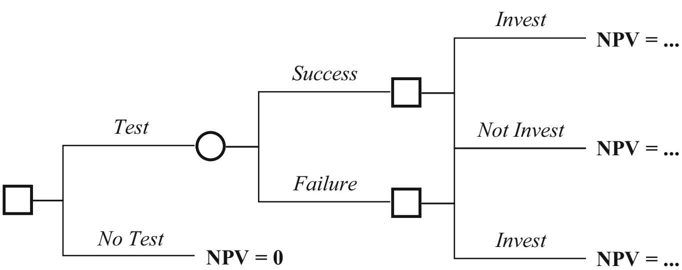

Fig. 9.15 Diagram of the investment with the option to evaluate the opportunity to continue investing, at a later time

$$\begin{matrix}\text{Exp.payoff}\\ \text{at time 1}\end{matrix} = \left[\begin{pmatrix}\text{Prob.of}\\ \text{success}\end{pmatrix} \times \begin{pmatrix}\text{Payoff}\\ \text{given success}\end{pmatrix}\right] + \left[\begin{pmatrix}\text{Prob.of}\\ \text{failure}\end{pmatrix} \times \begin{pmatrix}\text{Payoff}\\ \text{given failure}\end{pmatrix}\right]$$

$$\begin{matrix}\text{Exp.payoff}\\ \text{at time 1}\end{matrix} = (0.55 \times 522,246.10) + (0.45 \times 0) = €287,235.35$$

The NPV evaluated at date 0 is

$$\text{NPV}_{PH.1} = -200,000 + \frac{287,235.35}{1.12} = €56,460.14$$

The NPV is positive; therefore the drug should be tested.

The limit of binomial trees is the dimension of the problem that can be handled. If one allows for stochastic costs, that would add another dimension to the tree, therefore increasing the number of nodes by the square.

Another numerical approach is offered by Monte Carlo methods, given their applicability to high-dimensional problems. For American styled real options, this application is somewhat more complex, although recent research combines a least squares approach with simulation, allowing for the valuation of real options which are both multidimensional and American styled.

The simulation involves the generation of thousands of paths the underlying asset value may take during the option life given the boundaries of the cone of uncertainty as defined by the volatility of the asset value.

At time 0, for every simulation, one must start with the expected underlying asset value S_0. The next step calculates the asset price at next time by using the following equation with a random variable function:

$$S_t = S_{t-1} + S_{t-1}\left(r\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right)$$

where:

S_t is the value of the underlying asset at time t.

S is the value of the underlying asset (investment).

σ is the volatility of the underlying asset (investment).

ε is the simulated value from a standard normal distribution, usually taken with mean zero and a variance of 1.0.

The algorithm must be repeated many times to generate prices until the maturity of the option. The resulting project values are then discounted back to present value at the risk-free rate. One way to obtain a random number from a normal distribution is given by the general Excel command

$$= \text{NORMINV}(\text{RAND}(), \mu, \sigma)$$

which becomes, for a standard normal distribution,

$$= \text{NORMINV}(\text{RAND}(), 0, 1)$$

There is also a way to obtain a good approximation for a standard normal variable is obtained by generating 12 uniform random numbers minus 6.0, through the formula

$$\begin{aligned} &= \text{Rand}() + \text{Rand}() + \text{Rand}() + \text{Rand}() + \text{Rand}() + \text{Rand}() + \text{Rand}() + \text{Rand}() \\ &\quad + \text{Rand}() + \text{Rand}() + \text{Rand}() + \text{Rand}() - 6.0 \end{aligned}$$

After simulating all asset prices, the price of the option at expiration is calculated according to the standard payoff profiles, namely, $Max(0, S_T - K)$ for a call option and $Max(0, K - S_T)$ for a put option where K is the exercise price and ST is the asset price at expiration.

This produces one possible option value at expiration. You then repeat this procedure many thousands of times, take the average value of the call at expiration, and discount that value at the risk-free rate.

Monte Carlo simulation is a good tool for European options, but it becomes cumbersome to use then applied to American options because it requires to simulate all possible exercise dates.

It becomes an even bigger challenge when dealing with sequential options, because each decision leads to a new path. This can involve millions of simulations, which can be an enormous computational task even with today's fast computers.

Partial differential equations (PDEs) can be also used for real option valuation. In this case the finite difference methods for option pricing represent the best approach for valuation.

The use of PDEs is not very common given its high level of mathematical sophistication. One example is given by the finite difference method, as illustrated in the following paragraphs.

In this case it is possible to use the standard no-arbitrage conditions to derive a partial differential equation for the value of the contingent claim. Recall that the Black and Scholes partial differential equation for the value of the real option $R(S, t)$ is

$$\frac{1}{2}\frac{\partial^2 R}{\partial S}S^2\sigma^2 + rS\frac{\partial R}{\partial S} + R_t = rR$$

where:

$R = R(S, t)$ is the value of the real option.

The boundary condition at maturity is

$$R(S, t = t_1) = \max(PV_C - L_1, 0)$$

The finite difference procedure of discretizing all state variables was first proposed by Schwarz in 1977, and it involves setting the boundary conditions for the value of the claim, using a finite difference approximation of the first and second derivative and using a discretized version of the PDE to be solved backward.

The value of the project is a function of two state variables: output price, S, and time to maturity, T. Time is discretized into m intervals and price S into n intervals. The Black and Sholes differential equation is then replaced by the following difference approximations:

$$\Delta S = \frac{S_{\max} - S_{\min}}{n}$$

$$\Delta T = \frac{t_1}{m}$$

$$\frac{\partial R}{\partial S} = \frac{R_{i+1,j+1} - R_{i+1,j-1}}{2\Delta S}$$

$$\frac{\partial^2 R}{\partial S} = \frac{R_{i+1,j+1} - 2R_{i+1,j} + R_{i+1,j-1}}{\Delta S^2}$$

$$R_t = \frac{R_{i+1,j} - R_{i,j}}{\Delta t}$$

where:

$S_{\max}$ is the maximum value of the underlying asset.

$S_{\max}$ is the minimum value of the underlying asset.

Once these approximations are substituted into the differential equation, we obtain, for every node (i,j)**,**

$$a_j R_{i+1,j-1} - b_j R_{i+1,j} + c_j R_{i+1,j+1} = R_{i,j}$$

with

$$a_j = \frac{1}{1 + r\Delta t}\left(-\frac{1}{2} rj\Delta t + \frac{1}{2}\,\sigma^2 j^2 \Delta t\right)$$

$$b_j = \frac{1}{1 + r\Delta t}\left(1 - \sigma^2 j^2 \Delta t\right)$$

$$c_j = \frac{1}{1 + r\Delta t}\left(\frac{1}{2} rj\Delta t + \frac{1}{2}\,\sigma^2 j^2 \Delta t\right)$$

It follows that if one knows the value of the claim at time $i + 1$, it is possible to obtain the values at time i. From the boundary condition giving the value at time $i = M$ is then possible to work backward until $i = 0$.

9.4 Summary

Financial derivatives are widely used by investors to hedge the position on the underlying asset and also by speculator in that they are a very risky type of investment, when used for speculation.

Financial options have a very peculiar payoff profile, and they can be understood by carefully analyzing it. The structure of an option is such that different market conditions may lead to different outcomes.

It is also possible to combine options in order to build investment strategies and combinations. The relative payoff can be represented on more complex graphs, and each combination aims at satisfying specific assumptions about the underlying asset.

Option valuation is an interesting task to accomplish and can be done in several ways. The purpose is to find a theoretically fair value for the option so that it reflects all the inputs used to price it.

Binomial trees model the price of an option graphically, and they correspond to a numerical approach to the valuation issue. By working the trees back from the payoff expected at maturity, it is possible to price the option at each step of the tree.

The Black-Scholes-Merton model on the other hand is an analytical approach, corresponding to the equivalent in continuous time of the binomial trees. By taking several inputs related to the underlying asset and the market, it yields a risk-neutral valuation of the option.

Real options are the equivalent of the application of option valuation to real business investments, rather than financial assets. The underlying is therefore a project or any other type of investment.

The options in this case allow the investor to act on the project so to extend it, close it, etc. The valuation of real options can be done analytically or through numerical approaches.

As for the financial options, a tree approach is possible, and such a method is especially good when valuing real options on commodities. Alternatively, partial differential equations can be used as well.

Problems

1. Consider a stock meant to pay a dividend of €0.10 per share in 3 months and 9 months from now. With no other dividend payments within the next 12 months, and the interest rate is 5% with continuous compounding.
   (a) Calculate the 12-month forward price of the stock if the current market price is 20€ per share.
2. Using the data in exercise 1, assume an investor entering a forward contract to buy 100 shares of the stock in 12 months.
   (a) What is the value of the forward contract today?
   (b) What is its value in 6 months, provided the stock is then traded at 18€ per share?

3. Assume the current stock price is €27, and a 3-month call with a strike price of €28 costs €2.65. You have €6, 500 to invest.
   (a) Describe two alternative speculation strategies, one in the stock and the other in the option on the stock.
   (b) What are the potential gains and losses from each strategy?
4. Prove that the price of a call option satisfies the BSM PDE.
5. Prove that the solutions for the BSM Greeks satisfy the PDE.
6. Companies A and B have been offered the following rates per annum on a 5-year loan of €20,000,000:

	Fixed rate	Floating rate
Company A	12.0%	Libor + 0.1%
Company B	13.4%	Libor + 0.6%

Company A requires a floating-rate loan, while company B requires a fixed-rate loan.

(a) Design a swap contract netting the 0.1% per annum to the intermediary bank, and that will appear equally attractive to both companies.

7. Consider a binomial world in which a stock, over the next year, can go up in value by 20% (by a subjective probability of 55%) or down by 10% (by a subjective probability of 45%). The stock is currently trading at €10. The risk-free return is 5%. Consider a call that expires in 1 year, with a strike price of €11.
   (a) What is the value of the call option?
   (b) If the call option was trading for €0.32, can you find an arbitrage opportunity?
   (c) If the call option was trading for €0.61, can you find an arbitrage opportunity?
8. Consider a European call option, which is written on a stock whose current value is €8, with the strike price €9 and expiration date in one month. Assume for simplicity that one month later, when the option can be exercised, the stock price can either appreciate to €10 or depreciate to €6. Assume that the risk-free interest rate is zero.
   (a) What is the current price of the call option?
9. Suppose that the strike price of an American call option on a non-dividend-paying stock grows at some constant rate g.
   (a) Show that if g is less than the risk-free rate, r, it is never optimal to exercise the call early.
10. You would like to speculate on a rise in the price of a certain stock. The current stock price is €29, and a 3-month call with a strike price of €30 costs €2.90. You have €5800 to invest. Identify two alternative investment strategies, one in the stock and the other in an option on the stock. What are the potential gains and losses from each?

11. Consider a binomial world in which a stock, over the next year, can go up in value by 50% (subjective probability of 60%) or down by 33.33% (subjective probability of 40%). The stock is currently trading at €50. The risk-free return is 5%.
    (a) What is the value of the call option that expires in 1 year with a strike price of €55?
    (b) Calculate the expected return from the stock.
    (c) Is the expected return on the call option higher or lower?
12. For a European put option on a stock with 1 year to expiry, the stock price is 50, the strike is 50, risk-free interest rate is 4%, dividend is 4%, and the price of the option is 6.22. Determine the implied volatility of the stock.
13. Prove that, using this binomial tree, we can find the price of any contingent claim. In other words, any contract paying $h(S_1)$ where h is any function of S_1.
14. Consider options on two different assets (asset 1 and asset 2). Would you prefer to have two different options, one on each asset (with the same strike price K), or would you prefer to own an option on the portfolio of the two assets with a strike of 2K?
15. A company is considering investing in a project. The present value (PV) of future discounted expected cash flows is either €12,000,000 in growth or €3,000,000 in recession next year. The objective probability the market will go up is 45%. The appropriate risk-adjusted cost of capital is 18%. The initial capital investment required at time 0 is €7,500,000. Risk-free rate is 2.5% per year.
    (a) Determine the PV of the project at time 0.
    (b) Determine the NPV of the project at time 0.
    (c) Should the company invest in this project?
    (d) The company can abandon the project and liquidate its original capital investment for 55% of the original value. It can also expand operations, which will result in twice the original PV. To expand the company will have to make an additional capital expenditure of €3,500,000. Determine the real option analysis value of this project with exibility.
16. Consider this multistage project. The first stage requires an initial investment outlay of $I_0 = €17,500,000$ and expected cash inflows over 2 years of

    $C_1 = €3,500,000 \quad C_2 = €6,500,000$

    The second stage will become available in year 4. It will require an additional investment of

    $I_2 = €125,000,000$

    The expected cash inflows over the subsequent 2 years are

    $C_3 = €37,500,000 \quad C_4 = €75,000,000$

    The cost of capital for both stages is 16%. The risk-free rate is 3%. The volatility of the second stage's project value is 48%.
    (a) Explain how real options analysis would characterize this project with exibility using the language of financial options analysis. Be specific with numbers.

(b) Use a binomial lattice to determine the real option analysis value of this project with exibility.

17. The present value of a project without exibility is 50 million. The project pays no dividends. The project without exibility follows a binomial lattice with $u = 1.25$ and $d = 0.8$. Risk-free rate is 3%. The required investment is

 €6,000,000 at time 0
 €18,000,000 at time 1
 €27,500,000 at time 2

 Investments must be made to continue the project for the upcoming year. At any time the company has the option to default on these planned investments at which point the project is terminated.

 (a) Determine the real option analysis value of this project.
 (b) Describe the optimal stopping policy in words.
 (c) Determine the value of the installment option.

18. A company owns a parcel of land on which it can build either a 4-unit or a 8-unit condominium. The current real-estate price for a 1-unit condominium is €250,000. Next year the price will either rise to €225,000 or decline to €180,000. The chance the market will move favorably is 55%. The construction cost is flat over time at €155,000 per unit for a 4-unit building or €190,000 thousand for an 8-unit building. Assume construction is instantaneous. Rent covers operating expenses, so no free cash flow is generated. Risk-free rate is 2.5%. What is the value of the land if the company has a 1-year delay option?

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10 Long-Term Financing

Previous chapters of the book have underlined how the capital structure of a firm normally involves both debt and equity sources, and how important it is to find the right mix between the sources to maximize the value and reduce the risk.

This chapter deals with the description and in-depth analysis of the various sources of long-term capital, equity, and debt, and how it is possible to raise the funds in either form through appropriate procedures.

Both long-term equity and debt constitute the spine of the company and determine its value and also the capacity of producing goods and services at the appropriate volume and speed.

It is therefore important to understand what are the various sources of equity capital at all stages of the company's life and also understand how a company issues debt instruments for external financing.

The issue is the trade-off between economic convenience, which translates into a lower return to be paid to the investor, and risk, which is born when accessing riskier forms of capital.

Given the geographical peculiarity of some capital structures in the world, the chapter also aims to give a regional perspective on the problem, by analyzing the policies for fund raising in different parts of the world.

After studying this chapter, you will be able to answer the following questions, among others:

- What are the different sources of equity capital? How do they differ according to the stage of the company's life?
- What is a rights offer and how does it differ from direct shares offer?
- What are the different types of debt available to the firm for external financing?
- How can a debt issue be protected through covenants and early repayment?
- What are the approaches for long-term fund raising in different parts of the world?

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The first section of the chapter deals with the various types of equity financing, from the start-up stage of the company to a more mature stage of growth and establishment in the market. The second section deals with the debt financing sources and how the company handles the issuance of debt securities. The final part is devoted to a geographical analysis of the long-term fund raising issue, with a deep look into the American and European markets but also to the reality of the emerging markets.

10.1 Early-Stage Funding

Learning Outcomes

- Explain the role of angel investors in the early financing of a firm.
- Explain the role of venture capital in the early financing of a firm.
- Explain private equity and its role in equity financing.

10.1.1 Angel Investors

Business angels are investors with high net worth investing in entrepreneurial ventures. Their noninstitutional nature, combined with sufficiently high funds available, enables them to invest in high-risk, high-return start-ups. In return they ask for a share of voting, income, and, ultimately, capital gain.

Angel investment represents the very first stage of external independent funding for a company, and it normally involves early-stage ventures where the founding team has exhausted their personal savings and sources of funding from family and friends.

The start-ups targeted by angels are normally so embryonal that they cannot independently stand on their own. The outlook for these companies is normally between failure and takeoff.

They are not mature enough to gain private equity funding, and the management typically lacks experience in a growth venture, and the business needs not only the additional funding but also mentoring to take it to the next stage of development.

There are drawbacks in investing in early-stage companies, which makes the investment unattractive to banks and other institutional investors. Angels are therefore individuals who can afford to lose the money and/or are willing to wait some years before they see a return on their money (Clarysse et al. 2007).

The features of early-stage businesses make them unique compared to mature companies and corporations. First of all, the shares are not traded and there is no market for them.

Anyone investing in start-ups must wait for the company to go on sale or be publicly listed to trade the shares. Novel business concepts and inventions are often associated with emerging and untried markets.

Angel investment is associated to a higher risk than normal corporate investment. The business is subject to a high level of uncertainty, with products or services under development.

The design and knowledge of the product depend on some employees, with usually little or no business experience.

The knowledge of the product and its design may be highly dependent on a small number of key staff, who may not necessarily have proven business experience. Due to their small size and lack of presence in the market, start-ups are generally susceptible to changes in market conditions.

The survival of the early-stage firm depends critically on timing, and the quickness in releasing products, as well as the ability to achieve revenue steps, can make the difference.

The access to funds other than the ones provided by angels is very complicated for a start-up. This is further complicated by the fact that these ventures typically have little collateral to pledge for loans.

There is limited access to further finance if the business encounters delays or undertakes operations which require additional funds. A high cash burn rate characterizes the business, given that they have not yet reached the ability of self-funding.

Expanding the business is also complicated given the limited amount of funds. Given the little or null history of financial performance, it is very hard to give a valuation of the business.

Minority shareholders have little power unless it is through an investment agreement. Even if they disagree with management actions, they have little power and can't sell their shares easily (Lane and Mifflin 2011).

Several types of angels are available in the market. Some of them have direct experience in the industry of reference, and they can help with business development, recruitment, sales, strategy, contacts, and so on.

Some entrepreneurs are so specialized in angel investment that they reinvest the amount cashed up into new ventures and add their experience on the previous successful business to the new one.

However, they may not have experience in the industry in which the firm operates. Wealthy and/or retired corporate executives often make investments in new ventures within their industry.

They can provide not only the money but also the contacts, introduction to other investors, recruitment, and risk assessment services. In some cases, they are simply wealthy individuals with a desire to invest in the private sector, and money is their only contribution to the business.

Analyzing the typical profile of an angel investor is not easy given that there are no public data available about this type of funding. Angels in fact are most likely to stay out of the public eye and refuse to speak about their experience.

A profile of the typical angel investor has been set up by the Center for Venture Research at the University of New Hampshire, in a study solely focused on the US market.

The survey of the centre depicts the angel investor as a capital supplier focused on a specific geographic area, very limited. Individual angels rarely invest more than a few hundred thousand dollars in total.

Angel investors belong to the wealthier social class and are quite old, but not necessarily millionaires. Financial figures include a required return of about 25% and an expected rate of failure on the investment of a third.

They reject seven out of every ten deals that cross their desks, for a variety of reasons, including poor growth potential, overpriced equity, and an inexperienced management team.

Angel investors are the initial bridge to more sophisticated sources of funding like venture capital. At the angel capital stage, the focus is on the capital really needed to start the operations and on the connections made available to expand the business.

10.1.2 Venture Capital

Venture capital is the source of funding provided by outside investors, to fund a growing company. The risk associated to such an investment is high; therefore a stake of the shares in the company is requested in exchange for the capital.

It is an important source of funding for start-ups and other companies that have a limited operating history and don't have access to capital markets. The target of venture capital firm (VC) are small businesses with a high growth potential.

Most venture capital firms are structured in the form of limited partnerships. They pool investment capital to be invested in some companies and range in size from a few millions of dollars to many billions of dollars' equivalent value invested in start-ups.

Venture capital is provided either by groups of investors or branches of commercial or investment banks, as well as some insurance companies. In all cases the aim of the VC investor is to employ its business knowledge and expertise to contribute to the growth of the companies. The process is assumed to yield a substantial return on the VC's investment, generally within 3–7 years (Bloomfield 2008).

The trade-off between risk and return on a VC investment is quite pronounced and the failure rate is quite high. On the other hand, a successful investment in VC can yield a return consisting of three to ten times the initial investment.

On top of a stake of equity, the VC also demands decision rights in the management of the target business. The involvement in management and planning decision can be very robust.

The VC offers its managerial contribution through the expertise of its general partners who may be former CEOs, bankers, or experts in a particular industry. It is common that one or more general partners of the VC take positions in the board of the financed company as well. They may also help recruit key executives to the portfolio company.

Matching the needs of the growing business with the right VC partner is one of the main duties the young entrepreneur has to accomplish. Finding the right financing partner can make a difference in the future of the business.

Not all the VCs invest in the same type of companies. Some of them invest small amounts of capital for very early ventures while others focus on early or expansion funding. Other VCs prefer to invest at the end of the business cycle, to finance buyouts and operations of capital restructuring.

In terms of portfolio diversification, VCs may be generalists and invest in multiple industries and locations or (most commonly) specialize in a particular industry. It is therefore very important for a firm that is seeking funds to make sure the target VC is the right choice also in terms of industry specialization.

In addition to industry preferences, VCs also typically have a geographic preference. Being in the same general location as a portfolio company allows the VC to better assist with business operations such as marketing, personnel, and financing.

Not all new businesses can raise VC's attention. Investors are in fact very selective and only those companies that qualify are taken into consideration. The main feature VCs look for is high growth potential.

They aim for companies with very high profitability allowing them to exit with a higher than average return in a time frame of roughly 3–10 years, depending on the type of investment. The consequence of the strict selection criteria is that most funding goes to companies in rapidly expanding industries like technology and life sciences.

Recall that the typical venture capitalists is keen to invest in companies with high growth potential. The future outlook of the company is actually more important than the actual size of the business.

A target company for a VC is one that may be capable of becoming a large market leader in its industry due to some new industry opportunity and competitive advantage. There are some standard factors that lead the choice of the VC when screening a proposed investment opportunity.

First of all, the company must present a product that is commercially viable, one that can be efficiently produced to generate revenue. The market must be clearly identifiable, and once a need for the product is spotted in the consumer market, the company must show a clear plan to meet the identified need in an efficient, revenue-generating manner.

The strength of the management is also important, as well as the level of confidence inspired by the leadership of the company. Management is expected to have the vision and expertise to take the business to the next level.

A very important factor of success for a business, and also the best way to impress a VC, is to show that the business has a competitive advantage. The idea behind the business should be such to establish significant barriers to entry that will inhibit others from encroaching upon its market (Draper 2011).

Obtaining the funds from VC entails a detailed process, aimed at disclosing the necessary information for the investors to be able to make decisions. First of all, a detailed business plan should be submitted, as well as an executive summary of the business proposal.

The VC receives the documents and starts discussing internally about the investment opportunity. If a positive decision about proceeding is taken, the business in need of funds goes to another stage. This first step may take a few weeks to be completed.

Every VC receives a huge amount of proposal, and only a very limited percentage of them is invited to meet the VC's partners for a more formal discussion. An average of 0.3% of businesses demanding VC funds gets the money.

The second stage involves an introductory meeting between the company and the VC. The purpose of the meeting is to discuss the business in more details once the general idea has been accepted.

The first approach is usually via phone, and if the phone conversation is successful and a mutual fit is seen, the entrepreneur is usually invited to visit the VC for a meeting to discuss the opportunity in more detail. After this meeting, the VC will determine whether or not to move forward to the due diligence stage of the process.

The third phase is the due diligence, which is dependent upon the features of the presented business proposal. The process can be lengthy and involves continuous contact between the company and the VC.

Contacts in this phase include phone calls, emails, management interviews, customer references, product and business strategy evaluations, and other such exchanges of information during this time period.

Upon satisfactory due diligence, the company is commonly offered a term sheet, which is a document that does not bind any party but is meant to list and explain the basic terms offered for the investment. The term sheet is normally negotiable, and it is followed by the legal procedures for documents and due diligence, before the funds are made available (Peterson 2003).

The term sheet contains also the assumed valuation of the target company, set by the VC after a thorough process that involves the analysis of several factors. Factors driving the valuation process are the management team, the market and competitive advantage in the marketplace, and the earning potential.

The amount invested in the company sums up to the above pre-money valuation in order to give the final post-money valuation, which includes the new money invested by the VC.

As for any other deal in life, the best option for the company in search of funds is to have access to more than one interested VC, which would allow to obtain much more favorable conditions due to competition.

The term sheet negotiation is based on a couple of main issues that are very important for all VCs. First of all, the financial aspects of the deal must be satisfactory in terms of the return offered in exchange of the funding.

The second important aspect is control, meaning how the VC will be able to exercise control over your company's decisions. The pertinent negotiations will revolve around these two issues.

The several stages of funding for a new born company are commonly denominated with letters from A to D, with A being the first angels, to D meaning more mature stage of capital raising. After that, the final rounds take the company to the IPO that officially makes the business public and the shares traded on financial markets.

A VC may specialize in providing just one of these series of funding or may offer funding for all stages of the business life cycle. Knowledge of the preferences of the VC is essential, as well as clearly articulating what type of funding the business is looking for (Ryan 2013).

When a business is at the very beginning, with no development of product or organization, it is said to need seed capital. There are not many VCs that give seed capital, and the amount invested is generally very small.

Start-up capital instead is given to companies that already have a sample product available with at least one principal working full time. Such type of investment is also rare, and the money is used for additional recruitment and finalization of products or services.

A third level of funding is called early-stage capital and comes at a later stage of the company's life, after a few years from commencement. The company has a proper management team in force, and sales are increasing. This is the stage where the funding offered by a VC fosters the increase in sales to the break-even point, improves productivity, or increases the company's efficiency.

A later stage is the expansion capital, and it comes for companies that are well established and looking for help to get the business make a decisive further step. Funding at this stage may enter new markets or increase marketing efforts.

The late-stage capital is the final step before the IPO, and the company at this stage has reached an impressive level of sales and profit. The purpose of additional funding is to increase capacity, foster marketing, or increase working capital.

At this mature stage, approaching a VC is also a good way to find a partner to help find merger and acquisition opportunities. There are VCs that focus on this end of the business spectrum, specializing in initial public offerings (IPOs), buyouts, or recapitalizations.

VCs can help planning an IPO as well as developing a mezzanine or bridge financing plan. Such a short-term source of funds can help with the costs the company incurs to go public.

Again, a key factor for the VC will be risk versus return. Investing in a company at earlier stages entails higher inherent risks and a longer time period before the VC can exit the investments. The earlier the stage of investment, the higher the return expected by the VC.

The exit opportunity represents a way for the VC to cash out an investment in a company that is not performing as expected. VCs hope to sell the equity they hold in a funded company, after a maximum of 7 years, ideally through an IPO.

When the IPO takes place, the shares of the company become very liquid and the VC can cash a lot from selling them, therefore making the huge profit they aimed to when they decided to fund the business.

While an IPO may be the most visible and glamorous form of exit, it's not the most common. Most companies are sold through a merger or acquisition event before an IPO can occur. If the portfolio company is bought out or merges with another company, the VC receives stock or cash from the event.

10.1.3 Private Equity

Private equity represents a more advanced stage of long-term committed share capital, aimed to companies that are not quoted yet. When a business is in need to start up, expand, buy into a business, or revitalize a company, private equity can help.

Private equity is very different from raising funds through a bank loan or other form of debt. Private equity is invested in exchange for a stake in your company, and the return on the investment is dependent on the growth and profitability of the business.

Private equity capital is assigned upon very selective criteria. Many small companies are "lifestyle" businesses whose main purpose is to provide a good standard of living and job satisfaction for their owners. In these cases, the PE firms will not step into the business, given the lack of the feature of growth aspiration which is mandatory for a business to qualify for PE capital (Carter 2004).

Entrepreneurially focused businesses can be distinguished from others by their aspirations and potential for growth, rather than by their current size. Such businesses are aiming to grow rapidly to a significant size.

The usual term for the business to show its full potential of a high turnover is normally set at 5 years. If the business is expected to take much longer than that to fully exploit its potential, PE will be unlikely to invest in it.

As in the VC case in fact, PE investors are only interested in companies with high growth prospects, led by experienced and ambitious managers who are capable of turning their business plan into reality.

PE investment comes with some good advantages. Companies funded by private equity have shown historically to grow faster than other types of companies due to a combination of capital and experienced personal input from private equity executives.

The goal of a PE company is to turn a business into a success, by increasing the value to its owners, without taking day-to-day management control. They demand a high stake in the firm's equity, but they can boost the value of the remaining equity to the owners much more than they would realize without PE capital.

Private equity firms raise capital based on a competitive market, same as for the business looking for them to fund their operations. PE firms have to demonstrate a good track record and the prospect of producing returns greater than can be achieved through fixed interest or quoted equity investments (Cummings 2010).

PE firms normally access funds for investment from institutional investors, and when they obtain their funds mainly from their parent organization, they are also known as captives. When captives raise funds from external sources as well, they are called semi-captives.

Funds raised by PE from external sources can have several types of structure. Most commonly they are structured as limited partnership with a fixed life of some years. The funds will have to be returned to the original investors after that period.

This generally requires the investments to be sold, or to be in the form of quoted shares, before the end of the fund. Some funds are structured as quoted private equity investment trusts, listed on major stock markets, and as they have no fixed life span, they may be able to offer companies a longer investment horizon.

In the United Kingdom, for example, there are Venture Capital Trusts, which are quoted firms collecting funds for small businesses, with the incentive for the investors of tax cuts and deferrals in return for a 5-year investment commitment. The funds obtained through a trust come at the expense of limitations on the future development of the corporate activities, for an initial period of a few years (Dermot 2008).

The first step in choosing a PE firm is to decide whether or not to hire an adviser. Then a few possible funders must be selected, in particular those that match with the business idea and proposal.

The fit with an appropriate PE firm is based on the stage of the company's development or the type of private equity investment required. Other important factors are the industry sector and amount of finance needed and the geographical location of your business operations.

As for the VC funding, there is a match between the stage of the business life and the type of PE firm to be contacted. From seed capital to the established companies, the range of available opportunities is very wide for both PE firms and funding-seeking businesses.

On top of the above seen reasons to acquire capital, there are also management buyout (MBO), management buy-in (MBI), and institutional buyout (IBO). Each of these strategies can be financed through PE capital.

MBO is an operation through which the management of a company and investors can purchase a significant share of equity in the business they manage. The range of MBOs goes from small family-owned business acquisitions to complicated multi-millionaire buyouts. The amounts concerned tend to be larger than other types of financing, as they involve the acquisition of an entire business.

The MBI is a similar operation that involves external managers and investors who are interested in acquiring control of a company they are not currently managing.

The IBO consists in a PE firm acquiring a business through institutional procedure. Immediately after the acquisition is made, the incoming management will itself acquire a stake of equity (Jefferson 2001).

This is a relatively new term and is an increasingly used method of buyout. It is a method often preferred by vendors, as it reduces the number of parties with whom they have to negotiate.

Hybrid types of operations are also possible through the funding provided by a PE. Secondary purchase, for example, is the acquisition of existing shares in a target business, from another PE firm or other shareholders.

Replacement equity consists of existing investors buying the stake of another investor. Rescue investment is meant to finance a company in trouble. PE's funds can also be used to redeem the debt of a company so as to reduce its leverage.

As mentioned above, the industry sector of a firm seeking for funds plays an important role in the choice of the right PE, with sectors like biotech, information technology, and computer related being the most targeted.

The amount of investment is also very important, and the majority of the PE firms normally invest high amounts of capital, with only a few of them investing in ventures that require little amount of funding.

Companies initially seeking smaller amounts of private equity are more attractive to private equity firms if there is an opportunity for further rounds of private equity investment later on (Ochtel 2009).

Whether the amount to be invested is low, high, or extremely high, the effort in the screening required to a PE is more or less the same. The private equity firms have to spend similar resources in appraising the business proposal prior to investment.

This is the main reason why investments of large size are more appealing to PE firms, because in size terms (as opposed to return terms), the interest received on a large investment is likely to be greater than for smaller investments and should more easily cover the initial appraisal costs.

10.2 Initial Public Offering

Learning Outcomes

- Explain what an IPO is and the road to it.
- Explain how the firm can issue stocks through an IPO.
- Explain the issue of IPO underpricing.

10.2.1 Types of Offering

An initial public offering (IPO) can be defined as the process of selling a security to the general public for the first time, with the expectation that a liquid market will develop for trading it. There can be several debt or equity securities as target of an IPO. The most popular are however those involving issues of equity by operating companies.

The early-stage sources of capital examined in the previous section involve investors putting money in the equity of the firms, with no liquid market existing if these investors wish to sell their stock (Cohan and Hungry 2012).

If a company prospers and needs additional equity capital, at some point the firm generally finds it desirable to "go public" by selling stock to a large number of diversified investors.

The enhanced liquidity due to trading in public open market allows the firm to access a wider range of investors ready to deal on more favorable terms. This is much better than compensating investors for the lack of liquidity associated with a privately held company.

The benefits of having access to open-market transactions are balanced by some additional costs. These costs are associated with the need to supply information on a regular basis to investors and regulators for publicly traded firms.

There are also costs that are linked to the IPO, which can be direct or indirect. Direct costs include legal, auditing, and underwriting fees. The indirect costs include the time spent on the operation by the management.

Another indirect cost is given by the dilution of capital due to the sale of shares at a price that is generally below the immediately post-IPO market level. These direct and indirect costs affect the cost of capital for firms going public.

The IPO is subject to market conditions, so that it is hard to predict what the reaction of the market will be and how the price will fluctuate immediately after the offering. This link to the trends of the market involves a high stress for the entrepreneur.

The preliminary part of the IPO goes through several steps, and the procedure must fulfill the general provisions from the regulators in the various markets. As an example, the following are the steps of the procedure in the American market, as regulated by the Securities and Exchange Commission (SEC).

- In the first step, the board of directors approves the offering. Also sometimes it is necessary to increase the number of shares sold, and this must be directly voted by the shareholders.
- The second step involves the preparation of a registration statement to be filed with the SEC. This is a general requirement with some exceptions relating to low-value issues. A registration statement normally consists of many pages of financial information, including a financial history, details of the existing business, proposed financing, and plans for the future.
- In a third phase, the SEC examines the documents and the company starts distributing a preliminary prospectus, which synthetizes the information in the statement. Unless the SEC recommends changes to the registration statement, this becomes effective after 20 days from filing. The SEC does not consider the economic side of the operation at this stage but only its complying with the rules and regulations. Also, the SEC generally does not check the accuracy or truthfulness of information in the prospectus.

After the above steps are fulfilled, the IPO goes into the stage of valuation. Valuing IPOs is in principle not so different from valuing other stocks, and both discounted cash flow (DCF) approach and comparable firms analysis can be used.

Most of the recent IPOs involved high-technology young firms. Since a young firm has a short history of accounting information, it is often impossible to project future profits and cash flows in an accurate manner.

For valuing the IPO, it is therefore necessary to rely on the information about comparable firms. The market often offers comparable firms in the same line of business, for which accounting information is available. In other cases, it may be difficult to find publicly traded "pure plays" to use for valuation purposes.

In practice, the procedure for pricing the IPO involves a pricing meeting to be held the same morning as the SEC clearance to go public is expected to be received.

The outcome of an IPO is generally very uncertain. Many high-expectation IPOs in the past with companies assumed to raise many millions of euros or dollars having to withdraw for exogenous factors.

There is empirical evidence that companies issuing stocks on an IPO in US market historically prefer to have a target stock price ranging from $10 to $20. That is why new companies usually tend to stick to the range, by splitting or reverse splitting the stocks.

Stocks with a price below $5.00 per share are subject to the provisions of the Securities.

Enforcement Remedies and Penny Stock Reform Act of 1990, aimed at reducing fraud and abuse in the penny stock market.

In the case of a cash offer public issue, an underwriter is involved. Underwriting is an important line of business for many large investment firms. These companies offer services like modulation of the issuance method, pricing of new securities, and the actual sale on the market (Kotler 2004).

The underwriter typically buys the issuing securities for a price that is lower than the offering price, thus bearing the risk of a nonsuccessful sale on the market, with a relatively huge loss.

The risk is therefore very high, and that is why usually underwriters combine to form an underwriting group called a syndicate to share the risk and to help sell the securities, so to share the risk and profit.

The offering is then co-managed by one or more managers, with a leading one who takes responsibility for the issues related to dealing with issuer and pricing of the securities. The other underwriters in the syndicate distribute the issue and produce research reports later on.

The compensation received by the underwriter is represented by the determined offering price and the price for buying from the issuer. It is called gross spread, and it is not always in cash. Sometimes it happens in fact that on smaller deals, the underwriter is compensated with warrants and stock in addition to the spread.

The choice of an underwriter entails either an offer on highest bidding or direct negotiation. Companies usually choose to use the negotiation as a method, but in some specific cases like public utility holding companies, competitive underwriting is mandatory (Nakamura 2005).

The use of competitive underwriting has been proven to be cheaper, and the main reason behind the standing dominance of negotiated underwriting in many countries is still debated.

There are two types of underwriting involved in a cash offer.

- Firm committed underwriting involves the sale of the entire issue from the issuer to the underwriter, who tries to resell it to the market. It is the most common method of underwriting in many countries, including the United States. In case the underwriter cannot sell all of the issue at the agreed-upon offering price, it may have to lower the price on the unsold shares. The risk is mitigated by investigations of the market, to test the reception to the offer, which happen before setting the offering price. Moreover, the price is not set in good advance to the actual sale of the issuance, so that the profit is not well known until that time.
- In a best effort underwriting scheme, the underwriter is legally bound to make all the possible effort to sell the securities at a price previously agreed on with the

issuer. There is no other commitment from the underwriter, and no amount of money is guaranteed to the issuer on top of that. This form of underwriting has become rather uncommon in recent years; firm commitments are now the dominant form.

When the price is set and the offer is made, the issue takes place and the investors buy the new securities on the market. The period after a new issue is initially sold to the public is referred to as the aftermarket.

This time is characterized by the sale of securities at not less than the offering price. Also the (main) underwriter can buy the securities in case the market price falls below the offering price. Such a strategy supports the sale on the market, by stabilizing the price against temporary bullish trend.

After some determined time, usually 30 days, if the issue remains unsold, the members of the underwriting syndicate are allowed to leave and sell their securities at the current market price.

In some cases, an underwriting contract may include a provision allowing the underwriter(s) to buy additional shares from the issuer at the offering price. This is called green shoe provision.

This provision is normally contained in all IPOs involving equity issues, while being not present in ordinary debt offerings. The reason for such a provision is to cover excess demand and oversubscriptions.

The green shoe provision is usually set on no more than 15% of the issuance and lasts about 30 days. It represents a cost to the issuer and a benefit to the underwriter, because it can be used when the market price goes above the offering price within 30 days, allowing the underwriters to buy shares from the issuer and immediately resell the shares to the public (Tsuruoka 2004).

Another type of collateral provision on IPOs is the lockup agreement, which specifies the period insiders must wait after an IPO before they can sell some or all of their stock on the market.

The features of a lockup agreement are pretty standardized, and the lockup period is normally set at 180 days. After the IPO underwriters have 6 months before they can cash out.

The provision ensures that they maintain a significant economic interest in the company going public. It is also important because it can happen that the number of locked-up shares exceed the number of shares held by the public.

The consequence of such a concentration is that at the end of the lockup period, a large wave of sales may hit the market and strongly decrease the price. As shown by the evidence, venture capital-backed companies are on average more exposed to a loss in value on the lockup expiration day.

10.2.2 IPO Underpricing

One of the main responsibilities of the underwriter(s) in an IPO is to set the offering price at which the stocks of the company will be sold. Setting a fair price is very important to ensure a smooth IPO and placement of all offered shares. In addition, it is important to ensure the right amount of proceed to the issuing company.

When the price is set to ensure all stocks offered can be sold, the underwriting fees are deducted and the remaining amount goes to the issuer as proceeds of the IPO. When the stocks are finally ready for trade on the secondary market, the supply and demand will determine the equilibrium price for the following time.

Looking at the history of IPOs, even if the underwriters normally excerpt a lot of effort in determining the right offering price, normally IPOs tend to be underpriced in terms of demanded price.

This unfortunately looks clear only after the stocks have been issued, when the market prizes the issuance by raising the price already in the days after following the IPO. On the other hand, the fact that the issuer cashed the offering price instead of the subsequent market-traded price results in a loss.

Underpricing is then defined as the situation when a stock is issued at a price below the market (fair) value. The underpricing is temporary, due to the action of market forces that set the price to the fair value in a relatively short time.

Factors like the fundamentals of the firm and the aim to marketable price normally drive the analysis and calculation of the IPO price. Factors like cash flows, growth perspective, and discount rates are at the basis of models like discounted cash flow (DCF), dividend discount models (DDM), and relative valuation models (RVM) analyzed in other chapters of this book.

One should always recall that investors normally perceive an IPO as a type of investment with a higher degree of risk compared to investing in an already public company. This is mostly due to the lack of public information that normally surrounds private company, especially in terms of financial data (time series of past stock prices).

The resulting asymmetry of information leads the issuer and underwriter to opt for an issuing price lower than a fair price would be. In this way, the issuance is more attractive to investors who benefit from a price discount as a compensation for the higher risk born.

There are normally hundreds if not thousands of IPOs run in the world every year. When companies get to this stage, it generally means they have no other way to raise the funds needed to support a fast growth or a change in scale of the business (expansion).

The literature and practitioners have indicated several reasons why stocks in an IPO are normally subject to underpricing at first issuance. Some concentration of investing power among big investment banks underwriting the issuance has been brought up as a reasonable explanation to that.

On the other hand, other analysts have pointed to the regulatory aspects of an IPO as an important aspect of the deal. The already mentioned asymmetry of information is another important aspect.

Commercial banks in the United States are not allowed to be involved in corporate equity underwriting. This leaves investment banks with a monopoly in the field that may naturally generate a tendency to offer lower prices to the target companies.

In addition, if one looks at the fact that normally the investment banks involved in underwriting of the biggest IPOs in the United States belong to a restricted group, the argument of monopoly becomes even stronger.

Some important acquisitions of investment banks by nonfinancial multinationals like General Electric among others have reduced the monopoly effect by introducing some level of competition in the underwriting sector.

Still, the level of concentration is so high that the issue still stands, and the spreads earned by the underwriters over the issuing firms are generous compared to what it would happen, if the market of investment banks were more competitive.

10.3 Debt Financing

Learning Outcomes

- Explain the different types of debt financing available to firms.
- Explain the difference between bonds and bank loans.
- Learn what leasing is and how companies can use it.

10.3.1 Types of Debt

The class of debt financing includes all the funds that are borrowed by a company with the promise to make interest payments in regular instalments, plus repayment of the principal.

Repayment happens in accordance with an agreed upon schedule. Along with equity (common stock plus retained earnings), debt represents one of the two major sources of capital for business enterprises.

The first source of debt capital to analyze is the money market. It is the market for debt instruments with maturity less than 12 months. An example of money market is the commercial paper.

Commercial paper is issued by financial firms and reliable industrial corporations, either directly or through a broker. It represents an obligation of pure discount in the short term.

When looking at long-term debt sources, the major class of instruments is represented by corporate bond issues. A firm can issue a bond either in its home country or in another country. It is obvious that raising money abroad involves being subject to the rules of the country in which it does so.

The bonds that are sold into foreign markets, to local investors of that specific country, are called foreign bonds. According to recent reviews, the US market is the largest market for foreign bonds, followed by Japan and Switzerland.

The international market for long-term bonds is quite well developed. Groups of underwriters form syndicates to sell the bond issues, and most of them are located in London Exchange.

Many international banks and security dealers, from the United States and other countries, have their branches located in the United Kingdom and make deals in major currencies. The US dollar has been the most popular choice, but a high proportion of international bond issues are made in the euro, the currency of the European Monetary Union.

In the early 60s, the American government wanted to discourage the outflow of capitals. The goal was achieved by taxing the interest made on foreign security so to equal it to the interest on domestic securities.

As a consequence, both European and American multinationals were forced to tap an international market for capital, named as the Eurobond market, not to be confused with the concept of Eurobond (which may be in any currency) with euro-denominated bonds.

The taxation was then removed in the first half of the 70s, and nowadays capital export is not controlled or hindered anymore. American companies can choose where to borrow their funds with freedom.

As a consequence, the interest rates in the United States and London markets, just to give an example, started converging and are usually very similar. On the other hand, the international bond market is not subject to American regulation, and financial management should be aware of small differences in the borrowing costs between markets.

The contract between a bond issuer and a bondholder is legally binding, and takes the name of indenture, or trust deed. It specifies all the important features of a bond, such as maturity, interest payments, interest calculation, optional features and so on.

On top of that the contract also contains all the conditions regarding the bond issue, as well as the financial covenants related to the issuer, and the formulas to be used for calculation of covenant matching.

The indenture is a reference in case of litigation, and should conflict arise between the two counterparties, it is the only document with legal and binding value. This is why it is carefully compiled and it contains all the features of the bond issue.

In times of normal markets, the indenture is not so important and the chance of conflicts is very low. The document becomes very important when there are events that put the issuer in a position at risk of violating a covenant.

The indenture will then be scrutinized closely to make sure there is no ambiguity in calculating the financial ratios that determine whether the issuer is abiding by the covenants.

When the bonds are registered, their ownership is recorded in the company's books and payment for the interest and final principal amount is directly made to the registered owners.

It is a general practice to register the bonds, but in some countries, the alternative way is used, which is called the bearer form. The certificate is the primary evidence of bond ownership.

The bondholder is then entitled to claim interest payments through coupons and then send the certificate at maturity in order to claim the final repayment of principal. International bonds allow the owner to hold them in bearer form. That is why in some countries like the United States, for example, the government has tried in the years to disincentive the residents from buying them.

Dollar bonds are usually expressed with a face value of $1000 while European bonds have a face value of 100 €. The price is expressed as a percentage of that value, and net of any accrued interest. It follows that the buyer of a bond is supposed to pay the price plus any accrued future interest amount.

It is very important that the company keeps paying the interest regularly, so that lenders are not encouraged to claim their money back instead of waiting until matters may have deteriorated further.

It follows that the interest represents a sort of protection against a loss in value. In some cases in order to offer this type of protection the bond issuer may offer a lower interest rate but a larger discount on the face value.

The investors then receive a significant part of their return in the form of capital appreciation. The special case is the simplest bond, called zero coupon bond, which pays no interest at all, but only the face value at maturity.

The interest rate offered on a bond can be either fixed or floating. A fixed rate ensures that all interest payments will be equal. In some cases the payments vary with the general level of interest rates.

A typical example of floating rate is the bonds tied to some Treasury bill rate, or those linked to the London Interbank Offered Rate (LIBOR), the rate used internationally by the banks for lending to each other. Often these floating-rate notes specify a minimum (or floor) interest rate or they may specify a maximum (or cap) on the rate.

Another important distinction is between unsecured and secured bond issues. Most bonds are in fact issues as unsecured obligations, meaning there is no protection against a default of the bond issuer.

In case of secured bonds instead, if the company defaults on the debt, the trustee or lender may take possession of the relevant assets, and in case these are not enough to satisfy the claim, the remaining debt will be put in line with any unsecured debt, against the other assets of the firm.

Mortgage bonds are the major class of secured bonds, providing a claim against some property (building) or even secured against all of the firm's properties. The value of the mortgage depends on the use of the property put as collateral.

Securities can be also used by a company as collaterals for a loan. When a holding company wants to borrow funds for example, it can use the common stocks in subsidiaries as collateral.

The above securities represent a junior claim to the lender, given that they come after other claims, with a senior right. Therefore, the collateral trust bonds often include limitations to the subsidiaries in order to avoid excessive issue of debt or preferred stock.

Another form of secured debt is called equipment trust certificate, which is mostly used in the heavy transport industry. The agreement gives a trustee the ownership of all the equipment.

The company then makes a down payment on the overall cost of the equipment and a bunch of equipment trust certificates is issued in order to provide the balance. Maturities of the certificates normally range from 12 months to 15 years. When all the debts are paid, the company acquires formal ownership of the equipment.

Like other securities, the equipment trust certificates are rated by agencies like Moody's or Standard and Poor's. The rating of the certificates is normally one grade higher than the company's regular debt, showing their relative low riskiness.

Bonds are in line with a seniority grading, meaning that senior bonds represent claims that will be satisfied first in case of redemption, while junior bonds are subordinated to the senior bonds or to all other creditors.

So if the firm defaults on its obligations, the holders of senior bonds are repaid first. The subordinated bondholders stay in line behind the firm's general creditors, and just ahead of the preferred and common stockholders.

Another way of raising funds through debt is by borrowing money indirectly. Companies in fact sometimes bundle up a group of assets and then sell the cash flows from these assets. These securities are known as asset-backed securities (ABS).

The typical case is of a financial company or bank making a large amount of mortgage loans to home buyers. If such a company is in need of cash, but cannot wait for the loans to be repaid, the solution is simple.

The procedure for indirect borrowing involves creating a separate company that buys the package of mortgage loans from the lending company. The purchase is financed through sale of mortgage pass-through certificates.

The holders of the certificates then receive a share of the mortgage payments. Repayment of certificates goes parallel with the speed of repayment of the actual mortgages.

The indirect way of borrowing funds is not limited to the real estate sector, but involves many types of loans like automobile loans, student loans, and credit card receivables. They are often bundled together and re-marketed as a bond. The major investment banks are nowadays able to repackage every type of loan.

The major part of debt issues is placed publicly through specialized agencies. But in some cases the debt is placed privately with a small number of financial institutions. This is done because the costs of a private placement are lower than for a public debt issue.

There are several aspects under which a private placement differs from a public issue. First of all placing an issue privately with a financial institution (or two) can be usually through a simple promissory note.

This is just an owing declaration that lays down certain conditions to be observed by the borrower. In a public issue one must worry about representation of bondholders in negotiations and the procedures needed for paying interest and principal.

As a consequence, the public issue involves a more complicated contractual structure. Also, publicly issued bonds are very standardized in order to allow fast trading of them without checking the fine print in the agreement.

In a private placement the contract is personalized and tailored for companies with any possible need and problem. This type of contractual link makes the relationship between borrower and lender much more intimate.

In general, bond issues are structured to give protection to the lender through a series of conditions on the borrower. In the case of a private placement the conditions tend to be more severe. Such types of constraints are accepted by the borrowers who know it is easily possible to replace them in the near future, if some conditions are met.

This is not possible in the case of a public issue, where the contractual conditions, including the limitations to the activity of the borrower, are standardized, and it is therefore almost impossible to rework them at a later stage.

All these features make private debt placement an interesting source of financing and fill a particular niche in the corporate debt market, namely, loans to small- and medium-sized firms.

The target firms for private placement are those who face the highest costs in joining a public issue, which require detailed analysis and often require specialized, flexible loan arrangements. Even if optimal for small and medium companies, private placement is sometimes also used by large firms.

There are also some disadvantages, and a private placement does not come at no cost. First of all, the lenders ask for a higher interest rate, to compensate for the high illiquidity of the assets.

It is difficult to generalize about the differences in interest rates between private placements and public issues, but a typical differential is on the order of 50 basis points.

Another type of private loan is the project finance, which is linked to the outcome of a particular project and that minimizes the exposure of the parent. Such a loan is a signature product of large international banks.

The debt is supported by the project and not directly by the company. Anyway the debt ratios are usually very high because the loan is also supported by other assets and guarantees provided by customers or third parts.

The variability of debt is linked to the fact that two projects are hardly very similar. However, there are some common features that can be summarized and that define the nature of project finance debt.

First of all, the project is usually established as a separate company. The contractors and plant managers acquire majority of project shares, bearing the related risk. Also the project company signs a series of complex contracts distributing risk among the agents involved.

There is usually governmental agreement on providing the necessary permits and allowing the purchase of foreign currency, among other things. The governmental help, combined to the contractual bounds, allows to raise the capital for the project in the form of bank loans, or private placement.

Project finance is widely used in developing countries to fund power, telecommunication, and transportation projects, but it is also used in the major industrialized countries.

Another convenient use of it is for funding power plants, like electric utility companies. If such a company needs a new plant for feeding energy to another industrial plant, the latter can be a convenient incentive to the bank to lend.

In fact, the existence of a receiving plan guarantees that the energy produced in the coming plant will be fully employed and generate a revenue stream. Banks are then happy to lend a consistent percent of the cost of the project because they know that the cash flows associated to it are free of the most common risks affecting normal businesses.

One should however not ignore the regulatory implications involved in such a strategy. For example, usually the government sets the fees for energy supply to the public, therefore bounding the potential revenues.

The aim is to set charges that allow the plant to recover its cost of capital. The problem is that the cost of capital is not easily measured and is a natural focus for argument in regulatory hearings.

The accounting solution is to roll the cost of capital into the contract price at the time of sale of electric power to a utility, so to make the pass-through to customers less controversial.

Given that the value of the bond is strictly related to the creditworthiness of the issuing institution, sometimes the contract provides protective (restrictive) covenants that restrict the issuer from doing things that would make it less creditworthy.

An irresponsible behavior from the issuer in fact would greatly lower the value of the bond in the secondary market, with an increased probability of default in meeting both interest and principal obligations.

In some cases, the protective covenants may require the assigned trustee to insure the property put as collateral for the bond issue, or it may require that the collateral be well kept.

The protective covenants are set as to compromise between the freedom of the issuer and the need of protection of the buyer. Issuers in fact want to pay the least possible interest and have freedom.

On the other hand, the bond buyers would like to earn the highest possible interest given restrictions on the operations of the issuer, in order to maintain its creditworthiness and protect their investment.

In practice both make a step towards each other agreeing on a fair yield which is inversely related to the level of protection guaranteed to the buyer. As a consequence, the more protection, the less yield, and vice versa. This is consistent with general risk-return financial theories, stating that the greater the risk of the security, the greater its yield must be to attract investors.

Sometimes a company decides to repay part of the issue before maturity. In order to do so, it may collect regular payments into a bucket that is called sinking fund. If the payment is in the form of cash, the trustee selects bonds by lottery and uses the cash to redeem them at their face value.

Regulatory provisions around the world generally state that a company should keep a compulsory sinking fund, but also optional funds are possible if the company decides to satisfy them.

In case of low quality issues, if repayment starts after an adequate period of allocating cash in the fund, the money should be sufficient to redeem the entire issue in equal installments over the life of the bond. High quality bonds on the other hand require a large payment at maturity on top of the instalments paid from the fund.

The sinking fund is an important part of the lending agreement, because in case the company is not able to put money in the fund, the lender is entitled to demand the money back.

When companies are allowed to repurchase bonds in the market, the sinking fund becomes meaningless to give a picture of the solvency of the firm. The financial distress in fact makes the cost of repurchasing debt much lower.

Sometimes corporate bonds include call provisions. A call option attached to a bond allows the company to pay back the debt earlier than maturity. It is sometimes the case, when companies have periods of extra cash.

On the other hand, a puttable bond can be retracted giving the lender the option to ask for early repayment. There are also extendable bonds, that give the investors the option to extend the bond's maturity.

As for the standard options on equity, there are several conditions that make them very attractive. For example, if interest rates fall and bond prices rise, the option to buy back the bond at a fixed price can be very attractive.

It is possible in fact for the company to buy the bond back, and issue a new one with higher price and lower interest rate, making it costless to raise the debt capital.

The choice of the right timing in calling back a bond is a crucial task for a financial manager. Recall that the company aims at maximizing the value of its shares, which translates into minimizing the value of the bond.

As a consequence, the company should never call the bond if its market value is less than the call price. In this case in fact, the exercise of the option would just give an advantage to the bondholders.

So the company should call the bond when the market value is higher than the call price. The investors on the other hand know that the company could call the bond at some point.

It follows that, given the information that the bond will be called as soon as it is worth more than the call price, no investor will accept to pay more than the call price for the bond.

As a consequence of investors' behavior the price of the bond could therefore reach the call price, without going above it. This is the reason why the rule says that a company should recall a bond when, and only when, the market price reaches the call price.

Regarding the relationship between the price of a straight bond and the price of a callable bond with the same features, consider that, if the value of the straight bond is very low it is unlikely that the company will ever wish to call the bonds.

It follows that the value of the callable bond must be very similar to that of the straight bond. On the other hand, if the value of the bond is such to give incentive for recalling it, the value of the callable bond will be slightly less than that of the straight bond.

In case the trend of interest rates is declining, the price of the straight bond will proportionally rise, but there will be no investor willing to pay more than the call price for the callable bond.

It follows that the benefits of a call option on the bond come at some cost. The consequence of making the bond callable is a lower issue price, but still companies embed callable features in their bonds.

This is mainly due to the number of restrictions usually put in place by the indenture of the bond issue. The restrictions are accepted by the companies as long as there is a way out from them. The call provision provides the escape route.

Recall that puttable bonds provide an investor with the option to demand early repayment. The existence of such an option is justified by the uncertainty about every action that the company may take, that cannot be anticipated in the indenture.

Some of the actions in fact could harm the bondholder by reducing the value of the bond. In this case the bondholder can demand for early repayment in order to escape the consequences of the issuer's behavior.

All investors know there is some risk underlying a corporate bond. It is normal that they make sure the issuer behaves properly in order to limit the risk. They don't want it to gamble with their money or to take unreasonable risks.

It is therefore very typical for bond indentures to include restrictive covenants, meant as provisions to prevent the moral hazard of the issuer who may try to purposely increase the value of the default option.

A new bond issue always affects the original bondholders by increasing the ratio of senior debt to company value. The point is that original bondholders would be happy if the company issues common stock simultaneously to the new bonds, in order to keep the ratio fixed.

As a consequence, often the indenture states that the company can issue more senior debt only at the condition of keeping the debt to company value fixed, adjusting it within a specific limit.

The situation of the subordinated bond issues is obviously not of any interest for the senior bondholders given that the subordinated lender does not get any money until the senior bondholders have been paid in full.

The subordinated bonds are therefore similar to equity, from the point of view of the senior bondholders, and they would be happy to see an issue of either. On the other hand, the holders of subordinated bonds care about the proportion of total debt which is senior, therefore coming before them in case of early repayment. This is why in case of a subordinated debt issue, the indenture generally provides restriction on new issue of both total debt and senior debt.

In general, both senior and junior bondholders care about the company issuing more secured debt. This is why an issue of mortgage bonds often imposes a limit on the amount of secured debt.

Another type of restriction involves leasing. It is possible in fact for a borrowing company to buy an asset not only by borrowing the needed money but also through entering a long-term agreement to rent or lease it. From the point of view of bondholders, this is equivalent to an issue of secured debt. This is why the indentures also include limitations on leasing.

Recall that the value of a financial option is affected by dividend payments made on the underlying stock. If the company pays out large dividends to its shareholders the cash dries out and should be replaced by an issue of stock.

If this does not happen, their assets covering the debt are reduced. This worsens the position of the bondholders, and that is why many bond issues restrict the amount of dividends that the company may pay.

Historically, covenants have been widely used in the early 80s. But in the last 35 years the restrictions have been decreasing, and most institutions relaxed their requirements for lending to large public companies and accepted bonds with no such restrictions.

10.3.2 Loans Vs. Bonds

Large corporations raise the majority of funds through debt financing, reaching almost 95% of total external financing on market capitals in Europe, leaving only 5% to equity issuances.

Bond markets have become particularly attractive to both investors and companies, especially after the introduction of the common currency in Europe that enhanced the development of markets.

In parallel, the syndicated loan market has also developed, albeit more progressively, currently accounting for around one-third of borrowers' total public debt and equity financing.

Syndicated loans are the most viable financing alternative to bond issuance. In both cases firms can raise huge amount of capital, with maturities ranging from the medium to the long term.

When a company becomes very large, it tends to use only bond issuances and syndicated loans as a source of debt capital. The choice between the two methods of corporate financing is characterized by agency costs and asymmetric information.

The focus in syndicate borrowing has been put on the role of the banks in screening the borrowers. Financial intermediation is so focused on that given that screening comes at a cost for banks.

But such a cost also represents an opportunity that carry many advantages, in that the efforts banks put in lending to large firms get compensated by long lasting relationships, and a good amount of information acquired, as underlined by Boot and Thakor (2008).

The monitoring of borrowers becomes an unnecessary cost in the case of bond markets, where the investors can easily replicate the investment strategies of the monitoring investors, with major ease compared to the case of syndicate loans.

For this reason, banks can be the delegated monitors of depositors also in the syndicated loan market, where the uninformed lenders participating in the syndication delegate most of the screening and monitoring to an informed lender.

The flow of capital in the market is then primarily driven by bonds and bank loans, with private and public institutions using the funds to finance projects and support investments of all types.

From short-term to long-term investments, the range of possibilities for using the money raised is quite wide, and the money can cover a variety of needs including infrastructure development and corporate expansions.

While it is usually common to finance long-term debt capital needs with bond issuances and short-term needs with bank loans, in recent years, there has been a shift to the bond market as it offers liquidity, with lower costs and higher flexibility.

Access to bond markets is not so easy in that it involves high costs and some hurdles compared to bank loans. Aspiring companies must fulfill certain requirements and go through screening so that they can operate in the bond market.

Consider a company capable of using both funding sources and willing to finance the purchase of new machinery, a considerable large investment scheduled to pay off in 10 years.

The decision is on the CFO, who has to deliberate about the funding type to be used. This choice is mostly determined by the amount of capital needed, the cost of the operation, and the tenure involved.

Small and medium size enterprises tend to use the loan market almost for the total of their debt funding. Local banks are their main reference for financial support, and the providing of liquidity.

But there is a wide range of other banking services that is available to them on a daily basis. Eventually, banks engage in a long relationship with SMEs that beneficiates the latter whenever the need for capital arises.

When it comes to the agreement terms on a loan, such a small information asymmetry makes the deal more flexible. A bank loan often allows borrowers to pay off the loan partly or totally, at any time, with little or no warning.

The disadvantage of the system is that there are repayment charges that are directly related to the interest rates in the area of bank and firm operations, plus administrative fees that are charged for it.

The term of any loan can be easily manipulated by lenders that have the power to establish the conditions that mostly suit them. On the other hand, the borrowers are free to go to another lender if they are not happy with the terms offered.

The argument is very different on bond markets, where the company has to build a solid reputation in order to keep its credit score high and the yield it pays on the bonds low. It is mostly about communicating credit quality, which should determine the risk price of the investment.

The highly standardized terms and conditions on the bond indentures plus the public availability of the financial statement of the company make the information asymmetry issue very small in the case of bonds.

Standardisation plays an important role for companies. It enables the ability to reach a wider range of investors, contributes to lower the search costs and acknowledges liquidity.

There are operational costs involved in both borrowing from a bank and issuing bonds. However, there are differences between them that can be analyzed as follows. First of all, in the case of bonds, the expenses are decreasing in time.

It is the case in fact that entering the bond market might be rather difficult and expensive, but once the company is in the market, the costs of borrowing capital reduce significantly, especially from an administrative point of view.

The standardization of terms and conditions allow to reduce the expenses after the process for bond issuance has started. Borrowing from a bank on the other hand, implies being subject to the decision of the bank as well as dealing with sometimes uncertain conditions.

The amount of the money to be lent and the maturity of the agreement then become an important decision factor. The link about the corporate asset and liability is of primary importance for economic efficiency.

Syndicated loans are different from bilateral loans in that it is not issued by a single lender, but a group. The borrower is single, and lenders can be banks or any other type of financial institution.

The borrower gives to a leading bank the mandate to promote the loan to a group of potential institutions interested in financing the company. The lead arranger provides probable participants with a memorandum including borrower specific information.

In normal conditions, each lender is responsible for its share of the total loan, with no legal responsibility for the other participants. Syndicated loans lie somewhere between relationship loans and public debt. Sometimes in fact the lead bank may have some form of relationship with the borrower. But this is not likely to happen in case of a participation at a more junior level.

There have been developments in the syndicated loan market in the last decade, that made the distinction between bilateral loans and syndicated loans very clear. During the 90s for example the growth of the regulated secondary market supplied a lot of liquidity to the syndicated loan market.

In most recent years, syndicated loans rated by independent rating agencies have increased a lot, being another major factor for the establishment of a new standard. As proven by Armstrong (2003), the stronger secondary market activity, combined with independently rated syndicated loans, led to the recognition of these assets by institutional investors as an alternative investment to bonds.

The recent developments in volume, capacity to fund medium and long-term ventures, and the higher transparency of the contracts, have pushed the syndicated loan market to converge towards the corporate bond market, and further away from bilateral bank lending.

A long-term loan comes with several advantages. First of all, the company that has to raise the funds has access to a large number of lenders that can be interested in the deal. Moreover, each of these lenders may be willing to accept different terms for the loan.

Competition among lenders can lead to a discount on the interest rate or more flexibility in the repayment, which is something that cannot be obtained in the bond markets, where the conditions of yield and repayment are highly standardized.

Among the disadvantages of loans one should recall that many banks are not keen to issue loans to business at a fixed rate of interest for a long term. The borrower is then left with the only option to pay a floating rate that will fluctuate according to market conditions.

But bonds also have advantages, like the fact that the borrower has great certainty that the payment rate of the bond will not deviate over time. In case of a long-term loan, if the interest rate is floating or adjustable, the cost of the loan may rise suddenly. In addition, the baseline interest rate for bonds is often lower than for loans.

The main disadvantage of issuing bonds is that they have a very low flexibility. A company must issue the bonds according to very strict rules and regulations. Moreover, compared to a long-term loan, the conditions cannot be modified for any reason.

A company cannot generally modify the terms of a bond and the origination costs of a bond are often higher than those of a long-term loan, as the bond must be pitched to investors.

10.3.3 Leasing

A contract of lease sets an agreement between a user, called lessee and an owner, called lessor. The contract gives the user the right to use the asset owned by the lessor, in exchange of a periodic rental payment.

Leasing has some advantages compared to other forms of rental. First of all, the lessee does not commit any financial resources upfront. Assets that can be leased include real estate, durables, equipment and more.

The rental, which sometimes exceeds the purchase price of the asset, can be paid from revenue generated by its use, with a direct impact on the liquidity of the lessee. Lease instalments are exclusively material costs.

After some amount of payment is made, the user has the right to exercise a purchase option, that allows to acquire the leased asset at a convenient final price that represents the residual value of the asset.

In terms of general domestic economy, leasing opens many opportunities to consumers interested in state-of-art technology and expensive products not available otherwise, in that they are very expensive or impossible to acquire by loan arrangements.

There are also disadvantages in the practice of leasing. After all it is an expensive way of financing, in that the lessors want a return on their capital, while protecting themselves from any risk arising in collection of receivables.

Leasing is anyway a common and demanded form of rental, even if controversial under some aspects. For example, in less developed economies, the terms and conditions of the lease may generate inequality, with lessor's economic dominance and lessees signing contracts at unfavorable terms.

Depending on the country, sometimes leasing can also be undesirable because of taxation or for other fiscal reasons, depending on the taxation policy and restrictive legislation of the country.

Companies generally own fixed assets. However, it is the use of buildings and equipment that is important, not their ownership. In order to avoid the burden to raise enough capital to buy the premises, an alternative way to obtain the use of assets is by leasing.

The use of leasing is extensive in the health services industry, because of the high sophistication and cost of some types of equipment, and the extensive use of information technology.

Note that every lease transaction has two parties: the user of the leased asset is called the lessee, while the owner of the property, usually the manufacturer or a leasing company, is called the lessor.

A popular form of leasing is the operating lease, also called service lease. It is a type of leasing providing both financing and maintenance of the asset. This type of lease is typical for computers, office machines, cars, trucks and more.

Operating leases typically require the lessor to maintain and service the leased equipment, with the maintenance costs directly embedded in the lease payment. Operating leases are never fully amortized, and the payments required under the lease contract are not sufficient for the lessor to recover the full cost of the asset.

The lessor overcomes the issue because the length of the leasing is usually much shorter than the operating life of the asset, and the lessor can recover the costs by renewing the lease on it or reselling for an adequate price.

Leases sometimes contain a cancellation clause that allows the user to cancel the contract and return the asset before expiration of the lease. Such a clause helps the lessee in case the item becomes suddenly obsolete due to the development of improved technology.

The lease (rental) payments on operating leases can be structured in two different ways. Under conventional terms, fixed payments are made to the lessor periodically, usually monthly. The cost of the lease is therefore known and both the lessee and the lessor know how much money they are going to pay and gain.

Another type of structure is the procedure terms, under which the user pays a fixed amount each time the asset is used. In this case the cost to the lessee and return to the lessor are not known with some certainty but they depend on volume.

The second form of leasing is the financial lease, also called capital lease, which is generally quite different from operational lease. More specifically they are different under several aspects.

Financial leases do not provide for maintenance and the contracts are not cancelable. Moreover, the length of the lease normally coincides with the life of the asset, therefore being fully amortized.

In a typical financial lease, the lessee selects the item it requires and negotiates the price and delivery terms with the manufacturer. The leasing is then perfected through a lessor buying the equipment from the vendor and leasing it to the user.

The lessor benefits from full amortization of the investment plus a rate of return that is similar to the return to be paid on a secured term loan. A leasing contract is therefore very close to borrowing.

Loan secured agreements involve a lender receiving a series of constant payments comprehensive of capital repayment and interest payment. Lease payments are set up exactly the same way and the payments are just sufficient to return the full purchase price to the lessor plus a stated return on the lessor's investment.

The financial lease terminates with the transfer of ownership of the asset from the lessor to the lessee. In the specific case of a sale and leaseback, the user sells the asset to another party and simultaneously executes an agreement to lease the property back for a stated period under specific terms.

This is a way for the user to receive a quick lump sum payment for the sale of the asset, while then taking the obligation to use the same asset in return of a future series of lease payments to the buyer of the asset.

Nowadays leasing contracts are offered under a wide range of terms, so that the distinction between operating and financial lease is not as meaningful as it was decades ago. Therefore, in practice, leases often do not fit exactly into the operating lease or financial lease category but combine features of both.

The decision between buying and leasing is often based on the effects that taxes can play. For profit businesses, the full amount of each lease payment is a tax-deductible expense provided that the contract is recognized as a genuine lease.

A lease that complies with all of the requirements for taxable businesses is called a guideline, or tax-oriented, lease. In such a contract, the depreciation tax benefits accrue to the lessor and the payments made by the user are fully tax deductible.

If the lease does not meet the guidelines as stated by the regulation in the country where the deal takes place, only the implied interest portion of each lease payment is deductible by the lessee. By being the owner of the leased equipment, however, the lessee obtains the tax depreciation benefits.

In the United States, a tax-exempt lease differs from a conventional lease in that the interest part of the lease periodic payment is not taxable income; therefore it does not generate a tax liability for the lessor.

The rationale for this tax treatment is that the interest paid on most debt financing used by not-for-profit organizations is tax-exempt to the lender, and a lessor is actually a lender.

From the point of view of the financial statement, the lease payments are reported by the user as expense items on the income statement. In the case of a capital lease, the asset is depreciated every year, with the relative expense reported in the income statement as well.

However, under certain circumstances, the leasing payments can be taken off-balance sheet as can be shown by reporting the balance sheets of two hypothetical user companies.

Both parties involved in a lease evaluate it. The lessee on one side calculates the convenience of leasing an asset compared to buying it through financing. On the other hand, the lessor must decide what the lease payments must be to produce a rate of return consistent with the riskiness of the investment.

Among the two points of view, the most interesting is of course that of the lessee. Given the degree of uncertainty related to the choice between leasing and buying, complex decision models have been developed to solve the task.

Without even using these complicated models, a simple analysis combined with savior judgment is enough to prevent entering into agreements that are not interesting for the business.

There are several steps in the evaluation of whether to undertake a lease agreement. For example, the firm needs to acquire the use of new equipment. The focus of the lease analysis is to understand whether to acquire the item by leasing or purchasing.

The next issue is how to finance the acquirement of the item, deciding between excess cash, issue of new equity, and leased. Recall that a lease is comparable to a loan in the sense that the business is required to make a specified series of payments and that failure to meet these payments could result in bankruptcy. It is therefore appropriate to compare leasing to debt financing, rather than equity financing.

Lease analysis can be better illustrated by using an example. Assume company Xmed is active in healthcare and necessitates a new equipment for cardio surgery. The machinery comes at a cost of 100,000 €.

If the machine is purchased, the bank would lend Xmed the needed 100,000 € at a rate of 6% per year, on a 2-year, simple interest loan. Also assume that Xmed could depreciate the entire cost of the machine over 2 years for tax purposes, for an amount of \$50,000 in each year, and residual value zero after the 2 years.

If the tax rate is assumed to be flat at 33%, the annual depreciation produces an amount of tax savings equal to.

$$\text{Tax Savings} = 50,000 \times 0.33 = 16,500\ \text{€}.$$

The alternative to borrowing is leasing, and the company can lease for a yearly payment of

$$\frac{100,000 \times 0.06}{2} = 53,000\ \text{€}$$

In order for a decision to be made, the lease analysis goes through several steps. The first thing to do is to estimate the cash flows associated with borrowing and buying the assets.

Moreover, the cash flows associated to leasing the item must be calculated and a comparison must be made, in order to establish what method of acquisition involves the lower cost. It is good to show the cash flows from the buying option, given by:

	Cash flows if the company purchases (€)		
	Year 0	Year 1	Year 2
Equipment	(100,000)		
Loans	100,000		
Interest expenses		(6000)	(6000)
Tax shield		1980	1980
Principal			(100,000)
Tax savings from depreciation		16,500	16,500
Net cash flow	0	12,480	(87,520)

The cash flow statement shows that cash flows are positive at time zero and year 1 but negative in the second year. The cash flows associated with leasing, on the other hand, are given by.

	Cash flows if the company leases (€)		
	Year 0	Year 1	Year 2
Lease payment		(53,000)	(53,000)
Tax savings		17,490	17,490
Net cash flow		(35,510)	(35,510)

In both cases, a difference is made by how taxation affects the cash flows. Both methods of financing involve tax savings associated with interest expense, depreciation, and lease payments, as appropriate.

It is then possible to compare the stream of cash flows from buying with the stream of cash flows from leasing. To do that, a common discount rate is needed, and the best choice for it is the after-tax cost of debt. For Xmed such a rate is given by

$$\text{Discount rate} = 0.06 \times 0.33 = 4.02\%$$

The discount rate is needed to calculate the present value of cash flows for different financing options. By applying the calculated rate, the present value cost of buying is

$$PV_{\mathrm{B}} = \frac{12,480}{1+0.0402} - \frac{87,520}{(1+0.0402)^2} = -68,888.40$$

and the present value cost of leasing to be

$$PV_{\mathrm{B}} = -\frac{35,510}{1+0.0402} - \frac{35,510}{(1+0.0402)^2} = -66,956$$

In this case leasing is the best choice for Xmed, because it carries the lowest cost in terms of present value, and it is therefore the cheapest way of financing the asset. This simplified example shows the general approach used in lease analysis, and it also illustrates a concept that can simplify the cash flow estimation process.

10.4 Summary

Long-term financing provides the capital for the most expensive and important types of investment the firm can make. Fixed assets are in fact financed with borrowings at longer maturity, to match the potential life of the assets.

Every business starts from scratch and needs initial capital to function. The very first money (seed capital) is usually given by the entrepreneur himself and family or friends. But in order for the production to start, it is usually required that external people put additional capital, in the form of angel investors.

Angel investors are the first external source of funding for a start-up. The support of wealthy investors who are interested in businesses with high growth potential allows the firm to take off and make the first steps.

Venture capital comes after the angels, and it is usually structured in a form of pool of funds, to finance early-stage companies with an already structured idea and plan, but not able to make revenues yet.

Private equity is targeted to more mature types of business, where the financials are already good and show that the firm has started generating profits consistently, being ready for a more important investment.

Once the business has started working, through the intervention of intermediate levels like venture capital and private equity, the time usually comes for the firm to go public through an initial public offering.

Debt financing is the other side of capital structure. There are several types of debt financing, depending on the sources and the nature of the funds raised. Bonds and loans are together the whole of the financing for medium and large corporations.

It is important to understand the difference between loans and bonds in terms of the cost for the firm and the convenience for the management of capital structure.

Sometimes the two sources are mixed in order to achieve an optimal leverage at a convenient cost.

Leasing is another form of debt in that it entails loaning an asset against payment of a monthly fee on the value of it. As oppose as pure debt, leasing may lead to the final acquisition of the asset by the company.

Problems

1. What are angel investors and how can start-ups benefit from their support?
2. What are the main features of start-up firms that make them very attractive to early-stage funding providers?
3. Besides money, what other type of support do start-ups normally need?
4. What are venture capital firms and in what sense do they differ from angel investors?
5. What type of companies are normally targeted by venture capital?
6. Describe the process of obtaining funds from a venture capital.
7. What are the typical steps of venture capital funding for a start-up?
8. What is private equity and how does it differ from the other sources of early-stage funding?
9. What type of companies are normally targeted by private equity?
10. What are the main steps involved in the IPO process?
11. What are the costs involved in an IPO?
12. Explain the role of underwriters in the IPO process.
13. What are the types of debt sources normally available to the firm?
14. What are the differences between raising debt capital through bond issuances and raising debt capital through bank loans?
15. What is leasing and how does it differ from other debt sources?
16. What are the types of leasing available to the firm?

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Working Capital Management

11

After developing the issue of long-term capital management, it is also important to analyze the short-term side of capital management, which focuses primarily on the management of working capital.

The term working capital in fact resembles the group of short-term assets under the spot when it comes to complement long-term financial management and focus on the short-term needs of the company.

Managing current assets, credits, and inventory is a crucial task for the financial managers in that it directly affects the daily operations of the company, with a significant impact on the nature and magnitude of cash flows.

Companies often play with accounting recording, so that most of the sales and purchases are not immediately followed by a corresponding inflow or outflow to repay them.

This is why credits are issued to customers and received from suppliers, and the synchronization of them is important to ensure cash management is always appropriate.

Also inventory management contributes to cost reduction and efficiency.

Depending on the nature of the business, working capital management can be the key issue for success and profitability, and it is of primary importance to have a solid background of the topic.

After studying this chapter, you will be able to answer the following questions, among others:

1. What are the current assets, and what balance sheet entries can be included in the definition?
2. How should cash management be performed, and what are the implications on the profitability of the company?
3. What are account receivables, and how are they handled by the company?
4. What are account payables, and how are they handled by the company?

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5. What is an inventory? How does inventory management impact on the firm's profitability?

The first section is an introduction to cash management with definitions and methodologies. The second section focuses on receivables and payables and their management. The final section deals with the inventory management, and its impact on efficiency and profitability.

11.1 Structure of Working Capital

Learning Outcomes

- Explain what current assets are.
- Learn about the management of cash.
- Understand the significance of short-term financing.

11.1.1 Current Assets and Liabilities

Management of current assets is a crucial task in corporate finance. It involves managing the repositioning of cash flows, as well as managing current assets and liabilities.

Good working capital management (WCM) entails finding the right balance between the need of liquidity for conducting the business and the necessity to not tie too many funds in working capital (WC).

WCM involves managing the balance between a firm's short-term assets and its short-term liabilities, to ensure that the firm is able to continue its operations, with sufficient cash flow to satisfy both maturing short-term debt and upcoming operational expenses.

The topic at the center of WCM is the interaction between current assets and current liabilities. This is an important part of financial management, given the many interesting aspects it has.

The importance relies on different factors. In terms of time, WCM is a demanding task, and the financial manager allocates many hours to it. Moreover, WC represents a big portion of the company's assets.

There is also an aspect of credibility in that WCM has great significance for all firms, but it is very critical for small firms. Finally, the need for working capital is directly related to the firm's growth.

In terms of value, the main distinction is between gross WC (GWC) and net WC (NWC). With GWC, one refers to the company's investment in current assets. Those are defined as the assets that can be easily converted into cash within 12 months.

Items included in the current assets are stocks of raw materials, work in progress, finished goods, trade debtors, prepayments, and cash balances. Net working capital refers to the difference between current assets and current liabilities.

Current liabilities, on the other side of the account, are claims from suppliers and other people having credit toward the company. They include trade credits, accruals, taxation payable, bills payable, dividends payable, and short-term debt.

If WC is positive, it means the company is able to repay its short-term liabilities efficiently. On the other hand, a negative working capital means that the company currently is unable to meet its short-term liabilities.

The distinction in terms of time is between permanent and temporary WC. The permanent working capital is the minimum investment in current assets that is needed in turn around to finance basic activities of the company.

Temporary working capital is a part of the total WC that is required by a business in excess of permanent WC. It is also called variable working capital. The volume of temporary working capital fluctuates from time to time according to the business activities and is therefore financed from short-term sources.

The company runs its daily business by using both fixed assets and current assets. The raw materials are turned into products. Investment in cash and marketable securities are the least productive assets.

Such an asset is in fact not needed for manufacturing nor is it required for creating inventory or selling. It is therefore important to understand why firms hold cash and marketable securities anyway.

The truth is that daily activities of the firm also imply paying salaries, bills, and other duties. For all these expenses, a good cash balance must be maintained, so to speed up the transactions.

The tricky point is to establish how much capital must be locked in cash, therefore taken away from production. Often, firms prepare a cash budget through incorporating the estimates of inflows and outflows to know whether the cash balance would be adequate to meet the transactions. There are several specific reasons for a company to hold cash. It is crucial to analyze them.

One main issue for a company is to be protected from uncertainty about future cash required for transaction purposes. This is due to the possibility of an augment in expenditure or delay in cash collection or inability to source the materials and other supplies on credit foundation.

In order to get protection from all the contingencies, the company must hold additional cash. There is a precautionary balance to be kept in the form of short-term securities that can at least yield a small return. Securities are such that it is possible to market them promptly and collect cash rapidly.

Another motive for cash retention can be defined as speculative. It is possible in fact that sudden opportunities arise in the future. In this case the company must be ready to put cash on these opportunities to exploit them efficiently.

This is a very common issue in the commodity sector, which is characterized by the fluctuations of price for the same commodity. The success of the company heavily depends on the skill to source the material at the right time.

Surplus cash is also used for taking in the excess of other firms. Firms that intend to take advantage on the above counts stay big cash balances with them; however the same are not required either for transactions or as a precaution.

It must be also noted that some business sectors experience high seasonality in sales, and the companies involved experience an excess of cash flows in some specific periods of the accounting year.

The need of cash is therefore unevenly distributed over several points in time, so a cash flow must be addressed in order to not run out of liquidity in times of poor cash flows.

One solution is to repay bank loans in good times while taking on new loans when cash is missing. Also, when it is not clear whether a loan will be approved and available in a convenient lapse of time, the surplus cash is retained and invested in short-term securities.

The cash needs of a company are strictly related to the cash flows it generates, which depend themselves on the type of business. Many factors affect cash flows and can be broadly classified into internal and external factors.

Internal factors are those dependent on the management of working capital components and future growth plan. These factors are determined through the firm and arising out of management decisions.

There are also policies that are related to manufacturing process. The manufacture plans are determined and affect the purchase of raw materials for production, from the raw to the finished product.

Some companies manufacture on inventory, meaning that they sell the product on the market and consequently carry a high volume of material and other inventory in order to ensure smooth manufacture procedure.

There are also expenses that are not directly related to manufacturing. They show a degree of flexibility, so that they are not fixed in terms of time. R&D costs and advertisement are examples of these expenses that are called discretionary.

Requirements for discretionary expenses can be anticipated or sudden. The management policy on sanctioning discretionary expenses has a bearing on the cash flow. Flexible policies allow the expenses after seeing the current cash location, so that the pressure on cash will significantly decrease.

Cash is required in times of expansion and financing of new projects. Besides external sources of capital in the form of equity and debt, some part of the cash is expected to be raised internally.

Internal funding is expected to meet any delay in raising external sources. Companies which follow liberal dividend policy, for example, will put pressure on internal cash flows, since the dividend policy of the firm affects the cash flow.

The repayments of debt involve a cash expense that is in total higher than the nominal amount borrowed. It is therefore mandatory for the firm to have produced additional cash at the end of the operating cycle.

The main causes for the pressure of negative cash flows are due to the mismatch of inflows and outflows. Diversion of short-term funds for extensive-term requirements are another cause for this condition.

Firms that are profit creators periodically generate cash surplus even if they face pressure on cash flows at other times. The surplus cash must then be handled because an excess cash balance is the least productive asset of the firm and therefore should be minimized (Banker et al. 1993).

The common and more convenient choice is to invest in short-term funds that show to be liquid enough and give an even small return. After deciding on investment avenues and products, what is left represents the amount to be financially employed.

The securities chosen for the cash management are usually very liquid government securities. As an alternative, funds are usually also placed in certificates of deposits, commercial paper, and intercorporate deposits.

After liberalization of the economy, money and capital markets have become active, and the volume and diversity in the instruments traded have increased. The advent of money market mutual funds has broadened the scope for surplus cash investment.

Frequent use of this type of cash management through investment is anyway not recommended. The very little return can be in fact heavily offset by the transaction costs involved in frequent use of the investment strategy. Several models like the Baumol, Miller-Orr, and Stone are available to companies and help to choose the right amount to be invested.

Together with the forecast of cash flows and the planning of cash needs, good cash management also needs prompt collection efficiency, particularly speeding up the conversion of cash-in-transit to cash.

In some cases, as an example, check collection is not as fast, given the lack of a centralized electronic interbank system. In these areas of the world a payment by cash may take a long time to be collected, therefore creating a mismatch.

Summing up all the technical times at every step, it may take 2–3 weeks to complete the entire exercise. The buyer in this procedure enjoyed another 2- to 3-week credit, which is described as float.

A solution to the problem, when electronic clearing system is not available, is to force customers to pay by demand draft and send the draft through speed postage vehicles, net of courier charges.

Since customers may not agree to the proposal, not to lose the benefits of having a standing float, the solution is therefore to improve the general collection system of the company.

One way to do that is to target all the delays and assess them. An important section of the overall delay is at the banks' end, therefore making it necessary for the firm to select a bank which provides accelerated clearing facilities.

It would be beneficial to put some pressure on the banks asking them to speed up the clearing by using fast post to cut down the delay arising on explanation of postal transaction.

The sales-over-net-working-capital ratio gives an indication of the relationship between the working capital requirements and the amount of sales. Assuming the relationship exists, forecasts of sales can be used to forecast working capital needs.

One should anyway recall that the relationship between sales and working capital, even if existing, cannot be assumed to be stable. Since budgeted production is based on forecast sales, investment in inventories of raw materials, work in progress, and finished goods must be adapted to the economic cycle.

Determining needs from forecast is quite an efficient way to understand working capital needs of a firm. However, the actual level of the corporate activity may differ from the forecasted one (Bhattacharya 1997).

It is therefore important to review the capital needs regularly in the light of changing levels of activity. The cash conversion cycle also shows where managers should focus their attention if they want to decrease the amount of cash tied up in current assets.

In general, the working capital needs can be reduced by shortening the cash conversion cycle, through a decrease in the inventory days. This can be done by shortening the receivables collection period.

A more effective production planning and targeted outsourcing of some parts of the production process can lead to more efficient production times. Moreover, the amount of inventory within the production process can be reduced by increasing the responsiveness of production methods to change in sales.

A way to reduce the receivables collection period is to give incentives to customers for early payment. Good results can be reached by reducing the credit period to customers, chasing slow payers and improving the assessment of credit-worthiness of clients.

The minimum trade receivables conversion period is likely to be the credit offered by competitors. On the other side, the length of the payables period is not under the firm's control, but it is left to the judgment of the supplier.

11.1.2 Conversion Cycles

The operating cycle (OC) and cash cycle (CC) are important measures of the short-term financing management in the company. Standalone, or compared to the industry average, they give insights on possible gaps in the management of working capital.

The firm's operating cycle defines the length of time between the day inventory is purchased until the money from sales is collected, thus defining the whole timeline of the production and sales process.

By looking at Fig. 11.1, it is clear that the operating cycle is made of three main components, namely, the payables period, the inventory period, and the receivables period.

All periods are measured in days, and they normally consider 365 days in a year as normalizing factor.

One can conclude that the operating cycle is the sum of two components as:

$$\mathrm{OC} = \mathrm{IP} + \mathrm{RP}$$

where

IP is the inventory period
RP is the accounts receivable period.

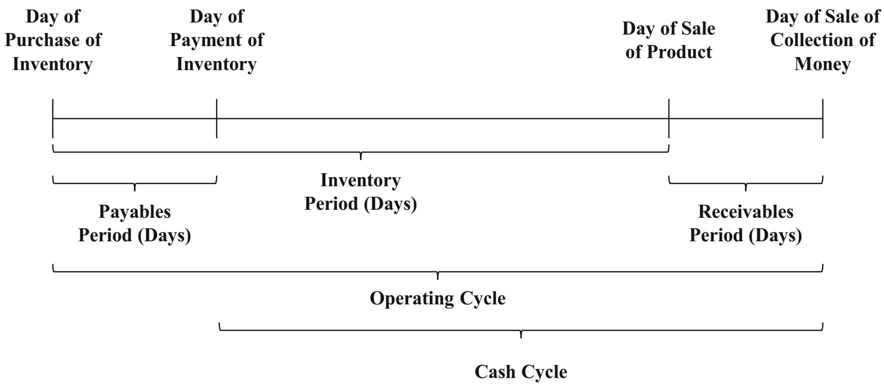

Fig. 11.1 Timeline of the cycles and periods in short-term financing management

Cash cycle instead is made of two components and can be also related to the operating cycle in that it is equal to the latter minus the payables period (days).

$$CC = OC - PP$$

where

PP is the accounts payable period.

All the three components on the timeline can be estimated by looking at the financial statements of the company. In particular it is possible to derive them as ratios of the relevant entries in the statements.

The inventory period (IP), for example, can be calculated by going through the calculation of the inventory turnover (IT) and then by dividing the 365 days by it, as

$$IT = \frac{CS}{AI}$$

where

AI is the average amount of inventory over the period considered (year)
CS is the cost of sales in the period considered.

The inventory turnover represents the number of times that on average the inventory had to be replenished in order to fulfill the overall cost of production in the period of reference (1 year in case we are using 365 as normalization factor).

$$IP = \frac{365}{IT}$$

The above two steps can be summarized as:

$$IP = \frac{AI}{\frac{CS}{365}}$$

The same logic holds for the receivables period, calculated through the receivables turnover as:

$$\mathrm{RP} = \frac{\mathrm{AR}}{\frac{\mathrm{Sales}}{365}}$$

where

AR is the average amount of accounts receivable over the period (year).

Finally, one has to calculate the same for the payables period, as:

$$\mathrm{PP} = \frac{\mathrm{AP}}{\frac{\mathrm{CS}}{365}}$$

where

AP is the average amount of accounts payable over the period (year).

All the information can be combined in an analysis that shows what are the problems in the management of working capital and also suggest the remedies to the gaps with the industry average.

Example 11.1 Consider a company with the following financial statements:

Balance Sheet (€ M) Company AAA					
	2015	*2016*		*2015*	*2016*
Current Assets			*Current Liabilities*		
Cash	1,130	1,330	Accounts Payable	86	102
S.T. Investments	200	250	Accrued Interest	4	8
Accounts Receivable	120	140	Other Current Liab.	230	300
Inventory	180	225	Tot. Current Liab.	320	410
Other Current Assets	40	52			
Tot. Current Assets	1,670	1,997	*Long Term Liabilities*		
			Notes Payable	95	168
Fixed Assets			Mortgages	3,500	3,650
PPE	10,850	11,250	Other L.T. Debt	1,135	1,264
Accum. Depreciation	(1,950)	(2,125)	Tot. L.T. Debt	4,730	5,082
Tot. Fixed Assets	8,900	9,125			
			Equity		
			Stocks	5,300	5,300
			Retained Earnings	220	330
			Tot. Equity	5,520	5,580
Tot. Assets	10,570	11,122	Tot. Liabilities	10,570	11,122

Income statement (€ 000s) Company AAA, 2016	
Sales . . .	1,000,000 . . .
Cost of sales . . .	(650,000)

In order to calculate the operating cycle and cash cycle of the company, the components of the timeline can first be calculated as:

$$IP = \frac{AI}{\frac{CS}{365}} = \frac{AI}{\frac{CS}{365}} = \frac{\frac{180,000,000+225,000,000}{2}}{\frac{650,000,000}{365}} = 114 \text{ days}$$

$$RP = \frac{AR}{\frac{\text{Sales}}{365}} = \frac{\frac{120,000,000+140,000,000}{2}}{\frac{1,000,000,000}{365}} = 47 \text{ days}$$

$$PP = \frac{AP}{\frac{CS}{365}} = \frac{\frac{86,000,000+102,000,000}{2}}{\frac{650,000,000}{365}} = 53 \text{ days}$$

Therefore the operating cycle and cash cycle are:

$$OC = IP + RP = 114 + 47 = 161 \text{ days}$$

$$CC = OC - PP = 161 - 53 = 108 \text{ days}$$

The management of the operating cycle and cash cycle can be done by directly changing the variables that are more under control of the company. Some of the variables involved in fact are not easy to manage in a short time.

So, for example, a company with a very long inventory period can manage the situation by reducing the average amount and age of the inventory in stock. An increase in the inventory turnover in fact may reduce the inventory period and improve financial performance.

On the side of the receivables a company with problems should try to reduce the collection period so to reduce the receivables period in a fairly quick way without losing customers.

On the side of the payables, the opposite obviously holds, and the company should be able to secure contracts with suppliers that allow for a longer payables period in order to improve the situation in terms of inflows and outflows.

For all of the above, a big role is played by the implementation of technology. Modern management systems can help in obtaining the results at the lowest possible cost for the company, thus improving profitability.

This is true for all aspects of working capital management, and the efficient management of current assets can overall reduce the amount of working capital needed to run the production.

11.1.3 Significance of Short-Term Financing

Every company needs an adequate amount of working capital for its operations to run smoothly. Working capital management is very important in that it is the basis of the long-term profitability of the firm.

Efficiency in short-term financing management can help to achieve the corporate overall goal of wealth maximization for shareholders, by assuring an efficient use of the corporate resources.

Shortage or bad management of cash may result in loss of cash discount and loss of reputation due to nonpayment of obligation on due dates. Insufficient inventories may be the main cause of production held up, and it may compel the enterprises to purchase raw materials at unfavorable rates.

Lack of working capital can lead to business failure even if the company has a good balance of assets and liabilities. This is the case when there is not likewise facility of credit sale, which is essential for sales promotions. Working capital works like a buffer in times when things do not go as good as they should, by offering a precious reserve of current assets.

However, excessive working capital would also work against profitability as inadequate working capital and would harm the financial health of the firm. The danger of excessive working capital can be summarized.

Too heavy investment in fixed assets as a consequence of excessive working capital may not be justified by actual sales, thus creating a situation of overcapitalization with deteriorating effects.

Also irrational investments in inventory may result in the stock having a very low turnover and the inventory becoming obsolete. At the same time, it may increase the cost due to mishandling, waste, theft, etc.

Excessive working capital may also enhance speculative tendencies of the business and increase profits and dividend distribution. However, the image of the firm would hamper when speculative loss may start.

Also, due to liberal credit, size of accounts receivables will also increase. Bad debts would then increase as a consequence with wrong practices, regarding delay in payments starting and becoming a major issue. Finally, an excess of working capital will normally lead the management to care less about costs, with obvious adverse effects on the profitability.

There are also consequences when working capital is not enough or even scarce. In this case, for example, it will be difficult to implement operating plans, with the business not achieving its profit targets.

In case of dividend payments due, these would be not covered due to lack of funds. Another fact is that bargaining capacity is reduced in credit purchases, and cash discount could not be availed.

The failure in meeting daily commitment would also have a negative impact on the reputation of the firm, and operating inefficiencies may creep in when a concern cannot meet its financial promises.

The lack of funds for new project would indeed also have negative influence on the growth that may stop. As a consequence, the firm will have to borrow funds at an exorbitant rate of interest in case of need.

Last but not least, an emergency situation may require the firm to reduce the prices of the products so as to boost the sales in order to raise cash for working capital needs, with again negative consequences on reputation.

From the above considerations, it is clear that the objective of good working capital management is to determine the amount of current assets to be held at any point in time by the firm and the mode of financing it.

Working capital funds are the circulating blood of the business organism, in that they are even more important than profits for a firm to survive and operate on competitive markets (Deloof 2003).

If a firm is not earning profit, it may be termed as "sick," but not having working capital may cause its bankruptcy. The alternatives are not pleasant.

The management of current asset has become more important in the recent years, and managers are nowadays called to ensure proper consideration of cash needs, especially in times of funds scarcity.

Well-managed working capital is translated in increased profitability, proper liquidity, and risk reduction. The key for a manager is to understand when to look for working capital funds, how to use them, and how to measure, plan, and control them.

Maintaining working capital is crucial for internal and external purposes. Many decisions related to sales, dividends, expansion, and new products strongly rely on the quality of the maintenance.

There are two main reasons why working capital management is so important, namely, the high proportion of average investment in current assets of companies and the fact that factors like sales can quickly push up or down the level of current assets and liabilities.

There are several differences between managing working capital and fixed assets, even if in both cases, the focus of the analysis is on the impact on profitability and risk. However, fixed assets management and working capital management differ in three important ways.

The first difference is about the use of the timeline on those assets. Fixed assets have a long life, and the discounting and compounding effect on the cash flows they produce is very important. This is not true for current assets, which are usually very limited in life, therefore not requiring particular care in terms of rules of time travel.

The second difference is that large holdings of current assets specially cash strengthen a firm's liquidity position (and reduce risks). However, profits can be overall reduced by excessive current assets. Finally, sales affect both current and fixed assets, but only the former can be adjusted by sales fluctuation in the short term.

The whole theory on working capital is based on the interaction between current assets and current liabilities. Good working capital management is about managing both of them properly and also taking care of the relationship between the two.

The goal of working capital management is to manage a firm's current assets and current liabilities in such a way that a satisfactory level of working capital is maintained.

There is no scientific equation in the establishment of the optimal level of working capital in a firm. This is because many factors influence the need for it, and proper management is an art more than a science. The remaining of this section is meant at describing the various factors that determine the optimum level of current assets.

One important factor is the nature of the business the company is running an important discriminant in how much capital is needed. The need will vary depending on the type of products or services offered.

Consider a company working on public services, for example. Such a business does not require much working capital, as opposed to an industrial company producing high technology devices.

A company offering electric supply has low need of current assets, firstly due to cash nature of the transactions and secondly due to sale of services. On the other hand, it requires high investments in fixed assets.

Another crucial factor in determining current assets expenses is the supply of inventory. If it is adequate and well managed, the update of inventory will require less resources. Investment in working capital will fluctuate in case of seasonal nature of supply of raw materials, spare parts, and stores.

There is also a strong link between working capital and production strategies. In case production is seasonal in fact, the sales will have a cyclical pattern and so will be for the requirement of working capital.

In order to avoid that, the sales department may follow a policy of off-season discount, so that sales and production can be distributed smoothly throughout the year and sharp, variations in working capital requirement are avoided.

The available supply of credit is an important factor that determines the needs of working capital for the business. If credit is easily available at a favorable rate and good terms and conditions, less working capital will be needed.

The problem comes when credit facilities are not so easy to approach, therefore requiring the firm to set more working capital aside in order to avoid unpleasant risks of shortage in the short term.

Credit policy also has an impact on the working capital needs, with different consequences according to whether the sales are paid at cash or at credit, with payments received only after few weeks.

A constant and good inflow of cash has an obvious positive impact on the need of current assets, and less investments in the short term are needed. In order to ensure that unnecessary funds are not tied up in book debts, the enterprise must rationalize its credit policy based on the credit of the customers and other factors.

An important factor that generates need of working capital is the expansion of the business, with the growth of the company and the expanded need for funds to finance an increased amount of sales.

The critical fact in this case is that the growth in business is not antecedent but consequent to the increase of working capital. It is clear that advance planning is essential for a growing concern.

Also the change in price level has an impact, in that in order to keep the same amount of assets, more working capital is needed, if prices are making the task more expensive. The rise in prices will have different effects for different enterprises.

The need for working capital is also connected to the life cycle of the capital itself, meaning the time required to complete one cycle from cash to material, through work in progress and finished products, to cash again.

One should also note that there is a negative relationship between liquidity and profitability. The increase of working capital consequent to the increase of sales, as mentioned above, will reduce risk and profitability on one side and will increase liquidity on the other side (Filbeck and Krueger 2003).

Another interesting relationship is the one that can be established between the riskiness of business and the working capital. The sense of the relationship is that an expansion of the volume of sales will generate more risk for the company.

This is not a direct effect, but it follows from the fact that if working capital is varied relative to sales, the amount of risk that a firm assumes is also varied, and the opportunity for gain or loss is increased.

The basic financial direct relationship between risk and return holds and if the level of working capital goes up, and the amount of risk goes down, and vice-versa, the opportunity for gain is likewise adversely affected.

To conclude there are three principles that defines the recipe for the right amount of working capital to be used in the business. First of all, the principle of equity position states that the working capital investment should be justified by a correspondent amount of increase in the net worth of the company, in the form of equity.

According to the principle of cost of capital instead, it should be remembered that the cost of capital moves inversely with risk. Thus, additional risk capital results in decline in the cost of capital.

Finally, the principle of maturity of payment states that the company should eliminate the mismatch between the maturity of payments and the internally generated cash flows. Some disparity should be there, because a greater return (and risk) is generated with greater disparity. However, a margin of safety should be provided for any short-term debt payment.

11.2 Receivables and Payables

Learning Outcomes

- Learn the rationale behind accounts receivable and accounts payable.
- Define and explain accounts receivable.
- Define and explain accounts payable.

11.2.1 Accounts Receivable

The management of the credits of a company toward customers is a very important task for the profitability and the correct WCM in general. Therefore, one of the first responsibilities of the management is to determine the credit policy of the firm.

The main factor affecting the credit policy is the trade-off between the higher profits on additional sales that arise due to credit being extended on the one hand and the cost of carrying those debtors and bad debt losses on the other.

The development of a credit policy goes through the determination of factors like the credit period, as the amount of net days of credit allowed, and the cash discount, which specified the rate of discount, the cash discount period, and the net credit period.

Another part of the job for the finance manager is to determine how risky it is to allow credit to some particular customer, as well as follow up debtors and decide on a suitable credit collection policy. It is very important to develop credit policies and execute them.

The main issue of maintaining receivables is that it comes at a cost, which is composed by several expenses. First of all, locking funds in receivables implies that the company needs additional funds, with arising opportunity costs and interest expenses.

Maintenance of a good credit policy also involves administrative costs, which include record keeping, investigation of credit worthiness. Also there are costs associated with collection of credits and potential default of the counterparty.

The credit policy determines both the quantity and the quality of accounts receivables. The size of the investment in receivables is determined by several factors that can be listed as:

- Relationship between credit and sales volume.
- The terms of credit.
- Cash discount.
- Policies for credit customers' selection.
- Customer behavior in repaying credits.
- Collection policy of the company.
- Efficiency in the administrative part of credit management.

The last point in the list is very important since the degree of operating efficiency in the billing, record keeping, and adjustment function, and other tasks would also have an impact on the size of the investment in receivables.

All firms decide for different credit policies from the more flexible to the more strict. Flexible credit strategies allow very liberal terms and standards on customers. On the contrary a firm following a stringent credit policy gives credit bases on a highly selective standard, allowing only customers with proper credit worthiness.

The increase in the volume of account receivables translates into high sales but at the cost of additional resources employed to support the strategy. Also the costs of credit investigations and collection efforts and the chances of bad debts increase.

There are also several factors that are under the control of the finance manager. For example, he supervises the administration of credit and contributes to the decisions of top management.

Moreover, there must be a clear list of criteria for selecting customers to allow credit to and a strategy to speed up the conversion of receivables into cash by a possibly aggressive collection policy (Grablowsky 1999).

About the sources of financing, the main categories are pledging and factoring. Pledging refers to the use of a firm's receivable as collateral for a short-term loan. Recall that receivables are the most liquid assets for a company.

In this case the lender analyzes the quality of the receivables owned by the company, and it selects the accounts. A line of credit is then created on them, by fixing it at a percentage of financing receivables which is usually around 50–90%.

The pledging of receivables to secure funds provides high flexibility to the borrower and ensures a continuous financing. The downturn is the pretty high cost of financing due to the costs associated to the loan.

Factoring is another way to finance account receivables. In this case, the company sells its receivables to another company or agency, called factor. The factoring lays down the conditions of the sale in a factoring agreement.

The factor gains the right for credit collection and manages the accounts for a fee. Normally the factoring arrangement involves a nonrecourse clause, so that in case of default, the loss is on the factor. But it is also possible to sign the agreement on recursive basis so that, in such a situation, the accounts receivables will be turned back to the firm by the factor for resolution.

The main advantage of factoring is the speed of conversion into cash and the control on the pattern prediction of cash flows. It provides liquidity without creating net liabilities and preserving financial solidity. Moreover, factoring is a flexible financial tool providing timely funds, efficient record keepings and effective management of the collection process.

Factoring cannot be considered as a loan, in that there is no debt repayment, no balance sheet worsening, and no delays associated with standard funding methods. It is a good way to grant the firm the cash for the growth needs of business.

Recently many innovative tools have been developed to increase effectiveness in accounts receivable management. Modern practices, tools, and methodologies allow for a better handling of the topic.

Some companies have registered a significant improvement in receivables management by simply re-designing their process. Due to the nature of receivables, the decisions made elsewhere in the organization affect the level of resources employed in the management of accounts receivable.

There are several aspects defining the opportunity to improve the management of accounts receivables. Some practices lead to efficient management of receivables and payables.

Centralization of high nature transactions of receivables and payables, for example, leads to improved efficiency. The same works for alternative payment strategies that try to overcome some burdens of traditional payment methods. It is observed that payment of accounts outstanding is likely to be quicker where a number of payment alternatives are made available to customers.

Other alternative methods for payment are convenient to the customers and therefore beneficial to the company. Several provisions are available to speed payment time, add customer service, and reduce remittance processing costs.

Direct debit is a method based on the customer authorization for the transfer of funds to the seller. Another method is the integrated voice response, with a computer-based system allowing customers to make payments over the phone.

Alternatively, the payment can be collected by an authorized external firm. The payments can be made by several means, and banks can be collecting agents of their customers and directly deposit the collections in customers' bank accounts.

Payments can also be done through the Internet platform, or lock box processing, where the latter involves an outsourced partner who captures payment data and forwards them to the client firm for processing in that firm's systems.

Every company should develop a credit management policy to support the maximization of expected profits. The policy should take into account both current and desired cash position and the ability to satisfy expected demand.

The level of optimal receivables is established by looking at the terms prevailing in the company's area of business, compared to the ability of the company to match and deliver those terms of sale.

Another important factor affecting the level of receivables is the pricing policy. There is in fact a trade-off between the price to be set for a good or service and the ease of the payment terms associated to it.

The effectiveness of trade receivables follow-up procedures used will also influence the overall level of receivables and the likelihood of bad debts arising. There are also administrative costs of debt collection to be taken into account.

The ways receivables policy can be implemented also represent a crucial factor, and the cost and effects of giving easier terms of credit should be offset by the benefits to be gained from offering credit to customers against the costs of doing so.

A higher turnover could be generated by longer credit terms. However, the longer terms increase the riskiness of the debt and should be balanced by the increased profits due to the turnover. In order to operate its trade receivables policy, a company needs to set up a credit analysis system, a credit control system, and a trade receivables collection system.

The decision about trading with another company or leaving is usually based on some necessary information. Proper screening of the creditworthiness of new customers is a good way to prevent bad debts (Kieschnick et al. 2006).

The preliminary screening is usually based on information coming from various sources. Sometimes the analysis may involve asking the new customers to produce both bank references and trade references.

Public information like financial statements and annual reports, the status of customers' accounts, as well as the experience matured with similar companies in the past help in forming a fair judgment on the creditworthiness.

For a fee, a report may be obtained from a credit reference agency, such as the US-based companies Experian, Equifax, or Callcredit. A credit report may include a company profile, industry profile, financial ratios and margins, analysis of trading and payment history, types of borrowing, and previous financial problems (if any).

After the terms of credit are agreed, the next issue is for the company to ensure that the client will hold the obligations and keep the terms of trade. Customer accounts should be kept within the agreed credit limit, and credit granted should be reviewed periodically to ensure that it remains appropriate.

The policy toward late payers should be very strict, and no client under no circumstances should obtain additional goods or services if they have exceeded their credit limits.

The risk of debt can be insured through brokers or intermediaries, and the insurance normally covers debts up to some amount, against the risk of nonpayment. Specific account insurance will allow a company to insure key accounts against default and may be used for major customers.

Factoring companies offer a range of services for the administration of sales and the collection of the cash from receivables. A factor can take over the administration of sales invoicing and accounting for a client company.

Factors also collect the amounts due from trade receivables and put pressure on late payers. A factor can offer a cash advance for buying the trade receivables from the client company, which gain access to cash as soon as credit sales are made.

The factoring companies also offer additional services for a fee. For example, they can collect the bad debts that may arise from nonpayment. These debts are acquired with no right of recourse of the factor to the company for compensation in case payments are missed.

A factor will normally advance up to 95% of the face value of invoices and will charge an interest payment on the sum. In exchange for accelerated cash receipts, a company pays an interest, which is comparable to the cost of short-term borrowing.

Additional fees are paid in percent of the annual turnover, and the benefits for the company will be in terms of reduced administrative costs and the access to the factor's expertise in credit management.

Factoring offers a wide range of advantages like prompt payment of suppliers and easy payment discounts. Moreover, it allows the company to reduce the amount of working capital locked in receivables.

Factoring also finances growth through sales, and it allows for the drastic reduction of administrative costs. There are also benefits arising from the factor's experience in credit analysis and credit control.

11.2.2 Accounts Payable

With the term accounts payable, one indicates the amount of money owed by a firm to its suppliers. They appear in the balance sheet as liabilities and are included in the bunch of debt.

When an invoice for payment is issued, the related account payable is registered. Approval for payment therefore makes it officially a debt of the firm. Usually the payable is recorded in the General Ledger.

Most common payables are the trade payables, which are written on physical goods, which are recorded in the inventory. Expenses payables on the other hand include the purchase of goods or services that are expensed.

Generally, the firm gets various offers in terms of payment options for an invoice. The terms may include a cash discount for paying an invoice within a defined number of days.

Example 11.2 2%, Net 30 terms mean that the payer will deduct 2% from the invoice if payment is made within 30 days. If the payment is made on Day 31, then the full amount is paid.

The supplier will ship a product and after that will issue an invoice, with later collection of the payment from the customer. The cash conversion cycle is defined as the time occurring between the payment of raw materials by the firm and the payment to the same firm of the receivables by the final customers (Capon et al. 1990).

Upon reception of an invoice, the purchases match it to the purchase order and the packing slip. After a check that everything is in order, the invoice is sent to payment, in a process called the three-way match. The three-way match can slow down the payment process, so the method may be modified.

In order to prevent the accounts payable staff from engaging in unethical behaviors, there are several checks that the firm can put in place. A common control is segregation of duties, in which a junior employee processes and prints a check, and a senior employee reviews and signs the check.

In most cases the software for processing the payables will not accept operations that overcome the limitations of functions assigned to a specific employee. So there is no way any one employee—even the controller—can singlehandedly make a payment.

In some companies the two functions of entering a new customer in the system and entering vouchers in the system are separated, so that no employee can make himself a vendor and write a check for himself.

The only way to do that in this case would involve colluding with another employee having access to the so-called master vendor file, namely, the repository of all significant information about the company's suppliers. In addition, most companies require a second signature on checks whose amount exceeds a specified threshold.

If a purchase order system is not in place, then an approving manager must be put in charge of approving payments. The staff should be familiar with the common problems related to it.

In accounts payable, in fact, simple mistakes can lead to large overpayment issues. Duplicate invoices are an example of such an issue, and sometimes an invoice may be temporarily misplaced or still in the approval status when the vendor calls to inquire into its payment status.

Audits of account payables can solve many issues related to missed or duplicated payments. Auditors often focus on the existence of approved invoices, expense reports, and other supporting documentation to support checks that were cut.

In order to get a tangible proof of the existence of any account, it is a normal practice to ask for a confirmation or statement from the supplier. It is also common to lose some of the relevant documentation by the time the audit rolls around. An auditor may decide to expand the sample size in such situations.

The work of the auditors involves preparing an aging structure of accounts payable, allowing the understanding of what outstanding debts are in existence over different periods, normally counted on steps of 30 days.

One should note that an account payable for the consumer company corresponds to an account receivable for the supplier. This creates a symmetry between the payables of the customer and the payables of the supplier.

The accounts payable process or function is immensely important since it involves nearly all of a company's payments outside of payroll. The process involves the intervention of a dedicated department or small team of employees depending on the size of the firm.

The mission of the accounts payable team, regardless of the size of the firm, is to make sure only the bills are paid and that all the recorded invoices are accurate. Before a vendor's invoice is entered into the accounting records and scheduled for payment, the invoice must reflect what the company had ordered and received, the proper unit costs, calculations, and other terms.

Internal controls must be put in place in order for the accounts payable process to safeguard a company's cash and other assets. Reasons for internal controls include the prevention of payment of a fraudulent invoice, or in the case of an accurate invoice, paying it twice. Internal controls are also in place to make sure that all vendor invoices are accounted for. Periodically companies should seek professional assistance to improve their internal controls.

Efficiency and accuracy are the keys for the accounts payable process to be successful, with positive impact on the accuracy and completeness of the firm's financial statement as well.

For example, if a repair expense is not recorded in a timely manner the liability will be omitted from the balance sheet, and the repair expense will be omitted from the income statement.

Another consequence of a bad accounts payable process is that discounts for early payments of some bills could be missed. By not paying the invoices when they become due, the commercial relationship with the supplier may also deteriorate.

Just as delays in paying bills can cause problems, so could paying bills too soon. If vendor invoices are paid earlier than necessary, there may not be cash available to pay some other bills by their due dates (Cheatham 1989).

A purchase order is normally prepared to communicate what the company is ordering from a supplier in an ordered and precise manner. It consists in a multi-copy form including all the information about the purchase and distributed to several people.

The employees receiving the purchase order are the person requesting that a purchasing order is issued for the goods or services, the accounts payable department, the receiving department, the vendor, and the person preparing the purchase order.

A receiving report is a document reporting that the good has been received from the supplier. It comes either in soft or hard copy, and it contains all the features of the received good, to be then compared to the information in the purchase order.

After the receiving report and purchase order information are reconciled, they need to be compared to the vendor invoice.

The supplier invoices the firm once the good has been received and controlled. The invoice is normally on credit, thus creating accounts payable. Each vendor invoice is routed to accounts payable for verification and approval.

The amount of the invoice will be then credited to the company's accounts payable account and will also be debited to another account, as an expense, for future registration in the financial statement of the firm.

The accounts payable process is normally exploited through a three-way match technique. The three-way match involves that all documents are in agreement in order for a vendor's invoice to be entered into the accounts payable account and scheduled for payment.

Example 11.3 Assume that a company needs 1000 packs of paper for their printers. The firm issues a purchase order to the supplier at 6 € per pack, for delivery in 7 days. One copy of the purchase order is sent to the supplier, one copy goes to the person requisitioning the cartridges, one copy goes to the receiving department, one copy goes to accounts payable, and one copy is retained by the person preparing the purchase order. When the firm receives the paper, a receiving report is prepared.

The three-way match involves comparing the following information:

- The description, quantity, cost, and terms on the company's purchase order.
- The description and quantity of goods shown on the receiving report.
- The description, quantity, cost, terms, and math on the vendor invoice.

After the match has been verified, it is possible to enter the invoice into the liability account accounts payable. The information entered into the accounting software will include invoice reference information, the amount to be credited to accounts payable, the amounts and accounts to be debited, and the date that the payment is to be made.

11.3 Inventory Management

Learning Outcomes

- Define inventory management.
- Learn the definition and purpose of an inventory.
- Understand the most common inventory models.

11.3.1 Definition and Purpose

Key functions of the corporate activity, like supply chain management, logistics, and inventory, are crucial to the business delivery functions and impact on the ability of the company to produce and deliver efficiently.

Inventory management is therefore an extremely significant function, because it determines the overall quality of the supply chain process and also defines the impacts of the financial health of the balance sheet.

For every company, optimal inventory is important, in that it should meet the necessities of the production, avoiding excesses of, over, or under inventory. In fact, mismatches can have a huge impact on the financial figures.

Inventory is a dynamic bundle for the company, and its management involves constant and cautious consideration of all factors affecting it, and appropriate management through scheduling and reviewing.

Most companies have a distinct department or inventory planners who continuously monitor, manage, and review inventory and interface with manufacture, procurement, and fund departments.

Inventory can be defined as a stock of physical assets with economic value. Items in an inventory are held in custody awaiting packing, processing, transformation, exploit, or sale in a future point of time.

There are usually large amounts of capital that are invested in inventories of raw materials and goods. The production process benefits from raw materials as a buffer to ensure the smoothness of production.

By using the inventories of finished goods, on the other hand, the sales department can satisfy customer demand reasonably quickly so to avoid losses. The optimal inventory level must be determined by comparing the benefits of holding inventory against the cost of it. The costs incurred when holding an inventory go from the insurance, rent, and utility charges to the replacement costs, including the cost of obsolete inventory.

Companies in manufacturing sector, or trading, selling, and servicing products of most kinds, must hold a stock of goods in order to allow production and foster future consumption.

In general, inventory is as necessary as unwanted by companies, since it represents a locking of funds, even if it is for production purposes. Companies hold inventories for several causes, ranging from speculative purposes to functional reasons.

Inventory can be in complete state or partial state. Inventory is held to facilitate future consumption, sale, or further processing/value addition. All inventoried possessions have economic value and can be measured as assets of the institutions.

Inventories may involve raw materials and production consumables as well as semifinished goods, at several stages of production. Both raw materials and finished goods that are in transit are also included in the inventory depending upon who owns the inventory at the passage stage.

There are several stocking points in a plant or factory where raw materials are stocked until they reach the production line. Besides raw materials and finished goods, institutions also hold inventories of spare sections to service the products.

Due to the recent developments of technology and procedures for handling inventory management and supply chain management, there have been revolutionary changes, fostered by the use of computing power.

Optimum inventory management is the goal of every inventory planner. An inventory in excess or under-planned has a financial impact on the health of the business as well as business opportunities.

The need of an inventory is clear if one thinks how companies are exposed and need hedging against internal and external factors and also looks at the inventory as opportunity and a requirement and for speculative purposes.

The plan for manufacturing is not fixed for any company but changes according to factors like sales, orders, and stocking issues. Accordingly, the demand for raw material supply for manufacture varies with the product plan.

One of the reasons why inventory needs to vary in time is the cyclicality of demand and supply, with seasonal dependency over some factors. In this case past sales data help companies to expect a vast surge of demand in the market well in advance.

Another reason for variable stocking is that sometimes it is convenient for companies to buy large stocks of raw materials every now and then, compared to buying small amounts more often. In such cases one buys in bulk and holds inventories at the plant warehouse.

Also expected changes in the price of the good sold can drive the production toward augments or diminishes, as well as the integration of sales in international markets and the impact of taxation.

When taxes and budget variations are expected to have a significant impact on the production costs, the companies tend to buy raw materials in advance and hold stocks as a hedge against increased costs.

Also there are convenience reasons for holding bigger stocks of raw materials than needed, especially when there are offers to buy in bulk to take advantage of the quantity discounts offered through the supplier. Of course this is convenient only if the discount enjoyed is higher than the inventory carrying cost.

If raw materials are imported from a foreign market, or if the distance from the vendor is substantial even if in domestic market, there are lots of savings related to buying in bulk to reduce transportation costs.

There could be a lot of factors resulting in shipping delays and transportation too, which can hamper the supply chain forcing companies to hold safety stock of raw material inventories.

Besides raw material and unfinished products, companies also hold finished goods inventory, from the time they leave the manufacturing process till the time they reach the final consumer.

During that time the inventories are either held through the company or given to intermediaries associated with the sales channels such as trading partners, selling points, distributors and dealers, etc.

The costs involved in inventory management range from the inventory procurement to the actual storage and the management of the stock. Inventory costs are simply categorized into various headings including ordering, carrying shortage, logistics, etc.

All elements of an inventory carry a value, and such a value must be recorded independently of the location of the inventory or its accessibility. To make it clear, not that even scrap has residual economic value attached to it.

Inventory holding policies differ from one company to another, with some holding inventories extremely high in value and some other holding inventories extremely high in volumes and number of units.

Inventories can occur both on site manufacturing plants or third party warehouse sites, depending on the demand for raw materials. Inventory controllers are in charge of managing inventory, by taking care of the many critical regions.

The inventory controllers must make sure that the minimum inventory is always in place, and also try to uphold the optimum quantity of raw materials, by determining the schedules for replenishment.

They also have to ensure that the balance of the inventory is granted all the time, especially in order to avoid excess inventory or lower inventory, which can result in damage to the business.

In the ABC classification of inventory, the logic is to use a Pareto principle in order to classify the stock based on the value of the elements that compose it. For example, when applying a 75/25 principle, the classification could be such that:

- Category a includes 25% of units and contributes to 75% of the cost.
- Category B includes 33% of units and contributes to 18% of the cost.
- Category C includes 42% of units and contributes to 7% of the cost.

There are several advantages in implementing this type of classification. First of all, it gives full control on the volume and allows to assign comparative priority to the right category.

Normally category A includes high value items, so the categorization allows to identify the related stocks as high value things, therefore assigned highest priority and the highest frequency of checks.

The prioritization then follows with category B which can be given second priority with lesser frequency of review and less tight controls with adequate documentation, audit controls in lay. Finally, category C can be supervised with vital and easy records.

The main disadvantage of the classification is to not reflect the actual frequency of movement of items in the inventory. This can sometimes mislead controllers, especially because lower categories might be neglected and left piling in huge stocks exposed to risks of all kind.

An inventory serves mainly five different types of use. The decoupling function is to decouple the manufacturing processes within the firm. Stored inventories may act as a buffer between two stages of production that depend on each other, so that delays in the previous stage would not generate delays in the following stage and the production as a whole.

Another important use is storing of resources, which is particularly important for the industry sectors that produce only in determined times of the year, with a demand of products that span over the whole year.

In these cases, inventory can be used to store these resources. In a manufacturing process, raw materials can be stored by themselves, as work in process or as finished products.

Irregular supply and demand also represent an issue to be solved through holding of inventory. When there are irregularities in the demand-supply relationship on the markets, storing certain amounts in inventory can be important.

Another use of inventory is to take advantage of quantity discounts. Many suppliers offer discounts for large orders, and purchasing in larger quantities can substantially reduce the cost of products.

Large quantities come at some costs, and there are disadvantages in holding a lot of inventory stocks. There will be higher storage costs and higher costs due to spoilage, damaged stock, theft, insurance, and so on.

Inventory is also used to avoid shortages or stockouts, given that it is not good for any company to go out of stock. The customers in that case would satisfy their needs through other suppliers.

11.3.2 Inventory Models

Depletion of inventory items, with subsequent replacement by the arrival of new stocks, is one of the most common situations faced when managing inventory in a company. There are several models for managing inventories. The purpose of this section is to look at the deterministic ones.

A simple model representing the situation typically faced in the life of an inventory is the economic order quantity (EOQ) model. The model assumes that units of the product under consideration are withdrawn from inventory continuously at a known constant rate.

Another assumption is that the inventory is replenished by purchasing or producing new items, when there is a need for ordering. A fixed stock of Q units is purchased, and all Q units arrive simultaneously at the desired time.

The costs involved in the model are the setup cost K for ordering one stock, the single unit cost c, and the cost h for holding each unit in the inventory per unit of time. The objective is to determine when and by how much to replenish inventory so as to minimize the sum of these costs per unit time.

The inventory is replenished on a continuous basis, when the level gets low enough. Shortages are not allowed, and the inventory is replenished when it goes to zero, so as to minimize the holding costs.

The pattern of the inventory is such that it starts with the level Q at time zero, to progressively reduce to a level zero after one unit of time. A new stock is then ordered, and the inventory stock goes back to Q, and then cyclically to zero again, and so on.

The process is repeated each time the inventory level drops back down to zero.

The model calculates an optimum order size by balancing the costs of holding inventory against the costs of ordering new supplies. A minimum cost procurement policy can be set based on the optimum order size.

Recall that the model assumes that, for the period under consideration, costs and demand are constant and known with certainty. The model is deterministic in that it makes these steady-state assumptions.

By assuming a constant for inventory, the holding costs are proportional to the quantity in stock, while the ordering costs decrease as order quantity increases, and the number of orders falls. The total cost is the sum of the annual holding cost and the annual ordering cost.

$$C_{\mathrm{TOT}} = C_{\mathrm{PCH}} + C_{\mathrm{AH}} + C_{\mathrm{AO}}$$

where:

C_{PCH} is the purchase cost.
C_{TOT} is the total annual cost.
C_{AH} is the annual holding cost.
C_{AO} is the annual ordering cost.

The addends in the formula can be decomposed by considering that the purchase cost is given by demand multiplied by price, as defined by

$$C_{\mathrm{PCH}} = D_{\mathrm{ANN}} c$$

where:

D_{ANN} is the annual demand.

Moreover it is possible to decompose the annual holding cost by considering that it is equal to the product between the average quantities in stock, multiplied by the annual holding cost, as

$$C_{\mathrm{AH}} = \frac{Q}{2} h$$

Finally, the annual ordering cost is equal to the fixed cost of the order multiplied by how many times per year the orders are placed, as

$$C_{\mathrm{AO}} = \frac{D_{\mathrm{ANN}}}{Q} K$$

The annual holding cost is the average inventory level in units multiplied by the holding cost per unit per year. The annual ordering cost is the number of orders per year multiplied by the ordering cost per order.

Consider

First order condition of the total cost equation with respect to Q shows that

$$0 = -\frac{D_{\mathrm{ANN}}}{Q^2}K + \frac{h}{2}$$

The point where the holding costs and ordering costs are the same corresponds to the minimum total costs. By putting holding costs equal to ordering costs, and rearranging, one obtains

$$\widehat{Q} = \sqrt{2K\frac{D_{\mathrm{ANN}}}{h}}$$

which is now the economic order quantity, i.e., the order quantity which minimizes the sum of holding costs and ordering costs.

The equation is the EOQ model, one of the simplest and most commonly used inventory management models. Other models have been developed to relax some of the classical model's assumptions.

The lead time is defined as the time between ordering and delivery. When both ordering and delivery are assumed to be constant, new stock will be ordered when the inventory falls to the demand level during the lead time.

Example 11.4 Assume that the demand for some item is of 100,000 units per year and the lead time for delivery of an order is 3 weeks on average, the amount used during the lead time is

$$100,000\left(\frac{3}{52}\right) = 5769 \text{ units}$$

When the level of the inventory in use falls to 5769 units, new stock will be ordered to replenish.

In case of stochastic demand or lead times, a firm may choose to hold buffer inventory to reduce or eliminate the possibility of running out of inventory. The EOQ model can still be used in this framework to determine the optimum order size.

By indicating with BQ the size of the order placed, considering the average annual demand, in the new framework, new orders are placed when the inventory falls to the OR level.

Recall that this is possible because lead time is known and is equal to ab. The company can meet unexpected demand during the lead time from the buffer inventory held, and the average inventory level is given by

$$I_{\mathrm{AVG}} = I_{\mathrm{BFF}} + \frac{Q_{\mathrm{RO}}}{2}$$

where:

I_{BFF} is the buffer inventory.
Q_{RO} is the regular order quantity.

Another popular approach to inventory management is the just-in-time (JIT) approach. The main purpose of the model is to minimize or eliminates the time which elapses between the delivery and the use of inventory.

The main requirement for such a policy to be viable is a strong relationship between the supplier and the company. It applies to a wide range of business operations for the purchase of raw materials and components.

The margin of inventory is so thin that the company must be assured on both quality and reliability of delivery from the supplier in order to avoid disruptions to the production process.

The advantage for the supplier is the possibility to benefit from long-term purchase agreements. Companies adopting JIT inventory management policy, in fact, tend to stick to suppliers who are able to offer goods at the required quality and time.

The purchaser will benefit from a reduction in the costs of holding, ordering, and handling inventory since materials will move directly from reception to the production line.

The minimization of inventory aimed to in the JIT approach can be obtained also by changing the production process in order to reduce the queues of work in progress and the size of production batches. A JIT strategy is successful only when the production planning is efficient.

11.4 Cash and Liquidity Management

Learning Outcomes

- Understand the importance of cash management.
- Learn how companies balance inflows and outflows.
- Understand the importance of float management.

11.4.1 The Management of Cash

The goal of cash management is to optimize the quantity of cash available to the treasury of the firm. The process involves maximizing the interest earned by spare funds not required immediately and reducing losses caused by delays in the transmission of funds.

The company faces a trade-off between holding cash for liquidity purposes and employing that amount of funds for productive uses. It is therefore important to assess the right amount of cash needed by the firm for daily needs (Bragg 2017).

Too much cash set aside results in missed investment opportunities, and too small cash balances will increase the risk of being unable to meet debts as they fall due, so an optimum cash balance should be found.

Every company holds cash for basically three main reasons. First of all, there is a transaction motive, for which companies need a cash reserve in order to balance short-term cash inflows and outflows since these are not perfectly matched.

The estimation of cash reserves can be done by forecasting cash inflows and outflows and by preparing cash budgets. Cash is set aside for both daily liquidity needs and anticipated cash outflows.

There is also a precautionary motive behind the need of setting cash aside. The forecasts of future cash flows are subject to uncertainty so that unexpected demands for cash may arise.

Companies building cash reserves for speculative motive want to take advantage of investment opportunities that may suddenly arise. Such type of opportunities involves mergers and takeovers.

Cash reserves that are not seen as needed for new investment opportunities, therefore lacking a speculative motive, may be then distributed to shareholders either as dividends or share repurchases.

The several motives and reasons a company has to hold cash define how difficult it is to determine the optimum cash level to be held, which varies both over time and between companies.

Factors determining the need of cash, other than the above mentioned motives, include the cash inflows and outflows forecast, the efficiency in managing them, the liquidity of the firm's assets, the debt capacity, and the company's risk aversion.

Cash flow problems for the firm may happen for a variety of reasons. Experiencing losses on a regular basis, for example, can lead to severe cash flow problems, to the extent of liquidation or hostile acquisition.

The growth process of a firm requires investment in long-term fixed assets and working capital. When there is no prompt availability of the funds needed for this investment, the cash flows can be compromised.

Seasonal businesses are particularly exposed to cash flow imbalances, therefore requiring accurate cash flow management. The control in this case must take care of the cyclicality of cash flow patterns.

Cash flow problems may also arise due to consistent lump-sum items of expenditure, such as redemption of debt or investment in noncurrent assets. One way to limit the impact of debt redemption is through sinking fund provision.

Recall that the sinking fund is a pool of money in which regular contributions of cash and accumulated interest combine to produce the required lump sum, although refinancing with new debt is more common.

There are several possible remedies available against shortages of cash when needed. Postponing unnecessary capital expenditures is one way to free up cash to be used for treasury purposes.

The cut in expenditures will normally accelerate the rate at which cash flows into the business. This can be also achieved by offering discounts for early payment to customers, therefore cutting the receivables period.

Another possibility is to collect cash by selling investments or projects that were financed by some excess of cash available at the time of investment. Finally, a company may be able to identify ways to reduce or postpone cash outflows (payments).

Another way to reduce cash outflows is to delay or undo a dividend payment decision. Dividends should be paid out when the excess of cash flows in the firm is sufficient to justify that. Missed payment may in fact send a negative signal to the market.

Budgeting is the key to successful cash management. Through budgets it is possible to show expected inflows and outflows over a period, in order to highlight expected cash surpluses and deficits.

Through careful cash budgeting, managers gather information about how to plan borrowing and investment, facilitating the control of expenditure. With the help of computers, analysts can analyze scenarios and anticipate possible cash flow difficulties as well as examine possible future scenarios.

It is important to continuously update the budget in order for it to be useful. Comparison of estimated figures with the current ones allows to spot significant variations from the benchmark and plan for opportune adjustments.

Correct management of cash flows is a challenging task, involving prompt collection of credits from clients, with subsequent quick banking of the proceedings. Prompt banking will either reduce the interest charged on an outstanding overdraft or increase the interest earned on cash deposits.

On the liability side, payables should be fully exploited and all the time given by suppliers should be used, so as to delay the need of liquidity as much as possible. The benefit from such a strategy should of course be greater than the benefit of taking any early payment discounts available (Pass and Pike 1984).

Another important factor to consider is the float, which is the time period between the initiation of a payment and the record of the cash in the firm's bank account. The float can be several days and consists of some delays.

There is a transmission delay for passing the payment side to side and a lodgment delay for banking the payment. On top of that clearance also takes some time. Delay can be minimized by using electronic payment systems. Good cash management will aim to keep the float to a minimum.

Another important point is how to invest the surplus cash, not immediately needed. The cash should be invested in assets with riskless profile and short-term maturity. This is needed to support a company's continuing working capital needs.

It is therefore very important to diversify the portfolio of investment, by depositing the amounts into different banks. Several factors affect the choice of the appropriate investment method.

First of all, the size of the surplus is very important, as some investment methods have minimum amounts. Another important factor is the ease of realization of an investment, as well as its expected maturity.

Factors like risk and return are of course very important as well, as are the penalties which may be incurred for early liquidation. One should not forget in fact that the purpose of investing cash in short-term asset is for the funds to be promptly disengaged if needed.

The range of instruments available for short-term investment of cash goes from money market deposits to certificate of deposit, treasury bills, and others. It is important to get quotations about every single investment opportunity before making a choice.

Cash can be put on term deposit with a bank to earn interest, with the interest rate depending on the size of the deposit, its maturity, and the notice required for withdrawals. Money market deposits on the other hand are indicated for companies that can predict their cash flow needs with a high degree of certainty.

In most countries certificates of deposit are negotiable bearer securities issued by banks and building societies. At maturity, the holder of a certificate of deposit is entitled to receive both principal and interest.

They can be sold before maturity, which makes them a highly liquid form of investment. As opposed to term deposits, they can be useful if a company's cash flows are not predictable enough for a money market deposit to be made.

The traditional view of working capital is basically static and is defined as the difference between current assets and current liabilities. An alternative view of working capital is dynamic.

Such a view defines working capital as an equilibrium between the income-generating and resource-purchasing activities of a company (Pass and Pike 1984), in which case it is closely linked to the cash conversion cycle.

11.4.2 Float Management and Collection

The difference between the cash amount a corporate has on balance sheet and the amount it has in the bank accounts is called Float, in its more general definition, and comes in different types.

When a firm writes a check, the cash deduction is immediately recorded in the books, but not as immediately the money is deducted from the bank account. This situation is defined as disbursement float (Ross et al. 2016).

The opposite case holds when a bank receives a check which increases the amount of cash in the book, but does not immediately increase the amount of cash in the bank accounts. This is called collection float.

$$F = C_{\text{bank}} - C_{\text{book}}$$

where:

C_{bank} is the cash in the bank accounts.
C_{book} is the cash in the books.

Items affecting both disbursement and collection float include mailing time, processing delay, and availability delay. Float is then created when a company receives or writes a check that is then processed through several steps that are summarized in Fig. 11.2.

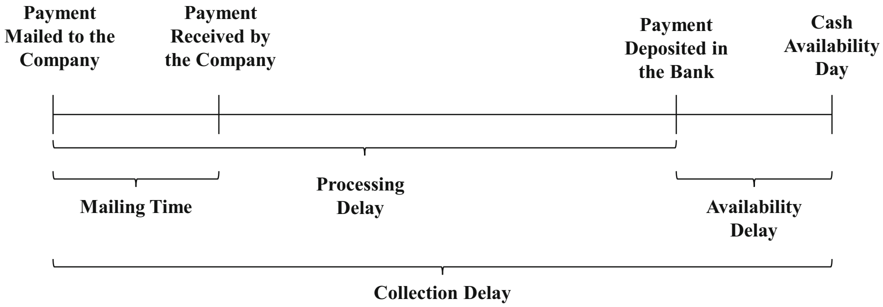

Fig. 11.2 Timeline of cash collection

Once both collection float and disbursement float are calculated for a company, the net float is simply the difference between the former and the latter and gives the measure of the net position of the company on unused cash.

$$F_{\text{net}} = F_{\text{dis}} + F_{\text{cll}}$$

where:

F_{dis} is the disbursement float.
F_{cll} is the collection float.

Example 11.5 Consider a company buying inventory and paying with a check of 50,000 €. The company book balance will be immediately reduced by the amount of 50,000 €, while the bank account will not. If the company starts with a balance of 200,000 € in both the book and the bank, the float scheme looks like:

	Bank balance (€)	Book balance (€)	Float (€)
Initial	200,000	200,000	200,000 − 200,000 = 0
After issuing the cheque	200,000	200,000 − 50,000 = 150,000	200,000 − 150,000 = 50,000

Example 11.6 Consider the same company as in Example 11.5, selling products and being paid with a check of 50,000 €. The company book balance will immediately increase by the amount of 75,000 €, while the bank account will not. If the company starts with a balance of 200,000 € in both the book and the bank, the float scheme looks like:

	Bank balance (€)	Book balance (€)	Float (€)
Initial	200,000	200,000	200,000 – 200,000 = 0
After issuing the cheque	200,000	200,000 + 75,000 = 275,000	200,000 – 275,000 = −75,000

Example 11.7 Consider the same company as in Examples 11.5 and 11.6. Given the collection float and the disbursement float calculated before, we can calculate the net float as:

$$F_{\mathrm{net}} = F_{\mathrm{dis}} + F_{\mathrm{cll}} = 50,000 - 75,000 = -25,000 \ €$$

The size of the float depends on the amount of cash involved in the transactions, and the days that occur between the book change and the bank account change. Float can be measured as a daily average, as:

$$F_{\mathrm{AD}} = \frac{F_{€,i}D_i + F_{€,j}D_j + \cdots + F_{€,n}D_n}{D_i + D_j + \cdots + D_n}$$

where:

$F_{€,\ i}$ is the monetary amount of cash of float i.
D_i is the number of days between book change and bank account change of float i.

Example 11.8 A company has a collection float of 2000 € outstanding for 3 days and will have a float of 1500 € outstanding for the following 5 days. The average daily collection float is then

$$\frac{2000 \times 3 + 1500 \times 5}{3 + 5} = \frac{13,500}{8} = 1,687.50 \ €$$

Example 11.9 A company has a disbursement float of 2500 € outstanding for 3 days and will have a float of 2000 € outstanding for the following 4 days. The average daily disbursement float is then

$$\frac{2500 \times 3 + 2000 \times 4}{3 + 4} = \frac{15,500}{7} = 2,214.29 \ €$$

Collection float is very important to the company, as it represents the amount of cash currently not available for other investments. The delayed collection of float results in an opportunity cost for not having the chance to use the cash, due to temporary unavailability. Cash can in fact generate returns if properly invested or even be invested in new projects.

In modern times the issue of float management has been relieved by the use of electronic banking, transfers, payments, and other means that speed up the process of cash collection, compared to the past.

The collection time is normally split into several stagers. The first moment is the mailing time, when the checks from customers are delivered. This time depends on the speed the postal system can deliver the checks.

The company is then responsible of the processing delay, which involves the time needed to process the payment and make it ready for collection. The bank finally is responsible of the availability delay, due to the time of processing the check in the banking system.

As mentioned above, the float represents an opportunity cost for the company. For a company having daily checks received of some amount C and average delay of n days, the average daily float will be nC.

The same amount represents the benefits that the company can get if the float is eliminated immediately. Reducing the average daily float to one day can bring a benefit of C.

The cash collection, as mentioned above, is made of three distinct components that should be handled to ensure the shortest possible collection time. Some companies even choose to outsource cash collection.

Location is one of the factors affecting collection in that the company may decide to have all checks sent to the same location or to different ones, based on criteria like business division, geographic location of subsidiaries, etc.

A very common approach to collection that is becoming more and more popular is the preauthorized payment arrangement. Dates and amounts are fixed beforehand, and the bank takes care of the transfer to be settled on the established date, at the right amount.

Back to location issues, having several different collection points for checks may entail having different banks and bank accounts taking care of the collection of cash into accounts.

Concentration of cash into one single location or few ones can normally speed the collection process by involving a single bank account or few ones. The concentration of cash into a single bank also offers the chance to negotiate better banking conditions on the amount.

Concentration banks are chosen to pool the funds from several local banks that handle a share of the company's payments. Tools allowing cash concentration include:

- Depository transfer checks (DTC), to transfer funds between accounts of the same company.
- Automated clearing house (ACH) transfers, as an electronic alternative to paper checks.

Reducing the float can be done in several ways and comes at a cost. The PV of eliminating the float is equal to the float itself, and therefore it is convenient to reduce it as long as the PV of the associated costs is lower.

Example 11.10 A company has an average daily float of 1,500,000 € and an average delay of 4 days. The total amount of float is then equal to

$$1,500,000 \times 4 = 6,000,000 \ \text{€}$$

Assume the company can reduce the float by 2 days at a cost of 2,500,000 €. The total float reduction will be equal to

$$1,500,000 \times 4 = 3,000,000 \ \text{€}$$

If the cost of reducing the float is lower, that means the company should reduce it and benefit from an NPV of

$$3,000,000 - 2,500,000 = 500,000 \ \text{€}$$

Banks offer lock-box services to directly receive and process the checks from the company's customers, so to speed up the collection and the crediting of the money on the company's bank account.

The service comes at a cost that normally involved a variable cost per number of checks processed and a fixed fee to be paid for the wiring of the amounts to the target account.

Normally the lock boxes are a postal service maintained by local banks, and some companies may even use plenty of lock boxes. Checks are collected many times per day and process them depositing the related amounts in the company's accounts.

Thanks to lock boxes the mailing time is normally reduced due to the direct mailing of the checks to the bank. There is also an electronic version where the customers can use telephone or the Internet to authorize payments of transactions.

Example 11.11 A company has to decide whether to require a lock-box service to process the checks externally. The finance office collected the following information about cash collection:

- Collection is planned to be reduced by 2 days on average.
- Alternative means of investment are available at a daily rate of return of 0.01%.
- The lock-box will receive an average number of 5000 checks per day.
- Each check will be of an average amount of $500.
- The bank charges a processing fee of $0.10 per check and a total fee of $10 to wire the funds.

Based on the information, the average amount of daily collections will be

$$5000 \times 500 = 2,500,000 \ \text{€}$$

A 2-day reduction of the collection period will then correspond to a total benefit of

$$2,500,000 \times 2 = 5,000,000 \text{ €}$$

The cost associated to reducing the float is

$$5000 \times 0.10 = 510 \text{ €}$$

Assuming the deal with the bank has not termination, we can assume the cost to be perpetual, with an associated PV of opportunity cost (given the missed opportunity of investing the above sum at the daily rate of return of 0.01%) of

$$\frac{510}{0.0001} = 5,100,000 \text{ €}$$

The NPV associated with the adoption of the lock-box system is therefore

$$5,000,000 - 5,100,000 = -100,000$$

And the lock-box should not be rented.

11.5 Summary

The management of short-term capital is as important as the long-term capital in that it determines the liquidity of the firm and therefore its ability to promptly pay for its short-term obligations.

Current assets are those with maturity below 1 year and resemble all the components of the capital that relate to the production in a strict sense. Inventory is a part of such a pool, and its management is also very important.

The management of cash is also of primary importance. Keeping a good balance of cash readily available for sudden payments is the key for a business to promptly react to unwanted needs and be successful.

Receivables and payables are in this sense a good term to judge the liquidity of the company and its overall health in terms of the cash flows. It is very important in fact to correctly balance the inflows and the outflows in order to not put the liquidity at risk.

Accounts receivable represent the credit given to the customers when selling products or services not on cash but upon delayed payment. Payables on the other hand represent the same chance taken by the company toward the suppliers.

It is very important that the number of receivable days and the payable days are such that there is no shortage of cash in the firm. Appropriate approaches can be used to make the flow smooth and efficient.

Inventory is a very important part of the current assets. Working capital management strongly relies on accurate management of the inventory, which is done through several types of model.

When the inventory is correctly managed, there will be no delays in the production and no useless storage of extra items that are not needed in a reasonable term, therefore resulting in an inefficient use of resources.

Problems

1. What is the concept of working capital?
2. Define working capital.
3. List out the various kinds of working capital.
4. What is meant by gross working capital?
5. What is net working capital?
6. What is meant by core current assets?
7. What is meant by variable working capital?
8. Explain the concept of operating cycle.
9. What are the stages of operating cycle?
10. Briefly explain the various advantages of working capital.
11. Explain the various disadvantages of working capital.
12. What are the disadvantages of excessive working capital?
13. Describe the various determinants of working capital in a business concern.
14. What are the sources of short-term working capital?
15. Discuss the new trends in financing of working capital by banks.
16. Discuss the various techniques of forecasting working capital of a concern.
17. Explain the procedure for computation of working capital.

Case Study: Conversion Cycles

Shortfin Ltd

The Case

ShortFin Ltd. is a European company active in the production of intermediate goods and services to be distributed to other companies. Established in 1932, the company is florid and runs a very successful business, and even though it reached a considerable size, it is still run as a family business.

Compared to competitors in the industry, the company has been running on lower leverage in the capital structure, which makes it easier for them to face the little interest payments due and also devolve much of the profits to equity.

The company has been historically relying to a buffer of cash that has never been to large, so to avoid opportunity costs due to missed investments, and missed returns earned on the money.

Recently the management of the company has started a thorough rigorous analysis of the company financials to assess whether the company is ready to face the competition in the new millennium.

The business sector is more competitive than decades ago, and it is very important that both the long-term financing and the short-term financing management are up to date and ready to sustain the company's future challenges. Given the nature of its business, the company must take good care of the short-term financing.

The company in fact operates in an industry where profitability is strictly related to the correct management of all current assets. In particular the company shows the following financials (average over 1 year, 365 days, in €):

Net sales: 1,200,000.
Cost of goods sold: 750,000.
Inventory: 250,000.
Receivables: 230,000.
Payables: 210,000.

You are hired by the company and assigned to a team in the finance office. The team is asked to take care of the analysis of the above financials and recommend possible solutions, if any problem is identified.

As a financial manager with years of experience you know that it is very important for a company to be solid and solvent in terms of liquidity, as well as it is important to be long-term solvent.

The focus of your current task is to analyze the short-term financing situation of the firm, and to accomplish your mission you are informed that the average numbers of the competitors (industry) are:

Operating cycle: 175 days.
Cash cycle: 76 days.
Receivables period: 55 days.
Payables period: 100 days.
Inventory period: 120 days.

All the information needed is on the tables, and you want to perform the task in the most efficient way, so to use appropriate measures and ratio, to identify issues, if any, and recommend solutions, as required.

Questions

1. How can you compare the financials of the company with the average in the industry?
2. Identify the source(s) of problem(s), if any.
3. What would you recommend to do in order to fix the problem(s), if any? Justify your answer with calculations.

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Financial Planning 12

Several topics are of primary importance to the firm. Growth management, for example, is crucial for new businesses, starting in the form of start-ups, with the aim of growing in a relatively short time.

The company crosses several stages during its life, from the early existence as a start-up to the maturity stage, eventually followed by the end of business. All these stages are peculiar and demand different plannings.

The intervention of multiple sources of early capital, like angels, venture capital, and private equity, allows newborn firms to access the capital that cannot be granted through ordinary channels, due to the initial stage of the business.

In this view, planning and forecasting has a crucial role in that it ensures the appropriate agenda for the corporation, with a global and medium−/long-term view about the business development.

Good management of all types of business must rely on appropriate methodologies for planning and forecasting, including both standard and advanced models, developed for management perusal.

Planning and forecasting involves both quantitative and qualitative approaches that can be used together to extract the best possible expectation about the future performance of projects and investments.

After studying this chapter, you will be able to answer the following questions, among others:

- What are the features of a start-up, and how does the company look in its early stage?
- What are the types of investors that intervene at the early stage of a company, providing the first sources of capital to the firm?
- How can the life cycle of the firm be described, and how is growth management handled?
- What is the importance of planning and forecasting for a series of projects?
- What type of models are available for planning and forecasting?

A. Corelli, *Analytical Corporate Finance*, Springer Texts in Business and Economics, https://doi.org/10.1007/978-3-319-95762-3_12

The first section of the chapter is dedicated to the growth management of early-stage business, from the start-up to the intervention of early-funding agents. The second section focuses on growth management and the life cycle of the company. The final section aims at introducing the topic of planning and forecasting and its role in investment decision.

12.1 The Birth of a Business

Learning Outcomes

- Define a business start-up and the first stage of a business.
- Define rights for equity purchase.
- Explain how rights can be converted into stocks.

12.1.1 Business Start-Up

It is quite difficult to give a fair valuation of a company in its early stage, due to the absence of history and the relatively high probability that the company will not survive the first stage.

The profitability of a firm is at the basis of its valuation, and the ability to generate cash flows is crucial as well as the uncertainty related to these cash flows. This is not the case of start-ups which in some cases have high values even if they are not generating consistent cash flows (Murray and Vidhan 2003).

It sounds like a contradiction of the paradigm of corporate value, which states profitability is the key to value. However, there are significant differences between a start-up and a more mature company.

A young firm in fact does not have significant investments in land, buildings, or other fixed assets and seems to derive the bulk of its value from intangible assets and human capital.

This is why when valuing young firms, the analysts prefer to abandon tradition valuation models, more focused on the presence of tangible assets, for new methods that can be used to justify investing in young firms.

The world of business is populated by young entrepreneurs trying to implement an idea and turn it into a successful company. In some sense, the entrepreneurs provide the energy for economic growth.

The most vibrant economies in the world are those where these young entrepreneurs actively promote their ideas in order to gather initial capital and support from experienced colleagues and other companies.

As mentioned above, at the beginning of any entrepreneurial adventure, there is an idea. In some cases, there is nothing more than that, and the entrepreneur is convinced that he or she can fill an unfilled need among consumers (Bodie et al. 2005).

In other cases, there is already something more than just the idea on the table, and the idea has been already converted in some product or service, with limited earnings and low visibility.

Still others have moved even further down the road to commercial success and have a market for their product or service, with revenues and the potential, at least, for some profits.

Any investor willing to put money in a start-up will be asking almost immediately about the value of the company. Most deals are lost at this stage, because the entrepreneur is not able to provide such information to the investor.

In most cases they have no answer or even quote a very large number that is not supported by adequate financial figures, thus convincing the investor that they don't even understand basic economics.

Early-stage companies face a very difficult challenge in determining their initial value and supporting it with good arguments. The company at the beginning owns very few assets and serves few customers, making the task even more difficult.

The founders need money but aren't ready to give up majority ownership, yet the investor needs to have ownership quantified to rationalize a traditionally high-risk investment.

The actions of the early-stage entrepreneur should be based on consciousness and realism. There is no point to claim a very high value if nobody then wants to invest in the idea.

According to the major consultancy firms, the rules for successful valuation and investment collection for start-ups can be summarized as a sequence of practical suggestions.

First of all, physical assets must be valued, at a fair value (asset approach). Start-ups generally have only few assets, so it is very important to include all of them in the valuation.

Sometimes founders forget to include all the computer equipment they bought or upgraded to get the business started. From this point forward, the valuation gets more subjective (Holton and Keating 1998).

Very often most of the value of a new business is in the idea and consequently in the intellectual property. Having filed patents and trademarks is a very positive step, even if their value is not certifiable, especially if you are only at the provisional stage.

The patenting of an idea gives an important push to the overall business and raises a lot the interest of the investors. Patents, Internet domain name, and some trademarks give a competitive head start.

At the early stage of a company, as well as at later stages, the value given by professionals and skilled employees is very important. In this respect, it is also important, when asking an investor for the right amount of money, to monetize the unpaid efforts of any executive or founder.

If, for example, the management team has successfully built a company like this before and has deep domain knowledge, a sizeable premium for "strength of the management team" should be added.

Existing customers and contracts also add value to the company and are proofs of the profitability of the business. The contracts not yet signed should be weighted by some probability. Also it is important to highlight any recurring revenues, like subscription fees, that don't have to be sold from scratch each time.

The valuation on cash flows can be done by using the methodologies encountered in the previous chapters. However, for a start-up the best methodology seems to be the DCF approach. The discount rate typically applied to start-ups usually varies between 30 and 60%, depending on maturity and the level of credibility attached to the financial estimates.

The key point is that valuing a business at its start is mostly about the ability to calculate an NPV of the potential future revenues. This is normally done in the form of a projection.

When the start-up is at a more mature stage, an interesting model that can be applied to its valuation is the multiple of discretionary earnings. If the company has at least 2 or 3 years of operations and managed to pass the breakeven point, the model applies.

It is then possible to estimate the company's valuation by just multiplying the EBITDA by some target multiple which can be read on some industry average tables or derived from scoring key factors of the business.

The average investor aims at getting back a return at least ten times their original investment. Roughly speaking, for a promising company, they will estimate a fair future value on a 5-year time span and then divide that figure by ten to arrive at the current valuation.

An effective way to impress any investor is to come prepared with a proper analysis of costs. In particular, when the business is being nonprofitable, such an analysis allows to measure the net value of the business by calculating how much it could cost for a new effort to replace key assets.

The replacement value can be used to estimate the current market value of a company, when the cost approach is implemented. The investors in fact tend to focus on the funds needed for an investment and its potential return (Kotler and Bloom 1975).

As mentioned many times, company valuation is an art more than a science. Also it was shown how sometimes it is so hard to get the right inputs for implementing the standard valuation model. In those cases, it is necessary to look at comparable firms.

The market approach is particularly important for early-stage firms, and it is possible to find online investor sites with the relevant information on recent funding activity. If the relevant information is not publicly available, one should ask a business advisor to query some investor friends.

In case a self-made valuation seems to be very difficult to reach, it is also possible to ask for professional service from valuation consultants who know how to get a feasible price for the company.

The size of the market is also important, in that it affects the growth projections of the business. When both market size and growth projection are high, the firm is obviously worth more.

There is a premium on the valuation price which is called goodwill, and it is usually set for start-ups on a target between $500 million and $1 billion, depending on the level of initial investment required.

Another important aspect of initial valuation is the correct analysis of the market in terms of competition. Competitors and barriers to new entries in the business can affect the potential profitability.

Investors in fact look for a big enough window of opportunity for a company to generate profits. In general, if there are more than three similar competitors, that's a bad sign and will negatively influence the valuation.

On the other hand, if it is possible to show to the investors that the market entails severe barriers to entry for possible competitors of the firm, this will raise the valuation, giving exclusivity to the business.

In summary, it is possible to conclude that start-up valuations always start with real financial data which you should be ready to provide. The analysis then extends into many subjective areas.

Most of the above approaches are indeed not very useful, if the investor is not keen to perceive the information given by them. In fact, their approach is usually very pragmatic, and most of the nuances will be ignored.

Rather than exact valuation, it is important to give the feeling of the potential of the business. Following all the above points gives a sense of credibility which is crucial besides the acceptance of them by the investor.

12.1.2 Rights

Previous chapters showed how the firm can obtain equity financing through multiple channels, once the start-up has come to a more mature stage. It is now important to also understand the meaning and use of rights issuance.

The issue of new shares reduces the proportion of equity held by the existing shareholders. This is why IPOs often contain a provision in the form of a preemptive right in the firm's articles of incorporation.

Such a provision obligates the firm to first offer any new issue of common stock to existing shareholders. In case the preemptive right is not included, the firm has a choice of offering the issue of common stock directly to existing shareholders or to the public.

The direct offering of new shares to existing shareholders is called a rights offering or privileged subscription. The firm issues rights to the shareholders, allowing them to buy a specific amount of new shares at a specified price, within a certain maturity. The terms of the rights offering are evidenced by certificates known as share warrants. The warrants are often traded on securities exchanges or on OTC markets.

The mechanics of the rights offering starts with financial management addressing several points like the price per share of the new issue, the number of shares to be issued, the limit of shares (if any) to be bought for each shareholder, and the likely effect of the rights on the existing shares (McGahan 2004).

The rights offer begins as a general cash offer, with the only difference lying in how shares are sold. Basically, the existing shareholders are informed that they own one right for each share of stock they own.

After that, the firm must specify what the proportion of rights per share is, so to determine how many rights are needed to buy one share. The rights must be exercised by filling out a subscription form and sending it, along with payment, to the institution acting as the firm's subscription agent.

Once the rights offer is made, besides doing nothing and letting the rights expire, the shareholder has two options. It is possible to exercise the rights and buy some of the target shares or place an order on the rights sold.

The right issue is offered to all the shareholders listed in the firm's records and can be fully or partially exercised.

In case rights are transferrable, the subscripted shareholders can sell them separately, either privately or on open market. A rights issue to shareholders is generally made as a tax-free dividend on a right-to-share ratio basis.

It must be underlined that, given that the exercise of rights with related sale of shares to the investor generates a cash inflow for the company, the rights issue can be considered as a source of capital in an organization.

Several issues must be addressed when issuing rights, including the engagement of a broker to manage the process, finding the right subscription price, determining the optimal rights-to-share ratio, and checking the effects of the offer on the current shares.

As for the securities in an IPO, the rights can also be underwritten, with an underwriter taking care of the procedure, to guarantee that the funds sought by the company will be effectively raised.

The rest of the agreement is pretty standardized, and it is set out in a formal underwriting agreement. Typical terms of an underwriting require the underwriter to subscribe for any shares offered but not taken up by shareholders.

The underwriter at some point will have a right to terminate its obligations, if specific events occur. There could also be sub-underwriters that bear the risk of the issuance through a purchase of a portion of the shares (Porter 1980).

The rights often come with additional possibilities attached to them. For example, the so-called oversubscription privilege allows the investors to buy shares beyond the quantity allowed by the rights possessed, if there are still shares available.

The additional shares will be usually limited to a number that is not higher than the actual shares purchased through rights. In case the shares are not enough to fulfill all the oversubscription rights of the investors, additional shares will be distributed on a pro rata basis.

Example 12.1 Consider an investor having 1000 shares of some company for a total investment of 50,000 €. Stock price is therefore 50 €, and assume it did not change between date of purchase and date of rights issuance. If rights give a 1 to 1 subscription right, at an offer price of 25 €, the investor can buy additional 1000 shares of common stock at that price. By exercising the rights, he will pay 25,000 €

to buy the new shares, and the average cost of the total portfolio per share will then be.

$$\frac{(50,000 + 25,000)}{2000} = 37.5 \text{ €}$$

Although the price on the stock markets should reflect a new price of 37.5 €, the investor is actually not making any profit nor any loss. The right price will converge to an equilibrium price so as to match the new stock price. In many cases, the stock purchase right (which acts as an option) can be traded at an exchange.

Example 12.2 From the above example, consider the point of view of the company, which has 10,000,000 shares outstanding. Given the current price of 50 €, the total capitalization is 500,000,000 €.

If all the shareholders of the company choose to exercise their stock option, the company's outstanding shares would increase by 10,000,000, with a new market capitalization (given by the sum of old capitalization plus the cash received from investors exercising the rights) of.

$$500,000,000 + (25 \times 10,000,000) = 750,000,000 \text{ €}$$

The EPS of the investors would then be halved, if the company does not do anything with the raised money. On the other hand, if the equity raised by the company is reinvested, the EPS may be impacted depending upon the outcome of the reinvestment.

From a capital structure point of view, the issuance of rights offsets the dilution caused by the issue of new share. This is why generally the rights offering does not need to be approved by shareholders in case at least the 20% of outstanding shares is offered at a discount.

The investor can however have different reactions to a rights issue, because it sounds like choosing between either investing more cash into the share or being subject to the dilution of the existing shares.

For this reason, rights offering is not very popular among market participants, and it is usually used only as a last resource for the company, in case the investor demand is really insufficient for the company's needs.

There is no tax applicable to exercised rights, and the actual taxation is only on the sale of the security. The cost basis of the shares is "the subscription price plus the tax basis for the exercised rights." The holding period begins at the time of exercise, and if rights expire without being exercised, they don't count as a deductible loss.

12.2 Growth Management

Learning Outcomes

- Understand the life cycle of the firm.
- Define the short-term capital needs of the firm.
- Learn how to develop good short-term financial planning.

12.2.1 The Life Cycle of the Firm

Every company goes through several stages during its existence. Every step of the business life cycle involves different challenges involving different financing needs and sources.

Different strategies can be in fact associated to different possible future operations, for example, market penetration, customer retention, marketing, and business developing, among others.

It is therefore very important to know and understand the many different and consecutive steps that a company will have to make in order to grow and stabilize from a simple start-up to a well-established business.

Everything starts from an idea of an ambitious entrepreneur who thinks his innovation can be of interest for many people and therefore turn into a business. People in this stage want to start a business and like the idea of it, without committing to becoming entrepreneurs yet (Loughran and Ritter 2004).

The first step is to test the business idea, by conducting research on the industry sector of interest, and also grasp the feedback from family, colleagues, and friends. Also contacting industry specialists is a good idea. At this point the entrepreneur is able to determine whether the business idea is worth pursuing, and if so it will be the birth of your new business.

While surveying the market, it is important to test challenges as the idea profitability, the acceptance from the market. Also it is important to work on establishing a good business structure and find appropriate accounting management.

At a following stage, the business is tested, and the entrepreneur decides to start the related business. The work entails building the market and making the offers. There are no customers yet or just a few.

The company is legally formed, and it is an entity with full capacity. During this stage, products or services are tweaked according to the initial feedback from the first paying customers (if any) and market demand.

The business model must be created and shaped so as to guarantee a high profitability and meet the expectations from customers and investors. By adjusting the business model, it is possible to set the business on the right track.

At the growth stage, there is an already developed business plan, and revenues are growing. New customers and clients have been acquired, and the business is now running at a good pace, even if not at full speed yet.

The business line is clear, and there is no more doubt about its feasibility. Once the initial stage has passed, the generated revenues help pay for operating expenses and open up new business opportunities.

In the growth stage, it is not yet clear whether the business is generating net profits or losses. Competition in fact may prolong the breakeven point, especially when tuning is needed on the methodologies, sales and marketing model, and operation model. The main challenge is to deal with increased customers and revenues.

After a while, the growth may become much faster, to establish a phase of expansion for the company. The business is ready to expand and spread its roots into new markets and distribution channels.

The fairly huge volume of sales generates significant cash inflows that help expand the operations in order to capture a larger market share. The rapid growth stage takes advantage from the proven sales model, marketing model, and operations model set forth from your growth/survival stage. Increasing market competition is crucial at this stage (Quinn and Cameron 1983).

A further stage can be defined of cruising and comes when the entrepreneur is finally able to fully unleash the potential of the business, by having achieved the targets of growth set in the previous stage.

The business is now running at full speed, and all the teamwork and support for the daily operation is put in place. All employees and management can now focus on their core competencies, or if they don't, they have a specific plan in place to get those resources.

The final stage is at maturity of the business, when the market has been dominated for several years and no more or little growth is still possible. The option at that stage is therefore to take a step back toward the expansion stage or to think of a possible exit strategy.

Every stage of the business life cycle brings new or pre-existing challenges. Solutions that may have worked for one stage may not work in another stage, which is why you should always adjust your business plan and operations accordingly.

Every stage is characterized by the need of specific sources of capital, to help face the challenges for the business. It is therefore very important to have proper accounting management, in order to constantly monitor the finances.

The monitoring of the financial situation of the company is also a good way to understand at what stage the company currently is. Details about the current situation of the company can help to face current challenges and forecast the upcoming ones.

The entrepreneur can undertake several actions to promote the business. They can speak in public or publish books. Other planned solutions can involve the sale of the brand or franchise, if the old business model is not valid anymore.

After the last stage is initiated, the natural end of the company comes naturally as a final closure. There can be several reasons for the entrepreneur to put an end to the adventure and various possible actions to be undertaken.

For example, there might be a loss of demand due to a competitor unexpected boost in market share, resulting in the loss of revenue due to lost market share. Also, it can happen that a company may lose important workers or managers.

In general, such a series of unexpected downturns can push the business backward of many of the above analyzed stages. It may also hamper their ability to deliver solutions to their customers or clients.

In some cases, the entrepreneur is just interested in a new adventure and decides to sell the business to start another. It is a choice, and some entrepreneurs specialize in multiple businesses. For some others it just happens, for many possible reasons.

Moving through stages is also possible. Sometimes the business is maybe at growth stage, but new opportunities arise, and new funding allows for significant changes that bring the business back to previous stages and start again (Rink and Swan 1979).

Much more difficult is to move forward skipping one or more of the stages, unless a very lucky coincidence happens that brings unexpected resources and visibility to the business and accelerates its progression.

It is hard for a serial entrepreneur to be identified in a stage, given that the companies he is running are probably not synchronized on the same stage. Businesses are at different levels of growth.

The individual businesses may have different needs than the entrepreneur, and it is usually the case that a company brand is disjointed from the entrepreneur's brand. It will then be necessary to separate what the business needs for growth versus what the entrepreneur needs for growth.

12.2.2 Short-Term Financial Planning

Companies need to meet budget and investment goals. This is why they develop short-term financial plans, which have a higher degree of certainty compared to long-term plans. Such plans can be therefore amended as financial and investment goals change.

Short-term cash flows of many companies are heavily hit by the elements of their working capital. These elements generally include raw or finished inventory, debtors, creditors, and cash.

There are cases where the variability in working capital creates large deficits that put the business at risk. Differences in cash cycles and account payables determine the above mentioned discrepancies. Recall that the accounts payable cycle is the time a company takes to pay for its inventories and the cash cycle is the time debtors take to pay for products.

Cash shortages occur for different reasons, like aggressive marketing policies allowing debtors to delay payments. Such a policy could be harmful to the finances of the firm, in some cases (Delurgio 1998).

In case of severe cash shortages, it becomes necessary to proceed to a cash flow forecast, in order to estimate the total cash flow collection and payment during each period, in different scenarios, from the worst case to the best.

The analysis involves knowing the difference between the total collections and total payments to ascertain whether there is a deficit in any quarter of the year. For each cash inflow and outflow item, you must account for all relevant increases and decreases.

The shortage in short-term funds can be covered through other short-term measures. For example, current liabilities can be increased, through negotiation of longer payment terms. Also the sale of certain unwanted assets and discounts to encourage debtors to make quicker payments are useful tools.

The short-term planning and forecasting begins with the estimation of future cash flows (FCF) of the firm. The analysis allows to determine whether a surplus or a deficit of cash will occur in the near future (Dickinson 2005).

Moreover, the management of the firm needs to assess whether the imbalance will be temporary or permanent, in order to determine what the impact on the firm's long-term financial decisions will be. Overall, short-term planning focuses on the cash surplus/deficits that are temporary. Short-term analysis is usually done at quarterly intervals.

One of the factors that affect the short-term cash of a firm is seasonality, defined as the seasonal tendency of cash flows to be high due to high sales in some months and low due to low sales in other months.

Some businesses generate so much cash during the favorable months that they are able to carry them during the bad months, therefore not experiencing any emergency in terms of cash flows.

Seasonality brings up two main issues. First of all, if it is true that the sale price follows the sales proportionally, the same is not true for the costs, which remain stable over time.

Moreover, the net working capital changes are more pronounced. Tracking seasonalities in the firm can bring to light patterns of occurrence giving the firm ample time to secure financial help during a deficit period.

Negative cash flows occur when there are specific circumstances that hit the firm and the cash flows get a temporary negative sign for some reason. The phenomenon relates to seasonality, and it creates temporary short-term cash needs. A common industry example of a negative cash flow shock is if a piece of machinery breaks and must be replaced.

On the other hand, positive cash flow shocks are also possible, and even if it is something desirable, it creates a short-term financing imbalance. A new expansion can create increased revenues but may create deficits beforehand due to increases in marketing expenses and capital expenditures.

Capital markets are not perfect, and they are characterized by frictions like transaction costs and other sources of disequilibrium. Firms can increase their value by adopting a policy that minimizes transaction costs.

According to the matching principle, the firm should finance its short-term cash needs with short-term debt and long-term cash needs with long-term sources of funds. The principle comes into play in the above situation which requires matching the maturities of the assets and liabilities.

The concept of permanent capital indicates the amount of capital a firm must keep investment in short-term assets in order to grant continuity to its daily operations. It is therefore a description of a long-term type of investment.

Following the matching principle, the firm should finance this long-term debt with long-term financing. This long-term financing would have lower transaction costs than short-term sources of funds.

The other type of working capital is the temporary working capital, calculated as the difference between the current level of short-term investments and the permanent working capital needs. It represents short-term needs and should, as the matching principle states, be funded by short-term debt.

Another choice for the firm is to implement aggressive financing policies, by financing almost all the working capital needs through short-term borrowing. Even if this gives an advantage in terms of term structure rates, there are some issues related to it.

The short-term debt savings is in fact mostly offset by the risk that the firm will have to refinance the debt in the future at a higher rate. This results in the rise of the equity cost of capital in order to offset any benefit from the lower borrowing rate.

Such a policy is useful in case the market is not perfect, and there are agency and transaction costs. The short-term debt is in fact less sensitive to the firm's credit quality than the long-term debt.

It follows that the value of the short-term debt is less affected by the actions of the management or the information hitting the market. The funding risk is therefore lower, and the company should be able to refinance itself with external capital.

Opposite to the aggressive financing policy, a firm can also adopt a conservative financing policy. Such a policy implies that no short-term financing is used; it is just used very sparingly to meet peak seasonal needs.

Short-term financing for a firm primarily comes in the form of bank loans, especially when the business is small. Bank loans start with the signing of a promissory note, stating the amount of the loan, the maturity, and the interest rate.

The simplest form of loan is the single, end-of-period payment loan, which requires the firm to pay an interest on the amount borrowed and the principal all together in a lump sum at the end of the borrowing period.

The interest stated can be either a fixed or variable rate. A variable rate is usually compared against the prime rate. The prime rate is the rate banks charge their most creditworthy customers.

Another popular bank loan is the line of credit, which does not represent a proper loan but the possibility for a firm to borrow any amount up to a stated maximum. The firm can then choose to draw upon the limit of credit offered, when needed. Firms use lines of credit to finance seasonal needs. An uncommitted line of credit is an informal agreement that does not legally bind the bank to provide the funds.

In a committed line of credit, there is a binding legal agreement obligating the bank to provide funds to the firm regardless of the financial condition of the latter. There are provisions about restrictions to be fulfilled by the firm (Hopp and Spearman 2001).

A revolving line of credit is a committed line of credit, which a company can use as needed, that is a solid commitment for a longer period of time. A revolving line of credit with no fixed maturity is called evergreen credit.

A bridge loan is a type of short-term loan which is commonly used to cover a gap of funding in the short run, until the firm can arrange for a long-term loan. There is a fixed discount rate, and the borrower must pay the interest in advance.

Loans are always accompanied by some fees that are related to the origination and the stipulation of the agreement. Commitment fees are associated with a committed line of credit increase of the effective cost of the loan to the firm.

A loan origination fee has to be paid for the credit checks and legal fees related to the origination of the loan. The firm pays this fee at loan initiation, which reduces the amount of usable proceeds that the firm receives, resulting in an additional interest charge.

Sometimes the bank requires a compensating balance requirement, to reduce the loan proceeds available to the firm. The firm is assumed to own a percentage of the loan amount in an account at the bank.

Most banks require that this balance be held in a non-interest-baring account, but others allow the balance to be held in an account that pays a small amount of interest to offset pay of the interest expense of the loan.

Commercial paper represents a popular source of short-term funding, in the form of unsecured debt normally used by large firms. It is a normally cheaper source of short-term capital compared to the bank loans.

Direct paper involves the firm selling the security directly to the investors, while the dealer paper is sold to investors in exchange for a spread (fee) for their services. This spread reduces the proceeds that the issuing firm receives, resulting in a higher cost.

Business can get short-term financing with a secured loan, a type of corporate loan in which specific assets are pledged as collateral.

Pledging of account receivables is another option for the firm, where the lender screens the invoices representing credit sales of the borrowing firms. The lender can then decide to accept them as a collateral to the loan and give it.

The amount lent to the firm usually corresponds to some percentage of the value of the accepted invoices. If the borrowing firm's customers default on their bills, the firm is still in debt to the lender (Harrigan and Porter 1983).

Another type of loan is the factoring of accounts receivables in which the firm directly sells its receivables to the lending entity, in exchange of the amount due from customers, who are instructed to then pay to the new creditor.

If the deal is arranged with recourse, the lender can seek payment from the borrower should the customers default on their bills. When the deal is without recourse, the lender's claim on the borrower's assets in the event of a default is limited to only explicitly pledged collateral.

A floating lien is an agreement where the full value of the firm's inventory is used as a collateral for the loan. It represents the riskiest form of loan from the point of view of the lender in that the value of the inventory is reduced as inventory is sold. Therefore, this type of loan bears a higher interest rate than the next two discussed. A trust receipt loan is also pledged by the firm's inventory, but in case of sale of the inventory, the proceedings are forwarded to the lender in repayment of the actual loan.

12.3 Planning and Forecasting

Learning Outcomes

- Learn the basic planning rules.
- Explain how planning are forecasting.
- Define and explain the strategic planning process.

12.3.1 Basic Planning Rules

Planning is an important component of good management and governance. It helps keep the company in line with the needs of the community and contributes to organizational stability and growth.

The firm must establish a basis for monitoring the progress of the business, focusing on results and impact. Good planning allows an organization to look into the future in an orderly and systematic way from a governance perspective.

Through strategic planning, the board sets policies and goals to guide the organization, providing the executive managers with a clear focus for program implementation and agency management.

Many organizations are forced by the investors to set priorities, organize work, and assess progress through an annual program of some kind. Most groups find it practical to define objectives for a 12-month period and to design strategies and programs to meet them.

Organizations can define their vision about the future operations of the business through a detailed plan, so as to determine systematically how to reach targets, understand obstacles, and figure out ways to overcome them (Doyle 2008).

Planning relies on organizational stability, and some confidence in the future of the business is needed. It must be the case that the key staff and its board leadership will continue to be affiliated with the organization, so that they have the time to plan.

In an unstable organization, the management may end up being fully employed in the operations, which means that they would be using every minute to carry out functions required for survival.

The agreement on some kind of planning starts from the consensus about the mission statement and the organizational goals of the company, which are a crucial basis for long-term planning.

Planning is sometimes difficult when the organization is very young or the leadership is new, therefore lacking a sense of the community and the understanding of the broader external environment.

In general, the longer the term of the planning, the more stable the organization should be, with the involvement of an organized, serious effort which takes time and energy.

Long-range planning is defined as the process used by the management of a company to determine the corporate objectives. It gives a view of how the company should look at the end of a specified period of time.

The planning horizon normally ranges from 3 to 5 years, and the period is filled with a series of multi-year goals and objectives, defining what the organization wishes to accomplish.

Goals are defined in terms of program development, tasks, and the timeline for achieving them. Long-range planning is based on forecasts of future environmental and internal conditions and plans how the organization can function within them.

Compared to operational short-term planning, the long-term planning is necessarily less precise and specific. It is developed as a more general framework with much wider objectives.

Operational planning in fact generates a work plan with detailed annual objectives, tasks, methods, timelines, and responsibilities. It is then focused on specific objectives and timelines rather than general strategic planning.

Some differences also exist between long-term planning and strategic planning, with the latter being a comprehensive process stating goals to achieve and how to reach them. Included are measurable goals which are realistic and attainable but also challenging.

Strategic planning implicitly states that future aspects of the business can be created or influenced by the firm. It is an ongoing process of self-examination, done by challenging difficult choices and prioritizing tasks (Hill and Jones 2010).

Strategic planning is therefore different from both short-term planning and long-term planning in that it is a dynamic and evolving process, taking into account the internal and environmental changes that affect the business.

It is a tool for organizing the business based on projections of the desired future, and it is necessary in order to define the road map of an organization from the actual point to a range of years in the future.

The plan must be simple, written, clear, and based on the real current situation and have enough time allowed to give it time to settle. It should not be rushed. Rushing the plan will cause problems.

As a plan, it must resemble all the thoughts and ideas of those who develop it and be flexible and practical, in order to be open to changes when needed. It should follow up after discussions and meetings about the overall structure of the document.

The implementation part is also important, and every unit within the organization should accept the plan, agree to its direction, and implement specific actions. Employees must work with team spirit in order to effectively and efficiently implement a plan.

12.3.2 The Process of Strategic Planning

The strategic planning process can be defined by many different models and involves several action steps. One approach assumes a cooperative effort between board and staff.

A strategic planning committee of board members and staff can be formed, for the effort. The following part of the section describes the several steps involved in strategic planning, along with some suggested approaches for carrying out each step. Frequently, some of the steps occur before the strategic planning retreat. First of all, there must be an agreement on the process, through a board meeting or other formal happening with key employees and external stakeholders.

Such a preliminary meeting aims to develop an understanding of the nature of strategic planning and the potential value added to the firm. Common goals and strategies are set in order to resemble a common vision.

Doing strategic planning comes at a cost, given the amount of resources involved, from staff to time. On top of that, one should consider the opportunity cost of the use of time for productive operations.

It would be quite difficult or unwise to enter into a strategic planning process if the business is in crisis or is financially or organizationally unstable and until the immediate problems and needs have been successfully addressed.

As mentioned above, not all businesses are ready for a long-term type of plan. In some cases, a good short-term plan to drive the company until the point of affording a longer plan could be the solution.

Only when strategic planning seems to be the right solution can the steps involved in implementing be considered. Processes and responsibilities for each step have to be established (Grullon et al. 2002).

It is common in medium–large-size firms to create strategic planning committees or task forces, with selected participants. The members must be committed to the process and willing to employ enough time to the planning effort.

Managers at different levels are usually included in the coordinating group, sometimes together with some representative of the support staff. In some cases, the group may also include representatives of stakeholders or older managers.

The strategic planning process may also be supported by external forces, and the group may decide to hire consultants to help with the preparation of the document. When internal staff is involved, their workload is normally adjusted to take the new task into consideration.

One of the first moves to make is to scan the environment in which the business operates. The relationship between an organization and the external environment it operates in is of crucial importance for the right development of the business.

The scan usually includes identifying and assessing opportunities and threats in the external environment. On top of that, there is an internal component to assess organizational strengths and weaknesses, in a process called SWOT analysis.

Both threats and opportunities must be identified in the target community and the broader environment. This allows to adopt flexibility in the management of the business, so as to face every issue properly.

Issues to be considered include the trends in the community and the political and technological forces. Also demographical and social changes, as well as the economical trends, are important to look at.

The needs of the community must be the focus of a careful observation, given that these needs have to be addressed by new products and services. This process allows to identify potential clients and beneficiaries of the organization's services and advocacy.

It is also important to consider the opportunities and the challenges related to the selection of fund providers, as well as keeping an eye on competitors operating in the same environment, which could compete for funding as well.

The exploration process is normally sorted by using marketing tools like interviews, focus groups, distance surveys, and informal discussions with clients and other community residents.

About the internal part of the analysis, the strengths and weaknesses of the organization must be carefully assessed. In order to do so, a number of components or approaches may be included.

The various aspects of evaluation of the internal performance include the human resources and the methods and practices, together with the outcomes achieved. When hard data are not available, a perceived performance can be partially determined through asking clients and stakeholders.

Identifying the critical factors of success for the business is also very important, even if such a task is normally not included in a strategic plan. It is about forecasting what factors will be important for the future development of the firm.

Factors like the relationship with the outside community, the available resources, and the planned strategies, as well as the managerial structure and the employees' behavior, can make a difference.

The organization might want to review or formalize organizational values and operating principles. Some organizations have written values and principles which guide their decision-making and their ongoing activities.

There are normally key issues arising from an analysis like the one described above. They could be specific strategic issues or issues to be addressed by setting priorities in terms of time and importance.

When there is no overall agreement about what to prioritize, it is useful to explore the issue priorities to identify opportune choices. In order to do that, for example, the board and staff might be asked to identify strategic issues from the environmental scan.

Issues can be many and involve the need to assess specific needs of a part of the community or the community as a whole. There could be also the need to expand the community of reference.

In particular, the choice could be to expand from the neighborhood to the entire city or county. Agreement would be also needed on whether the organization should consider merging with another group.

The first three steps should lead to an overall agreement about the issues or choices to be considered and decisions to be made as part of the strategic planning process.

After those steps, the organization is then ready to develop a strategic planning retreat agenda and schedule a retreat or a series of shorter meetings.

The overall agreement must be there also for the corporate values and mission, and the consensus about the rationale for the organization to exist should be full. The goals and outcomes, as well as contracts and grants, should be clearly defined.

The strategic plan should start from common organizational core values, a community vision, and the clearly stated mission of the company. These values, mission statement, and vision usually appear as an integrating part of the whole strategic planning session to be reviewed by the strategic planning committee or task force before the board's approval.

The vision about the community should lead to the development of a shared vision for the business, by assuming that such a vision may be dependent upon a shared vision of what society should become.

The process goes through defining the perspective of where the company wants to be in a few years and after that defining the strategies to be used to get there. The perspective should involve both broad terms and specific descriptions.

The task can also be assigned to diverse small groups, and the approach is useful, and then the board is illiterate or uncomfortable to speak the same language, being a multiethnic entity.

In this case the groups can pretend they want to be able to communicate the major corporate accomplishments to external people, in a few years from now. They can then share their vision with the full group and use it to develop a shared vision.

In doing so the small groups can put themselves into the shoes of either stakeholders or managers of any level, to develop statements about the company as they would envision in some years. The single visions are then merged into an aggregate one.

The common vision about the organization must be translated into key goals, in the form of statements describing the company. The goals can cover a wide range of categories like program, resources, relationships, and institutional development.

There must also be an agreement about the strategies to use to achieve the goals. Current and new programs must be emphasized, and the strategies should relate to specific goals in the list. The board needs to provide a broad view to guide this effort, while the planning group or staff can do much of the detailed analysis.

One possible approach is to look at the SWOT analysis results and use them to identify changes to be made in the current strategies. This could involve the identification of new strategies as well as changes in the priorities.

It is normal that the planning group presents several possible scenarios to the board, leaving to the top managers the decision about which one to use. Based on the decisions made using these scenarios, strategies will be determined.

The selected strategies should add value to the reach for goals and should be appropriate. Moreover, they should be feasible and acceptable in terms of their fit to the ideas of managers and stakeholders.

There is also a cost-benefit trade-off that must be addressed, and the benefits from applying a strategy should be to justify the costs in time and other resources. Related to timing, it is important that the strategy implementation is done at the right time, given the environmental conditions.

All the above steps lead to the completion of a preliminary draft resembling the results of the newsprint and notes from the retreat, the results of the environmental scan, and other relevant materials. Once a draft has been prepared, the next step can begin.

The implementation begins after all the objectives have been addressed. A specific work plan is needed which recognizes that strategies must reflect current conditions within the organization and its environment.

Detailed objectives are then uncommon to develop in a plan. It is usually the case for 1 or 2 years, and then the planning becomes more dynamic. However, some action plans are needed. Annual program objectives should be time-based and measurable. The annual plan may be a part of the strategic plan or may be an annual addendum to it.

Developing objectives and annual work requires input from both the board and employees. The staff usually takes a major responsibility for program-related goals and objectives once the board has defined goals and objectives related to governance.

The written strategy must be then finalized summarizing the decisions taken in the strategic planning process. The output of each major step should be included, and the format is normally free.

The final step is the monitoring and the eventual modifications of the strategy, due to changes in the external environment or the company itself. Progresses should be regularly monitored, and strategies should be revised if needed.

12.4 Summary

The genesis of a business normally starts from the early-stage start-up status and then continues like explained in previous chapters through several types of investment sources and companies.

On the other side, once the capital is set, there is also the opportunity for investors to benefit from rights issued by the company, in the form of options that allow old and new investors to purchase shares of the capital.

The management of growth of the firm is itself an interesting task, because it covers several stages of the firm's life and implies the understanding of the actual status of a company.

Several stages correspond to several times of the company's life, from birth to maturity, and the related management should be fitting the situation of either strong growth, or steady state, or decline of a business.

Planning is an important component of good management and governance The firm must establish a basis for monitoring the progress of the business, focusing on results and impact.

Several stages are involved in planning and forecasting for firms. The strategic plan should start from common organizational core values, a community vision, and the clearly stated mission of the company.

The process goes through defining the perspective of where the company wants to be in a few years and after that defining the strategies to be used to get there.

Problems

1. What are the facts that affect the valuation of a young firm?
2. What type of assets are the most valuable ones for start-ups?
3. What is the role of stakeholders in the value of a start-up?
4. Describe the typical preemptive provisions contained in an IPO.
5. Describe the process for rights offering.
6. Consider an investor with 10,000 shares of some company for a total investment of 750,000 €. Stock price is therefore 75 €, and assume it did not change between date of purchase and date of rights issuance. If rights give a 1 to 1 subscription right, at an offer price of 37.5, the investor can buy additional 1000 shares of common stock at that price. What will be the average cost of the total portfolio per share after exercising the rights?
7. Consider the point of view of the company in exercise 1, which has 25,000,000 shares outstanding. Given the current price of €75, what is the new capitalization of the firm?
8. Describe the stages in the life of the firm.
9. Explain how short-term financial planning differs from long-term planning.
10. Describe the factors that affect the short-term cash of a firm.
11. What are the various types of loan available to finance short-term corporate cash needs?
12. Explain the features that good planning and forecasting should have.
13. What are the typical factors that affect financial planning?

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International Corporate Finance

13

Globalization provides access to foreign markets where goods and services can be sold in order to improve profitability and diversify the risk associated with operations. It is therefore important to understand the sense of globalization.

The worldwide implementation of standardized policies and the attempt to achieve harmonized legislations led many companies to try the way of internationalization and to expand their business worldwide.

Many companies take the form of multinational corporations, and they expand their operations in many different countries. International trade follows specific rules and habits that are interesting to explore.

The understanding of international financial markets and international corporate finance goes through the basic knowledge of relevant variables and calculations aimed to clarify the impact of internationalization to the real economy.

Foreign exchange risk management and currency management are crucial tasks for companies aiming to enter the global markets and willing to optimize profits when cash flows denominated in several currencies are involved.

After studying this chapter, you will be able to answer the following questions, among others:

- What are multinational corporations, and how do they integrate in international markets?
- How can international trade be defined and explained in light of the globalization of economic and financial markets?
- What are the major variables of interest in the domain of internationalization?
- What are the main features of foreign exchange markets? How can foreign exchange risk be controlled?
- What is international capital budgeting, and how does it differ from domestic capital budgeting?

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The first section of the chapter is dedicated to the internalization of the firm, with insights on the nature of foreign business. The second section focuses on the basic financial variables of interest when dealing with international markets. The final section aims at introducing foreign exchange markets, with a look at the exchange rate risk and international capital budgeting.

13.1 Internationalization

Learning Outcomes

- Understand international trade.
- Define multinational corporations.
- Learn about globalization and the cost of capital.

13.1.1 International Trade

Most of FX trading takes place in US dollars, which is used to quote the price of many other major currencies. For this reason, such a price is called direct or American quote.

Close to that quotation, on another column, the so-called European exchange rate is recorded, defined as the amount of foreign currency per US dollar, therefore being the reciprocal of the American quote.

International corporations have those firms having consistent and significant foreign operations. Also called multinationals, such corporations must consider many financial factors that do not directly affect purely domestic firms.

The factors to be considered are mostly financial and include things such differentials in interest rates, methodology for foreign operations accounting, taxation overseas, and intervention of foreign regulators.

International corporate finance is governed by the same principles as domestic corporate finance. The companies seek to invest in projects with the aim to create value for the shareholders, at the lowest possible cost.

Therefore, the net present value principle holds for both foreign and domestic operations, although it is usually more complicated to apply the NPV rule to foreign investments (Ang and Tsong-Yue 1989).

The main complication of extending basic NPV method to multinational companies is the presence of international foreign exchange markets. These markets enlarge in fact the range of possible capital sources for the corporation, making capital budgeting and financial decisions more elaborated.

From a risk-return perspective, international markets open the possibilities of diversification for the company that can take advantage of them to hold portfolios that have a higher expected return for the same variance or have a lower variance for the same expected return.

In an integrated market, the investors hold diversified portfolio of risky assets. All investors are assumed to be the same, they invest their wealth in the same way, and the country they come from is irrelevant.

The bottom line is that all investors hold the same portfolio of risky assets, so that they hold the world market portfolio. This result resembles the sense of the CAPM for domestic economies.

As a result, the CAPM holds for all integrated markets together rather than on a country-by-country basis since investors hold securities from all these countries.

The new enlarged market must be considered as the global equity market, where the beta coefficient of any security is computed relative to the market portfolio for the global equity market, which we call the world market portfolio.

The consequences of economic and monetary integration of markets, like in Europe for example, can be direct or indirect. Direct effects are those immediately observable as a consequence of the integration, like the reduction of real risk-free rates, for example.

The convergence of inflation and interest rates, crucial for the good implementation of the integration process, has in fact effects on the real rates, implying that the opportunity cost of investing in equity decreased, reducing the cost of equity capital.

The European central banks tied their monetary policies to the German Bundesbank, therefore becoming inflation averse and managing to stabilize nominal interest rates and inflation at low levels.

This effect on the cost of capital in European countries, other than Germany, can be approximated by the difference in real short-term interest rates from the German rates prior to the period when a monetary union started appearing plausible.

The immediate consequence of decreasing barriers to trade and implementation of a common currency was the increase of risk sharing among European investors, with subsequent reduction of the risk premium and decrease in the average cost of capital.

The launch of a common currency in Europe did not come as a surprise. Therefore stock prices started reflecting the new scenario already before the implementation of the euro. After all, European stock market integration was a gradual process rather than a one-off event.

Required returns and cost of capital started being hit by the expectations on the elimination of barriers, since 1999, with the effect being as strong as the probability of euro occurring being higher.

Estimates of the total rebalancing of equity portfolios from domestic to pan-European portfolios are in the region of \$1.5 trillion (more than one-third of market capitalization) (Euromoney, August 1998).

13.1.2 Multinational Corporations

The reason for the internationalization of the biggest corporation is to be found in several factors, like the uneven geographical distribution of factor endowments and market failure.

Industrialized economies have experienced the development of big corporate giants, owning an amount of assets far larger than other countries. When the size of the company is such that it is not possible to fully employ the assets in just the domestic market, these firms can only successfully exploit these assets by transferring them across national boundaries within their own organizations rather than by selling their right of use to foreign-based enterprises.

Another wave of size enlargement has focused on the acquisition of foreign assets as a form of supplement to the nationally endowed assets, thereby making the national origins of the corporations less significant.

Manufacturing firms get the most advantages by becoming multinational, compared, for example, to the services industry. The value chain of manufacturing can be in fact divided across locations.

By locating several stages of the production process in areas of the world that guarantee a low cost for production, the overall costs for the corporation decrease. The same works for locating research and development in countries with high specialization in the field of research.

On the other hand, most of the value chain for service companies must be generated domestically, and there is little in the way of opportunity to centralize activities to low-cost locations. Moreover, services have to be usually tailored on the needs of each customer, making delocalization strongly unlikely to be advantageous.

There is also an issue of knowledge spreading and sharing among the corporate branches all over the world. While it is easy for a manufacturing company to make knowledge available through patented technologies, in the case of service firms, the transfer must be done through a learning process.

Nevertheless, with the liberalization of recent years, the share of services in foreign direct investments (FDI) has risen significantly particularly within telecommunications, utilities, investment banking, business consulting, accountancy, and legal services.

The advantages of multinationals over domestic companies are evident. First of all, there is a size factor that gives the opportunity to achieve vast economies of scale in manufacturing and product development (Blair 2000).

The higher exposition to several cultures and environments also gives the opportunity of innovative ideas and investment chances, regardless of where they occur. Their worldwide presence also gives to multinationals a bargaining power toward governments that tend to preserve investment and jobs.

But there are also downturns in getting global, and not all the multinationals have historically been successful. The Templeton Global Performance Index (Gestrin 2000) reveals that in 1998 the foreign activities of the biggest multinationals in the world accounted on average for 36% of their assets and 39% of revenues, and they only generated 27% of their profits.

Based on the report, it can be concluded that multinationals can be unsuccessful if foreign activities are not managed properly, in terms of acquisitions and development of competencies.

The decision of becoming an international player must be followed by the choice of how to appropriately organize the foreign activities. There are several types of strategy available for that, which are not mutually exclusive.

Different contexts may require different strategies or a mixture of them. The strategies available to the firm can be divided in non-equity modes and equity modes, depending on whether new equity is generated for the expansion or not.

Non-equity mode includes exporting, licensing, and franchising, all cases where the company can access new markets without making extra equity investments. Also contract manufacturing and provision of services abroad are included.

Equity modes involve the employment of new equity in order to realize the expansion project and include joint ventures with foreign companies and fully owned subsidiaries.

All of the above strategies involve different types of risk. Another difference is the organizational management and resource demand, as well as the amount of control that can be exercised over foreign operations. It is helpful to have a closer look at all of them, in order to understand their role in the internationalization of firms.

Exporting carries relatively low risk, as an entry strategy. The ease of exit in fact is quite friendly, and the up-front investment needed is relatively limited. As such, it is an obvious alternative for firms lacking in capital resources.

The main difficulty in being an exporter is to identify efficient and reliable distribution channels. It is difficult in fact to close a contract with a distributor, when signed, so that if dissatisfied not much can be done. Other critical factors are import tariffs and quotas as well as freight costs.

Also licensing involves a low risk level and a limited investment need. Such an entry strategy is particularly useful in economies regulated so to limit the market entry or having high tariffs and quotas, making it almost impossible to export in that country.

Such a strategy is also appealing when the culture of the target country is very different from the domestic environment of the company. A licensing agreement gives a firm in a host country the right to produce and sell a product for a specified period in return for a fee.

Licensing entails a lack of control over the licensee, making it difficult for the licensing company to keep control of quality standard, with potential danger of a negative impact on the brand in case of mismanagement.

There are also issues involved in monitoring the sales on which the royalties are calculated. There is also the risk that the licensee may steal the technology or competence underlying the product, to become a competitor eventually.

Licensing is therefore indicated for those products or services that are at a mature stage and embed an old technology. In other phases of a product's life cycle, direct ownership is a more viable strategy.

As an entry strategy, franchising is far more comprehensive than licensing. The franchiser gives to the franchisee a complete package, which include trademark, products, and services.

The franchisee pays a fee and royalty payments and also receives a complete set of operating principles, so that in the eyes of the customer, the standardization creates the impression of a worldwide firm (Chari and Henry 2001).

The contract manufacturing and service provision strategy is about distinguishing between functions like design, product development, and marketing, on the one hand, and product manufacturing, on the other.

Several big multinational companies, belonging to industries like textile, electronics, and more, contract out their manufacturing processes to countries like China, Indonesia, Thailand, and Vietnam, primarily for reasons of cost.

The main benefit of outsourcing production in these areas of the world is the absence of the problems of local ownership. Moreover, the company does not invest its own capital in manufacturing.

There are issues related to the externalization of production in low-cost areas of the world. First of all, over the years, some of the multinationals have become a focus for international scrutiny because of allegations of misconducts toward employees.

Moreover, it has been widely recognized that there must be some moral responsibility for the working conditions at contractor manufacturers, which usually turn out to be at the boundary of slavery, with a lot of working hours and very low salaries.

Following allegations to the major groups in the world, specialized bodies have examined the working conditions in the contractors, in order to improve the quality of the work environment.

Major mobile phone vendors, for example, have applied the same model to handset manufacturing. They outsource the production of handsets to Asian companies on a contractual basis while retaining control of research, design, branding, and marketing.

By not owning the production factories, vendors have the advantage to be more flexible. They can accelerate or slow down the production according to fluctuations in consumers' demand for their products, avoiding long-term capital expenses.

However, the lack of direct control in a crucial stage of the supply chain represents a big disadvantage. It is in fact more complicated to implement efficient quality control, and there is also a dependency on the contract equipment manufacturer (CEM) possessing or having access to the necessary parts.

Besides manufacturing, distribution can also be outsourced. This makes the multinational very different from the traditional definition of a firm. It becomes a vertically integrated organization, so much as a network of contractually determined market-based obligations that together constitute a complete supply chain.

There are new managerial challenges embedded in such an organizational form, with the management of contracts across borders being at the center of the analysis. Finally, it should be noted that contract arrangements are by no means confined to manufacturing.

As an example, consider the supply of IT services to the largest multinationals in the world. This is normally contractually accomplished by Indian service providers, because of their combination of low costs and advanced processing skills. The range or provided services go from simple system maintenance to more specialized development of applications and web services.

International joint ventures (IJVs) are contractual agreements between two companies, to realize a product or service together. As a strategy, it involves a higher investment compared to other strategies.

In an IJV strategy, a multinational contacts a local partner and shares proportions of equity with it in the investment. Equity proportions vary but usually relative ownership approximates to half-half. Many other combinations are anyway possible and are based on negotiations and individual contributions of the partners.

An IJV is usually set up with a managerial team of up to 10 managers. The negotiation about the control of the positions is another central issue. Particularly important is the position of general manager, usually assigned to the partner with dominant equity position. The partner that does not win the top position will argue strongly for other slots that guarantee the desired level of representation.

The members of the managerial board of the IJV are expected to commit themselves to the success of the venture. Moreover, they are delegates of the respective sponsor companies.

The IJV is structured like a legal entity with a board of directors deciding about priorities and the use of profits. Moreover, the board decides about investment policy and other critical things.

In some countries like India or China, the only way to enter the market was to have the participation of a local company in the venture, making IJV the only possible strategy to enter those markets.

However, even when local participation is not obligatory, an IJV may be appropriate because a local partner can provide intermediate inputs, such as local market knowledge, access to distribution networks and natural resources, as well as making the MNC an insider in the host country.

The IJVs provide a quick entry into markets otherwise very difficult to exploit. Another advantage is that risk is shared among partners and the economies of scale increase thanks to the increase of size.

In terms of power, it is common that the multinational overtakes the local partner, both in terms of technology and management skills. As a result, the local partner may end up being disturbed by the tendency of the multinational to protect its technology.

Another point of controversy is the imposition of control on the venture by the multinational on one side and the lack of trust of the latter in the local partner, due to the controversies.

Such a set of frictions is at the basis of the reason why in many cases IJVs are characterized by general dissatisfaction of both partners or even failure due to that. This behavior has been observed in something like half of the analyzed IJVs involving a multinational (Fey and Beamish 2000).

13.1.3 Globalization and the Cost of Capital

At the end of the Second World War, many of the currencies in the world were not convertible anymore, and the only way to invest in foreign market was to somehow get access to the scarce foreign currencies.

On top of that some countries had put in place restriction on foreign investments, making the problem of conversion even irrelevant. In some cases, foreign investors could not buy local shares. In other cases, domestic investors were not allowed to hold foreign shares.

The demand of foreign investors for shares in domestic companies was still in place, but very often they did not carry any voting rights, with limits on foreign ownership imposed by the local governments (Barro et al. 1991).

Most countries had created barriers to foreign ownership of local firms, introducing severe limits on foreign ownership. In addition to restrictions resulting from laws and regulations, there were obstacles related to other factors like political risk, accounting rule heterogeneity, and inadequate preparation of institutions to play internationally.

Barriers have progressively crumbled, with most developed economies reaching a high level of integration and emerging economies overcoming the previous barriers toward other countries.

Modern companies have the chance to raise capital from several international markets, and nowadays both offshore and onshore sources of funding are available. This also generated an obvious link in the expectations of investors among different continents, who are nowadays aware and worried about what is happening to some financial market on the other side of the world.

Markets, like New York, Tokyo, Hong Kong, and Shanghai, are all lined up, and they all benefit or suffer from the shocks happening in the others. Investors worldwide want to be constantly updated with the info coming from all over the world.

People in the financial academic sector have initially welcomed the globalization phenomenon, underlying the benefits to investors and companies. On the other hand, regulators and policymakers have questioned about how far the process could get before generating more troubles than advantages.

Someone argued that after the events in the last years and the crisis that spread worldwide due to the high level of interconnection of the economies, barriers should be reintroduced, at least to mitigate capital flows.

Recall that the cost of capital of a firm depends on its debt among other things and the taxation effect that the debt carries. The best way to grasp the effect of globalization on the cost of capital is then to focus on the cost of equity.

Taxation in fact varies from country to country, making the debt a very peculiar source of capital among different countries. Cost of equity instead is more standardized, with homogeneous features around the world.

The information asymmetry problem is the strongest argument for departure from the Modigliani-Miller theorem. Managers in fact tend to be more informed than investors about the company's cash flows. Besides the information asymmetry problem, managers have their own objectives that may differ from those of investors. This is the agency costs problem.

The aim of the management is the growth of the firm, and this makes the interest of managers conflict with those of the investors sometimes. Investors know that managers want to raise capital at any cost, so they will not trust their cash flow predictions.

This creates a cycle in which the managers are forced to not be able to raise enough capital for the projects they would implement. For existing firms, management may have to give up too much of the value of the firm to raise the capital it wants.

Therefore, one can envision a relationship between the cost of capital and capital structure of a firm and its governance. Information and agency costs problems impact firm value, and poorly governed firms experience the worst consequences from the two issues.

When a company goes international, the globalization process has an impact on its governance, among other things. The new shareholders, for instance, may have skills and information that enable them to monitor management in ways local investors could not.

Another effect of globalization is the access to capitals, which becomes more competitive. In closed economies in fact, the number of providers of funds is sometimes limited, therefore lacking competition among lenders.

In an international environment instead, capital can be raised in a more competitive environment, especially in case of large amounts of capital needed. The increased number of fund providers makes it easier to find the right lender and reduces the transaction costs associated with capital raising (Black 1974).

There is also a huge impact of globalization in the quality and the structure of corporate control. Sometimes companies with very large market share in their domestic market become one of many competitors in a foreign market. Also, takeovers may not be a problem domestically, but after internationalization the company may be more exposed.

The monitoring of managers is consequently increased, and both existing shareholders and potential acquirers are interested in the topic.

Another consequence of internationalization is connected to the issue of capitalization. Companies that are active on financial markets can benefit from extending their activities abroad. Funds raised internationally have the advantage of a higher diversification, with a corresponding decrease in the risk.

Some aspects of globalization relate to the fact that not all money markets are integrated. If this is the case, by traditional macroeconomic theory, the interest rate in a country has to be such that savings and investment are balanced.

The excess of one respect to the other in a closed economy would make the interest rate move quite suddenly, for any unexpected increase of one of them. In particular, the interest rate would raise for an excess of investments over savings and vice versa. The interest rate would react to place the economy back in equilibrium.

In a globalized economy, the internal balance between investments and savings does not have to be necessarily reached, and capital inflows and outflows can stabilize the equilibrium so that in general the interest rate is much less volatile than in a closed economy.

It is very hard to quantify the volatility reduction effect on interest rate due to globalization, even if some researchers have tried to measure it. However, for the purpose of this section, the rough idea of a lower volatility due to internationalization is enough.

The same reasoning for the interest rate can be applied to the risk premium demanded by the investors. In a country with barriers to internationalization, the risk of economic activities is normally shared among domestic investors only.

It follows that the demanded risk premium is strictly linked to the riskiness of the country's economic activities. The average risk aversion of domestic investor will therefore be a measure of the risk premium (Booth 1982).

When a country decides to open up its capital market to foreign investors, also allowing its domestic investors to invest abroad, the risks associated with the economic activities of the country are not born entirely by domestic investors anymore.

The risk is now shared with foreign investors, proportionally to the amount invested by them in the economic activity. For domestic investors, the benefit from bearing both domestic and foreign risks rather than only domestic risks is that some of these risks offset each other through the process of diversification.

The information that hits a globalized market is controversial. Some countries may experience a flow of bad news at a specific point in time, while other countries may have good news at the same time.

As a consequence, investors will be diversifying their portfolio by investing their wealth in different markets. They will be subject to shocks on several markets that most probably will offset each other, giving a diversification effect due to internationalization of the investment.

An example of how the effect is developed is assuming that the global economy is populated by a large number of small countries. Moreover, one can assume that for every country, the return and risk are the same and that markets are uncorrelated with each other.

When countries open up to the global market, they become interdependent. Following standard portfolio theory, investors are risk maximizers and risk minimizers. As a consequence, each investor will hold the world market portfolio, the one that ensures the maximum diversification benefit.

The expected return of the invested wealth of an investor does not depend on how his wealth is invested across countries, but the variance of the return of the investor's invested wealth falls with the number of countries in which he invests.

According to the assumptions above, for a very large number of uncorrelated economies, the overall risk for a fully diversified portfolio will tend to zero, therefore yielding no risk premium (Holland 1990).

In the context of risk minimization and profit maximization, the international investors hold an internationally diversified portfolio of risky securities, by measuring how they contribute to the variance of the return of the diversified portfolio.

The result is that the CAPM holds for integrated markets. This approach looks at an international market as a single aggregate market which we call the global equity market.

It is then possible to indicate a world market portfolio and to calculate the beta coefficient of a risky security relative to the market portfolio for the global equity market.

It is possible to formalize mathematically a more general case, considering a small country with equity market separated by the equity markets abroad.

Since the country is small, adding that country to the world equity markets does not increase the risk premium on the world market portfolio. To eliminate the impact of differences in risk aversion, the assumption is that all investors in the world have the same constant relative risk aversion.

It is possible to indicate the risk in the country before opening up to the world by a constant λ. Consequently, the risk premium on the small country before integration is given by

$$\lambda = \sigma_{\mathrm{sc}}^2 T$$

where:

σ_{sc}^2 is the small country's market portfolio return variance.

In order to consider the risk premium after internationalization λ^*, recall that the equity market of the small country becomes integrated in the global equity market. By denoting with $E(r_{\mathrm{w}})$ the expected return of the global market portfolio and with r_{f} the risk-free rate, the excess return on the world market portfolio is given by

$$E(r_{\mathrm{w}}) - r_{\mathrm{f}}$$

The CAPM model holds for the global market, and the expected return and risk premium on the risky asset depends on the expected return on the world market portfolio, as quantified by the beta coefficient. This can be described as

$$\begin{aligned} \lambda^* &= \beta_{\mathrm{sc}}[E(r_{\mathrm{w}}) - r_{\mathrm{f}}] \\ &= \rho \sigma_{\mathrm{sc}} \sigma_{\mathrm{w}} T \end{aligned} \tag{13.1}$$

where:

σ_{sc} is the volatility of the small country portfolio.
σ_{w} is the volatility of the world market portfolio.
ρ is the correlation coefficient between the return of the small country portfolio and the return of the world market portfolio.

The beta of the small country market portfolio with respect to the world market portfolio is defined as

$$\beta_{\mathrm{sc}} = \frac{\rho \sigma_{\mathrm{sc}} \sigma_{\mathrm{w}}}{\sigma_{\mathrm{sc}}^2}$$

By looking at Eq. (13.1), it is clear how the price of risk in the world equity market is the same for all investors. By comparing the risk premium before and after globalization, it follows that a necessary and sufficient condition for globalization to reduce the risk premium of the small country is that

$$\rho < \frac{\sigma_{\mathrm{sc}}}{\sigma_{\mathrm{w}}}$$

The condition holds in case an investor in the small country can form a portfolio with lower variance than the market portfolio of the small country, by moving some of the wealth from the small country market to the world market portfolio. It is possible that this condition would not hold for a specific country.

The effect of globalization on the domestic market of the small country could be an increase of the cost of capital. This happens because the country's risk premium depends on the covariance of its covariance with the return of the world market portfolio (Shapiro 1978).

For a high level of covariance, this means that the market portfolio of the country is risky from the perspective of the global markets. It is therefore expected to earn risk premium that exceeds the risk premium it would be expected to earn if it was segmented from the global markets.

If there is no correlation between the return of the market portfolio of the small country and the return of the world market portfolio, the small country's market portfolio is not expected to earn a risk premium when it is integrated in the global market.

As a consequence, for not too high correlation between the small country's market portfolio return and the return of the world market portfolio, the small country's risk premium falls when it joins the global equity market. The same applies if the volatility of the small country's market portfolio is not too low.

13.2 The Variables Involved

Learning Outcomes

- Define and explain what exchange rate is.
- Understand the purchasing power parity.
- Understand the interest rate parity.

13.2.1 Exchange Rate

The price of the currency of one country, expressed in terms of the currency of another country, is called exchange rate. Exchange rates can be expressed in couples, but nowadays, all currencies are commonly valued against US dollar or euro as common benchmarks.

The ancestry of the exchange rate model was the monetary model, which assumed the current exchange rate to be determined by fundamental macroeconomic variables like the money supply or the GDP. By combining the macroeconomic variables with the expectations of the investors about future exchange rates, the model is supposed to yield the current exchange rate (Booth 1982).

Still nowadays, journalists of popular specialized magazines analyze the movements in the exchange rates using the results of the monetary model. The same holds for the analysts in various parts of the world, who typically resort to some variant of the monetary model.

The rationale behind the monetary model is that the exchange rate is basically determined by the relative level of the prices in the two countries. The ratio between the prices in two countries will determine the ratio of exchange rate between currencies.

Then it is interesting to investigate what are the determinants of the relative price levels in the countries. In the case of the monetary system, the analysis is based on the money supply which is assumed to be positively correlated with the level of prices.

Example 13.1 Assume that the money supply in Europe rises so that the level of prices in the euro area will rise as a consequence. If the price level in the foreign country remains fixed, it will take more euros to buy one unit of foreign currency. The price of a euro in foreign currency will rise, so that the euro will depreciate.

The level of real output in each country matters as well because it affects the price level.

Example 13.2 If the level of output in Japan rises, but other fundamental factors, including money supply, remain constant, there will be a decrease of the average level of prices in Japan. As a consequence, the Japanese yen will appreciate.

Also future macroeconomic data matter, in that they determine market expectations. Therefore, expectations on future exchange rate are directly related to the level of the current exchange rate.

If the market expects the dollar price of the yen to become higher in the future than it is today, the dollar price of the yen will tend to be high today. But if the market expects the dollar price of the yen to be lower in the future than it is today, the dollar price of the yen will tend to be low today.

Example 13.3 It is possible to use the monetary model to predict the euro-dollar exchange rate. The relationship between the fundamentals of the economy and the exchange rate is very important. If the money supply in the United States grows faster than in Europe, the dollar will depreciate. The opposite happens if the money supply grows faster in Europe.

The analyst needs to assess monetary policy in the two countries. Moreover, he must assess expectations about the future exchange rate. If the market's expectation of the future exchange rate were to change, the current exchange rate would move in the same direction.

All the fundamentals act simultaneously in determining the forecast on exchange rate. It is therefore useful for the analyst to use a statistical model allowing to combine the effect of all the variables.

The use of the monetary model in practice is made difficult by the lack of knowledge about the true value of the economic fundamentals, which is often ignored by the analyst.

The main variables of interest are in fact never known for sure, but they are normally forecasted given the current available economic data. In this sense, expectations about the future of the exchange rate are unobservable, therefore even harder to assess.

Assuming the monetary model is valid, the forecast about exchange rate is successful, and then the predictions about fundamental values are good. After early success, the model was proven to fail empirically, except in very extreme conditions.

After the failure of the monetary model, economists went to work developing other ideas. Rudiger Dornbusch developed a variant of the monetary model called the overshooting model.

The model assumes that the average level of prices is fixed in the short term, matching the assumption that prices in the real world change very seldom. The assumption has the effect to overshoot its long-run value as a result of a change in the fundamentals and then turning back to the long-run value.

Also the modified model was soon shown to fail empirically. In particular, the model lacks a strong statistical relationship between the fundamentals and the exchange rate. That relationship should in fact exist if the model were true.

The portfolio balance model is another extension of the monetary model, in which the determinants of the exchange rate are the supply and demand of foreign and domestic bonds together with the demand for foreign and domestic money.

The model itself has proven to fail, but further versions have been developed, taking into account an optimal solution for the choice of bonds and money in the portfolio. According to such a theory, the substitution rate of domestic for foreign bonds depends on the risk aversion of the investor and on the volatility of the returns on the bonds. Moreover, the correlation between the returns on the different bonds in the portfolio is also important.

It turns out from the empirical application that the three major models of the exchange rate, namely, the monetary, the overshooting, and the portfolio balance models, do not provide a satisfactory account of the exchange rate.

The news about the fundamentals are important drivers of the exchange rate, and market participants form expectations about the value of the money supply before the government announces the money supply figures.

These expectations drive the investment decisions of traders to whether buy or sell currency. Once the investment decisions are made, the interaction of those on the market determines the current level of the exchange rate.

Market participants trade currencies following the announcements about relevant macroeconomic measures, like the change in money supply. Thus, news about fundamentals, under this view, is an important determinant of the exchange rate.

13.2.2 Purchasing Power Parity

Several theories deepen the analysis among the financial variables involved in the interaction of worldwide economies. These rules are basic for the understanding of the dynamics of international finance and trade.

The purchasing power parity (PPP) stands as a popular theory for determining the exchange rate, allowing comparison between the relative average costs of goods and services between countries.

According to the PPP theory, the dynamics of the exchange rate are determined by the deals of importer and exporter, who act based on the price differences across different countries.

Basically there is a link between the current account of some country and the value of the exchange rate on the foreign exchange market. This contrasts with the interest rate parity theory which assumes that the actions of investors, whose transactions are recorded on the capital account, induce changes in the exchange rate.

The theory links to the law of one price, which is the foundation for PPP, by applying it to the global economy. For those who are not familiar with the law of one price, it is useful to briefly recall it (Errunza and Losq 1985).

The law of one price (LOP) states that in the absence of transportation costs and tax differential between two markets, the price of identical goods must be the same in both markets, in order to avoid arbitrage opportunities and profits on price differences.

İn case of a price difference, in fact, it is possible to make a profit through trade by buying goods on the cheapest market and selling them on the more expensive, if no other costs are incurred in the trade.

The idea between the law of one price is that identical goods selling in an integrated market, where there are no transportation costs or differential taxes or subsidies, should sell at identical prices.

The PPP theory implements the LOP by generalizing its statement to all goods in a market. The best way to show it is, mathematically, through the definition of several variables.

First of all let's define the variable $B_{€}$ to be the cost of a basket of goods in the euro area denominated in euros. A very efficient way to generate the basket is by making it a proxy of the consumption basket in the economy.

The basket is then determined by surveying the quantity of different items purchased by many different households. In fact, it can be determined, on average, how many units of different goods are purchased by the typical household.

Similarly, it is possible to define the variable $B_{£}$ as the cost of a market basket of goods in the United Kingdom denominated in pounds. Now if the law of one price holds for each individual item in the market basket, then it should hold for the market baskets as well. This can be mathematically written as

$$\frac{B_{€}}{e_{€/£}} = B_{£} \tag{13.2}$$

where:

$e_{€/£}$ is the exchange rate.

There are two forms of PPP, namely, the absolute and the relative form of the theory. In order to understand their meaning, it is useful to consider them one by one. The absolute PPP is obtained by reworking the right-hand side of (13.2) so to obtain

$$e_{€/£}^{\mathrm{PPP}} = \frac{B_{€}}{B_{£}}$$

The PPP is satisfied when the above relationship holds, with the consequence that the related exchange rate is equal to the ratio between the relative cost in domestic currency of the same basket of products in the two different economies.

As a logical consequence, the reciprocal relationship

$$e_{£/€}^{\mathrm{PPP}} = \frac{B_{£}}{B_{€}}$$

holds as well.

Because the cost of a market basket of goods is used in the construction of the country's consumer price index, PPP is often written as a relationship between the exchange rate and the country's price indices.

The relative PPP says that variations in the exchange rate between two economies is determined by the difference in the respective inflation rates. Put in formulas, the relative PPP says that the expected percentage changes in the exchange rate over the next year, defined as

$$\frac{E(e_1) - e_0}{e_0}$$

is given by

$$\frac{E(e_1) - e_0}{e_0} = i_{\mathrm{FC}} - i_{\mathrm{DC}} \tag{13.3}$$

where:

e_0 is the current exchange rate between domestic and foreign currency.
$E(e_1)$ is the expected exchange rate at time 1.
i_{DC} is the inflation rate in the domestic country.
i_{FC} is the inflation rate in the foreign country.

It is now possible to rearrange the terms in (13.3) to obtain that

$$E(e_1) = e_0[1 + (i_{FC} - i_{DC})]$$

In general, relative PPP says that the expected exchange rate at some time t in the future is

$$E(e_t) = e_0[1 + (i_{FC} - i_{DC})]^t$$

The absolute PPP is very strict, therefore not holding for most economies and basket of goods. The relative PPP is instead a good starting point for studying the dynamics of the exchange rates between countries.

As a matter of clarification, from now on the acronym PPP is used to mean relative parity, and the absolute parity is dropped.

13.2.3 Interest Rate Parity

When the market is free of covered arbitrage opportunities, there is a relationship between spot exchange rates, forward exchange rates, and relative interest rates. That relationship makes the market free of arbitrage.

To see what this relationship is, consider the following investment strategies. The first strategy is investing in a riskless domestic investment at a rate r_{DC} which gives a return of $(1 + r_{DC})$ for every dollar we invest.

The second strategy involving investing in a foreign risk-free investment gives

$$E(e_t) = \frac{e_0(1 + r_{FC})}{f_1}$$

where:

r_{FC} is the risk-free rate in the foreign country.
f_1 is the foreign exchange rate between domestic and foreign currency.

In order to avoid arbitrage, the above strategies must yield the same result. Therefore, the relationship

$$1 + r_{DC} = \frac{e_0(1 + r_{FC})}{f_1} \tag{13.4}$$

must hold.

It is then possible to rearrange the terms of Eq. (13.4), to get the interest rate parity (IRP) relationship that can be written as

$$\frac{f_1}{e_0} = \frac{1 + r_{\mathrm{FC}}}{1 + r_{\mathrm{DC}}}$$

The relationship states the connection between the spot interest rates in both economies and the forward rates as well. It illustrates very clearly what is going on and is not difficult to remember.

A slight mathematical work on (13.4) allows to represent the IRP in the form of a percentage forward rate premium (or discount), which is equal to the difference between the interest rates in the two economies, as defined by

$$\frac{f_1 - e_0}{e_0} = r_{\mathrm{FC}} - r_{\mathrm{DC}}$$

The interpretation of IRP is that differences in the interest rates between two economies, in the midterm, are balanced by a corresponding change in the relative value of the currencies, so that the market becomes free of arbitrage.

By reworking the IRP equation, it is possible to show the relationship for t periods instead of just one, in the form

$$f_t = e_0[1 + (r_{\mathrm{FC}} - r_{\mathrm{DC}})]^t$$

Another popular relationship, which is basic for the understanding of exchange rates, is the so-called unbiased forward rate (UFR) condition.

One should recall that from basic financial theory, the UFR is equal to the expected future spot rate. The relationship can be therefore written as

$$f_1 = E(e_1)$$

and, in t periods,

$$f_t = E(e_t)$$

If risk is not taken into account, the UFR condition generally holds. If the forward rate on some domestic currency is consistently lower than the future spot rate by some fixed amount, anyone who wanted to convert from foreign to domestic currency in the future would consistently get more of the latter by not agreeing to a forward exchange.

In order for investors to be interested in forward exchange, the forward rate should increase. The opposite holds in case the forward rate was consistently higher than the future spot rate.

In that case, anyone who wanted to convert domestic to foreign currency would get more value by not agreeing to a forward trade. The forward exchange rate would have to decrease to attract such investors.

What the future spot rate will actually be is uncertain, of course. The UFR condition may not hold if traders are willing to pay a premium to avoid this uncertainty. If the condition does hold, then the 180-day forward rate that we see today should be an unbiased predictor of what the exchange rate will actually be in 180 days.

Finally, it is possible to combine the various relationships developed in this section in order to explore the implications of them as a group.

For example, it is possible to combine UFR and IRP. Given that UFR relationship states that $f_1 = E(e_1)$, it is possible to substitute $E(e_1)$ for f_1 in the IRP to get the so-called uncovered interest parity (UIP), as

$$E(e_1) = e_0[1 + (r_{\mathrm{FC}} - r_{\mathrm{DC}})]$$

The UIP is very important for international capital budgeting purposes. Generalizing for t periods, the UIP becomes

$$E(e_t) = e_0[1 + (r_{\mathrm{FC}} - r_{\mathrm{DC}})]^t$$

The comparison between PPP and UIP, on the other hand, yields the popular international Fisher effect (IFE). By looking at the respective formulas, the equality of the two relationships leads to

$$e_0[1 + (i_{\mathrm{FC}} - i_{\mathrm{DC}})] = e_0[1 + (r_{\mathrm{FC}} - r_{\mathrm{DC}})]$$

meaning that

$$i_{\mathrm{FC}} - i_{\mathrm{DC}} = r_{\mathrm{FC}} - r_{\mathrm{DC}}$$

The information given by the relationship is very clear. The difference in the interest rates between two countries is equal to the difference in their inflation rates. Rearranging this slightly gives us the international Fisher effect as

$$r_{\mathrm{DC}} - i_{\mathrm{DC}} = r_{\mathrm{FC}} - i_{\mathrm{FC}}$$

It follows that real interest rates are equal across countries. It is a basic relationship in economics, if one recalls that a differential in the real interest rates between two economies would cause money to flow into the high-rate economy.

As a consequence, the price of financial asset into the flows-receiving country would rise, with returns falling. The opposite would happen at the same time in the country with lower real rates. Such a process would finally equalize returns.

It is obvious that the above discussion should be adjusted to risk and barriers occurring in world trading. Both factors imply that returns might be different in two different countries for long periods of time if money can't move freely between them.

Anyway, with the progressive internationalization of markets, any differences in real rates that do exist will probably diminish. The laws of economics have very little respect for national boundaries.

13.3 Foreign Exchange Markets

Learning Outcomes

- Learn about foreign exchange market background.
- Understand foreign exchange risk.
- Explain international capital budgeting.

13.3.1 Background

The money from one country to another is physically exchanged through the foreign exchange market, which is an institutional structure where the rate of exchange between currencies is determined and foreign exchange transactions are physically completed.

A foreign exchange transaction is an agreement between a buyer and a seller that a given amount of one currency is to be delivered at a specified rate for some other currency.

The FX market covers the whole world, and the trading takes place every hour, in different places, every business day. The market reaches most of its liquidity in the early European afternoon, when the markets of both Europe and the US east coast are open.

Foreign exchange trading in some countries is conducted by open bidding on an official trading floor. The official published prices are the closing prices for the day, and some transactions are then based on that closing price.

Through the FX market, a person or firm transfers purchasing power to other countries. Moreover, it functions as a way to obtain or provide credit for international commerce, and investors can also use the FX market to hedge against FX risk.

The transfer of purchasing power in particular is needed due to the involvement of different currencies in international transactions, given that the parties normally reside in different countries. Each party usually wants to deal in its own currency, but the transaction can be invoiced in only one currency.

About the provision of credit, the movement of goods between countries requires financing of the transiting inventories, given the time required for the transfer to take place. The minimization of FX risk finally is provided through hedging by transferring the risk to some other investors by using appropriate securities.

The FX market is composed of two major parts, namely, the wholesale market and the retail market. The wholesale market involves transactions of high volume worth multiples of millions. In contrast, contracts between a bank and its client are usually for specific amounts, sometimes down to the last penny.

The FX dealers, which are mostly banks, operate in both the interbank and the client markets. They profit from buying foreign exchange at a bid price and reselling it at a slightly higher ask price (Sager and Taylor 2006).

The benefit of the globalized competition among dealers is that the bid-ask spread narrows down. This makes the FX market efficient in terms of the requirements that qualify efficiency in all the other markets.

The biggest international banks have large FX departments, and their dealers act as market makers. They specialize in currencies and buy and sell them by maintaining an inventory position in those currencies.

The big importers and exporters are participants in commercial and investment transactions. On top of them, there are international investors, multinationals, and other users of foreign currencies. Some of these participants use the foreign exchange market to hedge foreign exchange risk.

There are also speculators and arbitrageurs in the FX markets. Speculators, like in other markets, aim to make profits from trading in the market, by buying at cheap prices and selling at expensive prices. They operate in their own interest, without a need or obligation to serve clients or to ensure a continuous market. On the other hand, arbitragers try to profit from simultaneous exchange rate differences in different markets.

Other players in the FX market are the central banks and treasuries. The former acquire or spend their foreign exchange reserves, and at the same time, they influence the price at which their own currency is traded.

Compared to other investors, the central banks are better off when making a loss on the FX transactions they perform. They are therefore driven by different motives and behavior.

FX brokers are agents who facilitate trading between dealers, but they do not become principals in the transaction. They charge a fee for the service and provide themselves with hundreds of dealers worldwide.

The broker must have knowledge of what dealers want to buy or sell currency, so that they can find a counterparty for the customer in a very short time, keeping the transaction anonymous until the deal is achieved.

Transactions in the FX market can be executed in three modalities, namely, spot, forward, or swap. A spot transaction involves the immediate delivery of foreign currency from the seller to the buyer.

In the interbank market, a spot transaction involves the purchase of foreign exchange with delivery and payment between banks to take place, normally, on the second following business day. The date of settlement is referred to as the "value date."

Most transactions on the FX markets are spot and take place in real time. Forward transactions require the delivery of a specified amount of currency for a specified exchange rate, at a specified date. The exchange rate to prevail at the value date is established at the time of the agreement, but payment and delivery are not required until maturity.

Forward FX contract are quoted for maturities of 1–12 months, but the actual agreements can last for longer. Outright forward transactions only account for about 9% of all foreign exchange transactions.

Swap transactions are based on the simultaneous purchase and sale of some amount of foreign currency, spread on two different value dates. A spot against forward, for example, involves a dealer buying currency in the spot market and selling it back (for the same amount) in the forward. The agreement is executed as a single transaction, so that there is no FX risk for the dealer. Swap transactions account for about 48% of all foreign exchange transactions.

When looking at quotations on the FX markets, an interbank quotation is how commonly professional investors exchange quotations. It is the basic format used to display the quotes on the trading screens worldwide.

In the United States, the European terms quote shows the number of units of foreign currency needed to purchase one USD:

EUR 0.99/USD

An alternative method is called the American terms. The American terms quote shows the number of units of USD needed to purchase one unit of foreign currency:

Generalizing for the rest of world markets and currency couples, direct quote is a home currency price of a unit of foreign currency.

An indirect quote is a foreign currency price of a unit of home currency. In Europe, a direct quote for the British pound is.

USD 0.6341/CAD

This quote would be an indirect quote in Canada.

Interbank quotations are given as bid and ask rates, with investors buying at the bid price and selling at the ask price. Bid and ask quotations are complicated by the fact that the bid for one currency is the ask for another currency:

The foreign exchange (FX) market is the market where currencies are traded, and it is one of the largest world financial markets. Most of the trading takes place in major currencies like the euro, the dollar, the British pound, and the Japanese yen.

It is an over-the-counter (OTC) market, meaning it is not officially regulated like a regular exchange. The market participants are located in the major commercial and investment banks around the world.

All the major traders use computer connection, telephone, and other sorts of communication networks to communicate trading decisions, and connection is maintained by the Society for Worldwide Interbank Financial Telecommunications (SWIFT), a company with headquarters in Belgium.

The SWIFT processing centers allow banks from all parts of the world to communicate easily and perform trading actions in real time. There are several types of participants in the foreign exchange market including importers, exporters, portfolio managers, FX brokers, traders, and speculators.

Regarding the types of transactions available on FX markets, there are two basic trading methods: spot trades and forward trades. A spot trade is an agreement to exchange currency "on the spot," meaning the transaction will be completed within two business days.

The spot exchange rate, as the name suggests, is traded on the spot, and it is the most typical type of transaction involving currencies.

On the other hand, a forward trade is an agreement to exchange currency at some time in the future. As for any other forward contract, the exchange rate that will be used is agreed upon today and is called the forward exchange.

13.3.2 Foreign Exchange Risk

Currency risk or FX risk is the risk related to a financial transaction which is denominated in a currency other than that of the base currency of the company. Also the foreign subsidiaries of domestic firms are subject to FX risk when they register entries in their financial statements, in a currency that is different from the one of the parent company.

The risk implied in FX investments is that an adverse movement in the spot exchange rate will change the amount of domestic currency required to buy units of the foreign currency, before the transaction date.

All the businesses that trade goods or services internationally or make foreign investments are subject to an exchange risk, which sometimes has severe consequences on the finances of the firm. However, like other types of risk, it is possible to manage it.

Transaction exposure happens when a company has contractual cash flows subject to changes in value due to unanticipated variations in the exchange rates, due to a contract being denominated in a foreign currency.

In order to assess the domestic value of the cash flows denominated in foreign currency, a company must exchange the foreign currency for the domestic currency. When contracts are negotiated with set prices and delivery dates with volatile exchange rates, firms face a risk of changes in the exchange rate between the foreign and domestic currency.

The application of international accounting rules impacts on companies having transactional exposure through a process called remeasurement. As the name suggests, the process involves remeasuring the current value of the contractual cash flows at each balance sheet date.

If the value of the currency of payment or receivable changes in relation to the firm's base or reporting currency from one balance sheet date to the next, the expected value of these cash flows will change.

Another type of exposure the firm is exposed to is the economic exposure or forecast risk, due to the fact that the market value of the company is influenced by unexpected changes of the exchange rates.

The movements in the exchange rate values can affect the market share position of the company and modify it in relation to its competitors. Also the future cash flows and the firm's value are affected as a consequence.

The way economic exposure affects the firm is through the modification of the present value of the future cash flows. Any transaction that exposes the firm to foreign exchange risk also exposes the firm economically.

Economic exposure can be caused by several different business activities and types of investment, which are not barely transactions, like future cash flows coming from fixed assets.

When fluctuations in the foreign exchange market affect the demand for a good or service in some country, the company selling that good or service will be affected as well.

The problem with economic exposures is that they cannot be hedged due to limited available data and it is costly and time-consuming. Economic exposures can be only indirectly managed through product differentiation, pricing, branding, outsourcing, etc.

Translation exposure refers to the extent to which a firm's financial reports are affected by fluctuation in the foreign exchange market. Firms are asked to prepare consolidated financial statements, meaning that the consolidation process for multinationals involves translating the foreign assets and liabilities to domestic currency. The same applies to the financial statements of the foreign subsidiaries of the parent company.

It is possible that translation exposure may not affect the cash flows, but their impact on the reported earnings and the stock price can be relevant anyway. Translation exposure is distinguished from transaction risk as a result of income and losses from various types of risk having different accounting treatments.

A firm has contingent exposure when it gets involved in foreign projects or negotiating other contract or foreign direct investments. The firm could face potential transactional or economic foreign exchange risk by undertaking those ventures.

Sometimes a company has to wait for acceptance of a bid on a project by an external institution. The acceptance of the bid would then result in an immediate profit to be received.

While waiting, the firm faces a contingent exposure from the uncertainty as to whether or not that receivable will happen. If the bid is accepted and a receivable is paid the firm and then faces a transaction exposure, a firm may prefer to manage contingent exposures.

When all the parties in the foreign exchange market works, so that the markets are in equilibrium and efficient, a firm does not need any protection against foreign exchange risk, and it is indifferent toward foreign investment decisions. Deviation from international parities generates an exposure to foreign exchange risk.

The standard deviation of the percentage returns given as rates of change of the spot exchange rate are the normal measure of risk on foreign exchange markets. In foreign exchange, a relevant factor would be the rate of change of the spot exchange rate between currencies.

There are several hedging strategies available to firms to reduce their FX risk. Transaction exposure, for example, can be reduced with the use of short-term instruments, FX derivatives, and operational techniques related to payables, receivables, and invoicing.

Companies can also go beyond financial management of risk, by implementing strong research and development activities and differentiating its products in order to achieve inelasticity and reduce their foreign exchange risk exposure.

The accounting standards adopted by firms affect the level of translation exposure and the translation methods required by those standards. In the United States, the Federal Accounting Standards Board specifies when and where to use certain methods.

The balance sheet hedge method for hedging translation exposure focuses on the discrepancies between net assets and net liabilities resulting from FX rate differences. Translation risk in fact arises from these discrepancies.

A firm could acquire an appropriate amount of exposed assets or liabilities to balance any outstanding discrepancy. Foreign exchange derivatives may also be used to hedge against translation exposure.

The most direct method of hedging FX risk is a forward contract. An exporter can in fact sell a set amount of foreign currency for an agreed-upon exchange rate, and the delivery time can go from a few days to 1 year in the future.

Example 13.4 Consider goods from United States to be sold to some European company for 100,000,000 €. To eliminate the FX risk, the exporter can contract to deliver 100,000,000 € to his bank in 30 days in exchange for payment of $110,000,000. Such a forward contract will ensure that the US exporter can convert the money regardless of what may happen to the dollar-yen exchange rate over the next 30 days. However, if the Japanese buyer fails to pay on time, the US exporter will be anyway obligated to deliver the 100,000,000 € in 30 days.

It is advisable for an exporter to enter into forwards with conservative delivery dates. In case the currency is collected sooner, the exporter can hold on to it until the delivery date or can "swap" the old FX contract for a new one with a new delivery date at a minimal cost.

In case of the future completion of some FX deal, it is therefore worth to consider FX options. The option allows the exporter to acquire the right to deliver the agreed amount of foreign currency to the lender in exchange for dollars at a specified rate on or before the expiration date of the option.

The option is similar to a premium paid for an insurance policy. If the value of the foreign currency goes down, the exporter is protected from loss. On the other hand, if the value of the foreign currency goes up significantly, the exporter can sell the option back to the lender or simply let it expire by selling the foreign currency on the spot market for more dollars than originally expected, but the fee would be forfeited.

13.3.3 International Capital Budgeting

The core business of a multinational is to sell its products or services in multiple countries, at a price denominated in the various domestic currencies. This involves a consistent exposure to currency risk.

After the fall of the Bretton Woods system of fixed exchange rates, the exchange rates became very volatile, and multinational corporations may have become increasingly vulnerable to exchange risk.

This is because generally the short-term movements in exchange rates are not accompanied by corresponding changes in prices in various countries of interest (Shapiro 1992).

Globalization and diversification through internationalization lead to a lower cost of capital, which means that more capital projects will have a positive net present value to the multinational firm.

The general APV model is not totally useful for the multinational corporations in analyzing the FX expenditure of some subsidiaries from the perspective of the parent company.

Sometimes a project may have a positive APV from the point of view of the foreign subsidiary, while it has a negative one from the parent's perspective. This could happen, for example, if certain cash flows are blocked by the host country from being legally remitted to the parent or if extra taxes are imposed by the host country on foreign exchange remittances.

Another reason for discrepancies could be a higher marginal tax rate in the domestic market, making the project not profitable for the parent company. By assuming that the foreign subsidiary is owned by the parent company, which in turn is owned by domestic shareholders, cash flows are dependent on the domestic currency.

It is in that currency in fact that the cash flow will have to be converted, in order to benefit the shareholders. Recall that maximization of shareholder's wealth is indeed the objective of the multinational firm.

The model proposed in this section, to overcome the limitations of the classical APV model, was proposed by Lessard (1985). The model is based on the fact that the cash flows will be denominated in a foreign currency to be later converted into the currency of the parent firm.

The model also accounts for non-ordinary and special cash flows that are usually encountered when performing the analysis of foreign projects. The model can be written as

$$\begin{aligned}\mathrm{APV}_{\mathrm{INT}} = &\sum_{t=1}^{T}\frac{E(S_t)C_t^{\mathrm{OP}}(1-\tau)}{(1+r_{\mathrm{WACC}})^t} + \sum_{t=1}^{T}\frac{E(S_t)\tau\delta_t}{(1+r_{\mathrm{D}})^t} + \sum_{t=1}^{T}\frac{E(S_t)\tau I_t}{(1+r_{\mathrm{D}})^t} + \frac{E(S_T)TV_T}{(1+r_{\mathrm{D}})^T} \\ &- S_0C_0 + S_0L_0^{\mathrm{RF}} + S_0D_0^* - \sum_{t=1}^{T}\frac{E(S_t)C_0^*}{(1+r_{\mathrm{D}})^t}\end{aligned} \tag{13.5}$$

where:

$E(S_t)$ is the expected spot exchange rate applicable for year t.
C_t^{OP} is the amount of operating cash flows.
τ is the tax rate.
δ_t is the depreciation at time t.
r_{WACC} is the WACC to discount cash flows once converted in domestic currency.
r_{D} is the cost of debt.

I_t is the amount of interest expenses.
TV_T is the expected after-tax terminal value.
C_0 is the initial investment at inception.
L_0^{RF} is the amount of restricted funds from existing operations.
D_0^* is the amount of concessionary loan at a below-market interest rate.
C_0^* is the amount of concessionary loan payments.

Equation (13.5) clearly shows how the cash flows are initially denominated in the foreign currency and then converted to the currency of the parent company. The expected spot exchange rate is the conversion benchmark.

The higher between the domestic and the foreign tax rate is taken as the marginal tax rate in the model. This is because the model assumes that the tax authority in the parent firm's home country will give a foreign tax credit for foreign taxes paid up to the amount of the tax liability in the home country.

It follows that, for a higher domestic tax rate, the difference with the foreign tax rate represents additional taxes to be paid in the home country. On the other hand, if the foreign tax rate is larger, there is a credit that offsets the domestic tax liability, and additional taxes are due.

The equation also indicates the discount rates with the subscript d. It indicates that once the foreign cash flows are converted into the parent's home currency, the appropriate discount rates are those of the domestic country.

The model entry C_t^{OP} represents the portion of operating cash flows that can be legally remitted to the parent firm. Parts of cash flows that are blocked by the host government are not considered given that they do not provide any benefit to the stockholders of the parent firm. The model does not account for cash flow remitted by circumventing restrictions as well.

The international version of the APV is based on the same rules of the standard model for what concerns revenues and costs. In particular, only incremental revenues and operating costs are considered in calculating the C_t^{OP}.

In order to understand the concept, consider the example of a multinational company with a sales affiliate in some country, which could raise merchandise from the parent company directly or indirectly through some facility.

The company may decide to put a manufacturing facility into operation in the foreign country, so as to increase the overall sales compared with just having a sales affiliate. The downside is that the former manufacturing unit will experience lost sales as a result of the new foreign manufacturing facility.

A process of cannibalization enters into force reducing the profitability of the formerly existing project. The incremental revenue is given by the total sales revenue of the new manufacturing facility net of the loss in revenue from lost sales.

In the different case when sales are lost because a competitor who is better able to satisfy local demand is gearing up, the entire sales revenue of the new foreign manufacturing facility is incremental sales revenue.

The international APV model incorporates additional terms that are usually encountered during the development of foreign projects. For example, the term S_0

L_0^{RF} represents the value of accumulated restricted funds (of amount L_0^{RF}) in the foreign land from existing operations that become available when implementing the proposed project.

There are funds restricted in the foreign country from previous project, and the above variable represents the share of those funds that become available thanks to the new project.

Examples are funds whose use is restricted by exchange controls or funds on which additional taxes would be due in the parent country if they are remitted. The value of L_0^{RF} is given by the difference between their face value and their present value in the best scenario.

Another interesting term to analyze is

$$S_0 D_0^* - \sum_{t=1}^{T} \frac{E(S_t) C_0^*}{(1 + r_{\mathrm{D}})^t}$$

It shows the present value, expressed in the currency of the parent company, of the benefits coming from borrowing at a favorable market rate, when raising funds in the foreign currency.

Concessionary loans at a low interest rate are sometimes available to a multinational firm, when it decides to invest and make capital expenditure in the foreign land. The host country normally offers such an advantageous type of loan in order to attract foreign capitals for economic development and the creation of employment opportunities for its citizens.

The multinational company enjoys the benefit of the difference between the face value of the favorable concessionary loan and the face value of the same loan when issued at ordinary interest rate conditions.

It is clear that taking the difference between an amount discounted at the lower (favorable) interest rate and the same amount discounted at a standard (higher) rate yields a positive NPV for the company.

It should be clear that the present value of the loan payments discounted at the normal borrowing rate represents the size of the loan available from borrowing at the normal borrowing rate with a debt service schedule equivalent to that of the concessionary loan.

The WACC is calculated based on the optimal debt-to-equity ratio. The single project should always be considered as a small portion of the firm, therefore being financed using the same capital mix as the whole company.

When the asset base increases because a capital project is undertaken, the firm can handle more debt in its capital structure. That is, the borrowing capacity of the firm has increased because of the project.

The choice of investing and financing is separate in the corporate finance theory. When the optimal capital structure for the firm is known, one can use it to determine whether a project is viable or not.

It is not true that each and every capital project is financed with the optimal portions of debt and equity. Some projects may be financed with all debt or all equity or a suboptimal combination.

The crucial thing is that in the long term, when it comes to the aggregation of the many undertaken projects over time, the company does not depart too much from the corporate capital structure.

Looking back at the Lessard formula, the interest tax shield term in the model recognizes the tax shields of the borrowing capacity created by the project regardless of how the project is financed.

If another approach was used, then the APV would have changed considerably according to the amount of leverage implied by the project. In international capital budgeting analysis, this is a very important point, given the large use of concessionary loans.

The international APV model equation does not include all possible cash flows encountered in analyzing foreign capital expenditures. The analyst should consider it as a framework to apply to the specific single case with opportune modifications.

By understanding the nature and functioning of the Lessard's framework, one should now have the knowledge to incorporate into the basic APV model terms of a more unique nature for specific cash flows encountered in a particular analysis.

One way to tune the model, for example, is by introducing tax savings and deferrals, due to multinational operations. Some revenues and expenses may be spread among the foreign affiliates so as to reduce the taxable income.

Operations such as transferring of income to low-tax countries, as well as strategies for repatriate as much as possible of the cash flows generated abroad, are examples of how the international APV model can be modified to fit different situations.

All the above cash flow terms, from tax savings or deferrals to the repatriation of restricted funds, can be easily handled in the Lessard framework. The analyst can first analyze the capital expenditure as if it did not exist.

After that, it is possible to consider the additional cash flows separately, with no need to be explicitly included in the model. Additional terms in fact become meaningful only if the initial APV turns out to be negative.

In this case, the analyst can calculate how large the cash flows from other sources needed to make the APV positive and then estimate whether these other cash inflows will likely be that large.

The implementation of the general international APV framework requires to estimate the future expected exchange rate. One method to do so is to recall the purchasing power parity relation.

By using the PPP, the analyst can estimate the future expected exchange rate for period t as

$$E(S_t) = S_0 \frac{[1 + E(i_{\mathrm{d}})]^t}{[1 + E(i_{\mathrm{f}})]^t}$$

where:

$E(i_{\mathrm{d}})$ is the expected long-run annual rate of inflation in the domestic country.
$E(i_{\mathrm{f}})$ is the expected long-run annual rate of inflation in the foreign country.

Recall that the PPP is a theoretical framework which is not likely to hold in real life. Long-term biases may result in a systematic underestimation of overestimation of the expected exchange rates. However, if there is no suspicion of such a bias, the PPP should prove to be an acceptable tool. Alternatively, the analyst may choose to use long-dated forward prices to estimate the future expected spot exchange rates or use an IRP forecast.

13.4 Summary

The internationalization of business is a further step in the growth of corporations, and it takes the business to another level. Specific conditions must be usually met in order for a firm to go international.

International trade has developed over the years, due to the progressive liberalization of cross-country exchanges of goods and capitals and also due to integration in several areas of the world.

Multinational corporations are big firms that establish their branches worldwide, following the demand for their products or services, after careful consideration of the implications.

Economic, social, and political factors usually drive the decision of getting globalized, and the management of a firm is challenged with important decisions related to the future of the business.

The cost of capital in an international environment is a controversial point of discussion. On one hand the enlargement of the geographical environment leads to diversification that should reduce the risk, making capital cheaper.

On the other hand, a multinational corporation is much harder to control, and it is usually subject to various additional sources of risk, on top of the classic ones, including political risk.

The exchange rate is the key to understand the dynamic of currencies around the world. Through instruments like the purchasing power parity and the interest rate parity, it is possible to establish theoretical relationships among the relevant variables.

The foreign exchange markets are characterized by a high volume of trading, and they define the level of the exchange rate in the world. Foreign exchange risk can be managed through derivatives written on the exchange rate.

International capital budgeting involves the introduction of the exchange rate and other variables into classical present value models. The result is a family of models that capture the value added by internationalization.

Problems

1. Why is capital budgeting analysis so important to the firm?
2. What is the intuition behind the NPV capital budgeting framework?
3. Discuss what is meant by the incremental cash flows of a capital project.
4. What makes the APV capital budgeting framework useful for analyzing foreign capital expenditures?

5. Relate the concept of lost sales to the definition of incremental cash flow.
6. What problems can enter into the capital budgeting analysis if project debt is evaluated instead of the borrowing capacity created by the project?
7. What is the nature of a concessionary loan, and how is it handled in the APV model?
8. What is the intuition of discounting the various cash flows in the APV model at specific discount rates?
9. In the Modigliani-miller equation, why is the market value of the levered firm greater than the market value of an equivalent unlevered firm?
10. Discuss the difference between performing the capital budgeting analysis from the parent firm's perspective as opposed to the project perspective.
11. Define the concept of a real option. Discuss some of the various real options a firm can be confronted with when investing in real projects.
12. How is international financial management different from domestic financial management?
13. How is a country's economic Well-being enhanced through free international trade in goods and services?
14. What are multinational corporations (MNCs), and what economic roles do they play?

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Special Topics

14

The previous chapters showed how standard corporate management can be carried on through knowledge of corporate finance tools and issues and how to exert the standard functions of corporate management.

Beyond that, there is a range of operations and actions involving extra skills and knowledge, which are still very important for the life of the business and must be taken into account for daily activity.

Mergers and acquisitions resemble the need for a business to expand both in terms of magnitude and in terms of the range of the activities to be performed. In this sense it entails the need of the corporation to survive through diversification.

For this purpose, knowledge of the corporate governance is also important, in that management is the key to success and it is crucial to acquire knowledge about the governance structure of any corporation.

Among the many functions of management, for the correct development of corporate core business, risk management is among the key functions to be addressed for a successful and profitable business life.

All of these topics are of primary importance in order to complete the picture about how the corporation interacts with markets and how the value creation principle of good corporate finance can be achieved.

After studying this chapter you will be able to answer the following questions, among others:

- What is a merger or acquisition and what is the process for it?
- How does a hostile takeover take place and how can a firm defend itself against it?
- What is corporate governance and how do concepts like management failure and remuneration schemes impact the daily life of the company?
- How can corporate risk management be defined and what are the methods of identification and mitigation?
- What are the possible risk response strategies available to the firm?

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The first section of the chapter focuses on the concepts of merger and acquisitions, with insights on the process behind it and the concept of hostile takeover and defense. The second section deals with corporate governance, explaining the management failures and remuneration schemes influencing the behavior of management. The last section is about risk management with the description of corporate risk, the process for identifying and mitigating it, and the possible response strategies.

14.1 Mergers and Acquisitions

Learning Outcomes

- Understand the reasons behind a merger.
- Explain the process of merging and acquiring.
- Define hostile takeover and defense measures.

14.1.1 Reasons for a Merger

In corporate finance, a merger is a combination of two or more companies into one new institution. An acquisition is very similar, with the only difference being the way in which the combination of the two companies is brought about.

In the process of merging or acquiring, the settlement can be done with cash, but in some cases the acquirer may offer either cash or its own shares in exchange for target shares.

Cash transactions offer shareholders an immediate potential profit, whereas shares offer a longer-term investment. Share transactions tend to be more attractive to shareholders in a buoyant market as the value of the shares is likely to increase more rapidly than in a stagnant market.

The reasons for a merger are various, and companies choose such a strategy based on many factors. From common literature findings, there are several rationales and drivers motivating a merge.

The rationales identify the particular framework of thinking that leads to the conditions under which a decision to merge could be made. On the other hand, the drivers represent mid-level-specific influences, from financial to operational, that contribute toward the justification or otherwise for a merger.

A company, for example, may decide to acquire another company for implementing an expansion strategy (and that qualifies as the rationale). In the specific, over- or under-capacity may justify the operation. As an example of driving factor, consider, for example, the desired capacity control in a specific sector.

It is very important to illustrate the main differences between a merger and an acquisition, because the two terms may sound very similar, and it is very easy to be confused.

First of all, consider that the word merger has a strictly legal meaning and is not related to how the merged companies will operate in the future and how the resulting company will be managed.

In a merger, a company combines with another, disappearing into another corporation. All mergers are statutory, meaning that they occur as specific formal transactions in accordance with the regulation of the states where the companies are incorporated.

On the other hand, an acquisition is the process by which the stock or assets of a corporation are owned by a purchaser. The transaction may take the form of a purchase of stock or a purchase of assets.

As one can imagine, the difference between a merger and an acquisition is not that clearly distinguishable. Sometimes in fact both terms are used as synonymous, but there is a slight difference between them.

In a merger there are two companies that form a new company together. After a merger, the separately owned companies become jointly owned and get a new single identity. The stocks of the two originating companies are surrendered, and new stocks in the name of the new entity are issued.

A merger usually involves two companies of similar size, in which case the operation is defined as a merger of equals. In the case of an acquisition, however, one firm takes over another and establishes its power as the single owner.

In an acquisition there is usually a bigger and stronger company taking over a smaller and weaker one. The relatively less powerful smaller firm loses its existence after acquisition, and the firm, which takes over, runs the whole business by its own identity.

In the process of acquisition, there is no surrendering of the stocks of the acquired firm, but they continue to be traded in the stock market. In reality the difference is very little and subtle.

The acquiring company, even if bigger than the acquired one, usually tends to announce the process as a merger anyway. Although, in reality an acquisition takes place, the firms declare it as a merger to avoid any negative impression.

Another important difference is that a merger is based on friendly terms between the companies involved, while the acquisition is an unfriendly deal where the stronger firm inglobates the target firm, even against its will to be purchased.

In particular, when an acquisition is “forced” or “unwilling,” it is called a takeover. In a takeover, the management of the “target” company does not agree with their target company being taken over.

That case is different from the one when managements of both companies agree both mutually and willingly, in order to proceed to the acquisition. In this case it is called simply acquisition or a friendly takeover.

Consider in that respect that an acquisition or takeover does not necessarily entail full legal control. A company can also have effective control over another company by holding a minority ownership.

The process for merging or acquiring is relevant to the overall profitability of the operation. In fact, the process of merger and acquisition can heavily affect the benefits derived out of the merger or acquisition. This is why the process should be designed in a way to ensure the maximization of the benefits of a merger or acquisition deal.

Technically, the process for mergers and acquisitions is made of several steps, in order to ensure the implementation of the operation in light of profitability for the acquiring company.

First of all, a preliminary assessment of the target value must be performed, in order to grasp its market value. Such a step involves the assessment of the current financial performance of the company, as well as the estimated future market value.

The second phase is the proposal, which is made after careful review of the target's financials is made. Generally, this proposal is given through issuing a nonbinding offer document.

At acceptance of the proposal by the target company, the exit plan phase plans for the right time to exit and be absorbed by the acquiring company. The choice is between full sale, partial sale, and others. The firm also does the tax planning and evaluates the options of reinvestment.

After the exit plan is finalized, the target firm operates a structured marketing plan in order to achieve the highest possible selling price. In this step, the target firm concentrates on structuring the business deal.

Rationales

The types of possible rationales for a merger or acquisitions can be classified according to the purpose they underlie. The strategic rationale, for example, provides a set of strategic objectives.

The abovementioned need to secure control of capacity is an example of strategic rationale. It is common in fact for a company that wants to enter a profitable market, but does not have the know-how to do it, to acquire a well-established company in the field.

Such a strategy allows the acquiring company to penetrate the new market through the acquired company, without the need to develop the technology and research needed in order to do it.

Developing a research and development division in the new market products in an attempt to catch up and overtake the more established players would in fact have obvious cost and time implications.

Another way to look at the strategic rationale is from a defensive point of view. Generally, in a business environment characterized by many mergers of companies in a particular sector, not merging would be a disadvantage.

A non-merged company may be pressured into merging with another non-merged company in order to maintain its competitive position. This is what happens in strategic sectors populated by very big players.

This is the case of the oil sector, where in the past it became a common strategy for the big companies in the sector to merge and create even bigger corporations. In this way few super-companies were created.

In this type of industries, the merger wave is usually driven by a need to respond to the merger activities of competitors and preserve the market share by enlarging the magnitude of the business, on a comparable scale with competitors.

Another type of rationale is named speculative, and it happens when the company to be acquired is intended as a sort of commodity. The acquired company could be, for example, a start-up in a new and developing field.

If the management of some company targets the company to be acquired as a source of potential profitability in the new field, the acquisition would give the benefits of exploring a new business area without directly being involved in it.

One way to achieve this is to buy established companies, develop them, and then sell them for a substantial profit at a later date. This approach is clearly high risk, even if the targets are analyzed and selected very carefully.

When performing a merger, the acquiring company may face the risk of loss of human capital. It is not uncommon in fact that a significant proportion of the highly skilled people who work for the target may leave the firm.

This can happen either before, during, or immediately after the merger or acquisition, and it is particularly true for small companies operating in highly specialized and technological fields.

In case the loss of specialized workers and managers is significant, the target company may lose a large share of its value, for the acquiring company, making the merger a less successful operation than initially expected.

There are several forms of speculative rationale, and another common one is the merger in view of splitting. This happens when the acquirer purchases an organization with the intention of splitting the acquired organization into pieces and selling these, or major parts of them, for a price higher than the cost of acquisition.

The speculative rationale is also high risk in that it is very vulnerable to changes in the environment. Apparently attractive targets, purchased at inflated (premium) cost, may soon diminish significantly in value if market conditions change.

Another type of rationale that can drive the decision of going for a merger is the management failure rationale. This happens when the acquiring company experiences management failures.

The management of a company decides for the right strategies to implement in the interest of the business. If the strategies are assembled with errors in alignment, or it is the case that market conditions may change significantly during the implementation timescale, the original strategy becomes misaligned.

In case of management failure, the company is no longer able to pursue the planned targets, and a huge variance is experienced in strategy. This can be due to many factors like customer demand, competitors, etc.

When the variance is so high that it is not possible to produce an endogenous change of direction for the business profitability, the only choice left is to acquire a company that will assist in correcting the variance.

Other rational types include financial necessity rationale, when financial reasons are at the basis of a merger. Sometimes, when a company loses momentum and trust from investors, a way to get rid of troubles is to merge with a more successful company.

Political rationale on the other side defines the impact of political influences in mergers and acquisitions, an expanding phenomenon in the world. In the last decade, many countries experienced the merger of a number of large government departments in order to rationalize their operations and reduce operating costs.

Government policy also encouraged some large public sector organizations to consider and execute mergers. These policies resulted in the merger of several large health trusts and universities among others.

Also big banks have been given incentives to merge domestically and also to embark on an aggressive overseas acquisitions policy. The international aspect is particularly true for those institutions prevented by domestic legislations to directly merge with each other.

Drivers

One of the drivers for a merger is the need of specialized human capital and resources, which leads a company to look for another company to acquire in order to absorb the specific skill or resource owned by the other company.

It usually happens in the framework of a big company targeting a smaller one, with the latter having developed some high-value specific skills over a number of years and where it would take an acquiring company a long time and a great deal of investment to develop these same skills.

In times of booming or recessing markets, the value of the shares of an acquiring company becomes another important driver, in that valuable shares can be used as a mean of payment for the merger or acquisition.

On the other hand, when a company experiences a sensibly low value of its shares, it becomes an appealing target for other companies that want to acquire, therefore becoming very attractive for a cash purchase.

Globalization of economies and markets is another driver that encourages mergers, due also to the growth and development of IT. In the modern era, the geographical distance and separation between companies in fact hinder good business.

Organizations have the incentive to work together as single entities, both within the same countries and across international boundaries, making it easier to access new markets. One way to achieve that is of course through international merger and acquisitions.

As a driver it is also worthwhile to mention consolidation, as both national and international levels, of compatible companies that are available for merger in the same local or international area.

Diversification is another important driver for the companies that want to diversify their business by entering new areas or sectors, in order to balance the risk profile of their investment portfolio.

It must be said in that sense that several studies by reputable scientists show that this type of diversification does not effectively reduce the risk profile of the corporation, given the assertion that the more diversified an organization is, the less it has developed the specific tools and techniques to face specific issues relating to any one of its range of business activities.

Among other drivers it is worthwhile to mention:

– Industry pressures, in specific sectors and specific time periods.
– Capacity reduction, so that the merger leads to increased control on a sector output.
– Vertical integration, with the merger of supplier and customer, etc.

14.1.2 The Process of Merging and Acquiring

According to the type of companies involved and the framework of evolvement, several types of mergers can be identified, all of them resembling the different rationales and drivers guiding the process.

It is possible to talk, for example, about horizontal mergers when two or more companies operating in the same business sector decide to merge. The companies run the same type of business.

The main purpose of this merger is to obtain economy of scale in production by eliminating duplication of facilities, reduction of competition, reduction of cost, and increase in share price and market segments.

There are some competitive issues attached to a horizontal merger. First of all, when the companies merge, competition is eliminated to an extent that can be significant, if the companies are big.

Moreover, the combined operations of the merging firms may create substantial market power and could enable the merged entity to gain a monopolistic position and therefore raise prices by reducing output unilaterally.

Finally, there is an issue of concentration of the market players other than the merged firms. The transaction may in fact strengthen the ability of the market's remaining participants to coordinate their pricing and output decisions.

Vertical mergers involve companies that operate at different stages of the production and distribution process of the same good.

The main objective is to increase profitability by the previous distributors. When a company combines with the supplier of material, it is called backward merger, and when it combines with the customer, it is known as forward merger.

The main benefit of a vertical merger is the integration of the production chain, with subsequent internalization of all transactions between manufacturer and its supplier or dealer and the conversion of a potentially adversarial relationship into something more like a partnership.

The internalization process is useful in that it gives the management more power and instruments to monitor and improve performance. On the other hand, the entrenched market may hinder new business from entering the market, making the vertical merger very anticompetitive.

The vertical integration following a merger in fact does not really change the number of players operating at one level of the market but acts as a mean of change of the general industrial behavior.

Whether a forward or backward integration, the newly acquired firm may decide to deal only with the acquiring firm, thereby altering competition among the acquiring firm's suppliers, customers, or competitors.

In terms of market efficiency, the downturn of a vertical merger is represented by phenomena like outlets deprived of supplies, competitors not able to access both supplies and outlets, etc.

Therefore, there is an issue of vertical integration as a means of market distortion, by foreclosing competitors through limited access to resources of market players. Vertical mergers may also be anticompetitive because their entrenched market power may impede new businesses from entering the market.

In the case of a conglomerate merger, the integration happens between two companies that operate in different business activities. Conglomerate transactions take many forms, ranging from short-term joint ventures to complete mergers.

A conglomerate merger involves companies that operate in different markets and can be pure and geographical or resemble a product line extension. There is no direct effect on competition for this type of merger.

Such a type of integration enhances supply to market and also the demand for new firms, thus inducing entrepreneurs to start new business. It also provides opportunity for firms to reduce capital cost and overhead and achieve other efficiencies.

On the other hand, a conglomerate merger may lead to a reduction of small companies in the market, while increasing the overall political power of the merged ones. In this way the democratic processes and independency of decisions are put at high risk.

In a reverse merger a company, usually a healthy one, is engaged in merging with another company that is financially weak, so that a new company comes to existence and the former is dissolved.

14.1.3 Hostile Takeovers and Defense Strategies

As mentioned before, when the acquisition is opposed by the management of the target company, it is defined as a hostile takeover. The management in this case advises its shareholders not to sell to the acquiring firm (Savela 1999).

The term hostile takeover also applies in the case when a bid is placed for the shares of the target company, and the management of it is not informed, so to be directly aimed to the shareholders (Damodaran 1997).

The purchasing offer can be in fact directly addressed to the shareholders, with or without having the management of the target firm giving the consent or going through negotiations.

The line is thin and the matter is controversial, because it happens sometimes that a takeover that starts as friendly and normal suddenly develops into a hostile bid, during the acquisition process.

Generally speaking, however, the terms hostile takeover identify that type of bids and offers which are generally aimed to the shareholders of a target company, in a way to get control over the company itself by overtaking the management.

There are several reasons behind the choice of making a hostile takeover, with the main one usually being the will to change the management. It is said in fact that the best way of replacing an ineffective board or management of a targeted firm is through a hostile takeover.

There are cases in fact of companies that operate very inefficiently, besides having a great potential of business growth. As a consequence the shares of the company are undervalued on the market, while the real potential value is much greater.

The aim of the acquiring company is to then replace the board and management, so to let the company achieve its full potential and profitability, with the consequence of increasing the stock value.

It is obvious that companies having a bad management which pursues personal aims rather than the corporate objective are more subject to become the target of a hostile takeover trough agreement with shareholders.

This type of behavior in some sense guarantees that all companies on the market are profitable in the long run. Short-run management inefficiencies in fact may be soon overtook by takeover and replacement.

As mentioned before, this could be due to the lack of economic growth potential the management has been able to realize or perhaps that the strategic plan of the board does not comply with the shareholders' view or expectations (Samuels et al. 1999).

It follows that the hostile takeover can in some cases be seen as a way to market effectiveness that helps achieving profitability and corporate value through simple management replacement (Weston et al. 2004).

There has been a lot of research done about the issue of positivity or negativity of takeovers. There is still no consensus about that, so still it seems that the beneficial or negative affects depend highly on whose angle one is viewing from.

Empirical research could not resolve the issue of the change in value of the acquiring and the target company. However, there is evidence that all the gains or positive effects tend to go to the shareholders of the target firm and that the acquiring firm pays a premium for their company.

Also, in most cases, the value of the target shares increases after a takeover, making it a profitable operation for its shareholders. In this sense, the term hostile is just limited to the reaction of the board and management to it. A takeover which takes the shareholders of the target firm in consideration first is called positive (Weston et al. 2004).

The bid on a takeover consists in an offer to be made to the shareholders of the target company. Such an offer is usually higher than the market values of the shares, in order to give incentive to a positive response.

The bid premium defines the difference between the offered price and the market price of the shares. It represents an extra on the market price, and its size depends on the willingness of the acquiring company to pay for the target company.

According to surveys done in the United Kingdom, average bid premiums of hostile takeovers in the United Kingdom are between 35 and 45% higher than the market price of the target company (Schoenberg 2003).

There are several possible reasons to develop a defense strategy against a hostile takeover. In one case the will is to prevent or block the takeover and prevent the bidder from gaining ownership over the company's equities or stocks. In another case, the defense strategy is aimed at staggering the bid and even increasing it, thus resulting in a higher bid premium received by the target firm.

When the acquisition from another company goes through direct contact with the shareholders of the target company, the operation is considered hostile by the board of directors, which is not asked about it.

As from Savela (1999), there is a genesis for the expression of hostile takeover that resides in the negative attitude of boards. The reasons for such an attitude can perhaps be explained by several reasons and not always related to the valuation of the actual bid.

There are several reasons behind a negative attitude from the board of directors of a targeted company. They range from the fear of a negative effect on the company growth to the preoccupation with losing their positions and jobs (Bebchuk et al. 2002).

It is not surprising then that boards of targeted companies are ready to implement all possible actions in order to prevent a hostile takeover, preserving their independence and ensuring that the hostile bidder is pressured to sweeten their bid further.

It is clear to the directors that sometimes the defense is very unlikely to be successful. However, it can make the acquisition more costly or time-consuming for the acquiring company, thus making the targeted company less attractive due to the rise in cost which follows.

Also called shark repellent tactics or antitakeover measures, these measures can be used in a reactive approach to hinder a presented hostile bid or proactively to make sure that future raids from targeting companies are hindered (Pearce and Robinson 2004).

The range of possible strategies for defense is wide and it depends on the single case. The right reaction must be tailored to fit the acquisition strategy used by the acquirer, and the motives the targeted board of directors.

What follows is a review of some of the most common defense strategies and their use for defense against a hostile takeover. The main categorization is between proactive and reactive strategies, depending on when a company decides to adapt it.

Proactive measures are those used to make the company less attractive to potential acquirers. They are taken right before the actual hostile bid presents itself, and the later one is implemented in connection to the hostile bid (Pearce and Robinson 2004).

The first way a company has to take control over another company is to insert its own people in the board of the target company. Attaining representation and voting power are important to influence other board members to accept the bid or influence the shareholders to take a more positive stance toward the takeover (Bebchuk et al. 2002).

One tactic the targeted board can use for protection is through a staggered board, which can be created by first contacting the shareholders and letting them approve its creation in a meeting. It is then necessary to get seats on the board, again through a shareholder meeting where the members are chosen with the support of the shareholders.

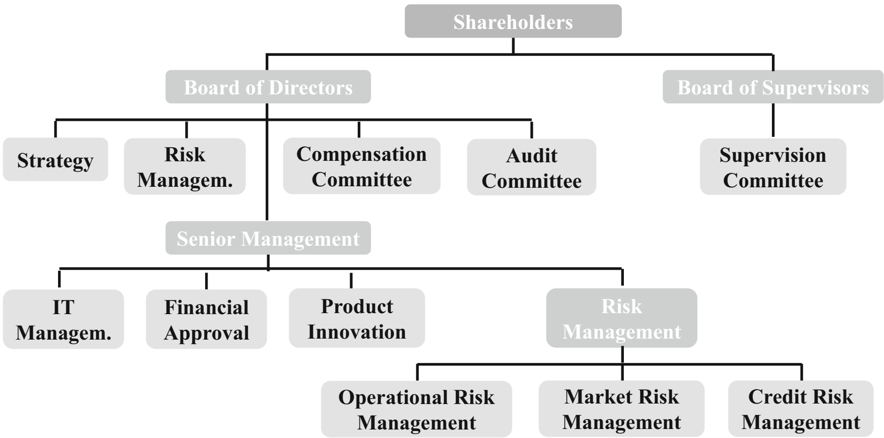

Fig. 14.1 An example of a typical board of a medium-big-sized bank

One way to infiltrate the board in this way is through the acquisition of enough shares to gain sufficient voting rights to elect one or more members to the board. The staggered board strategy is therefore a very time-consuming and expensive process of gaining influence and control on the board (see Fig. 14.1).

The practical method of running a staggered board strategy goes through several steps. First of all, a board is divided into groups. Replacing the board can take as many years as the number of groups created, not just one.

As a consequence, the acquiring company can start by acquiring a maximum number of seats equal to the number of the members in a single group and then wait for another year before having the chance to elect an additional amount of members. The process goes on until the acquiring company gains the majority of the board (Bebchuk et al. 2002).

The whole process clearly slows down a potential acquisition, making the staggered board a valid tactic against takeovers. It is an efficient method to prevent the acquirer from gaining control of the whole board instantly.

The large delay in gaining control, deriving from the defense application, results in increasing costs for the acquisition. Sometimes these extra expenditures make the target company less appealing, therefore discouraging the takeover (Bebchuk et al. 2002).

It was estimated that on average shareholders would get 8–10% less in acquisition premium from the bidder when adopting a staggered board. Though this tactic is moderately effective, it does not imply a definite protection against a hostile bid.

Obviously the staggered board strategy does not imply that the bidder cannot directly deal with shareholders by buying a large amount of shares, to gain control. In general, such a defense strategy is effective when combined with other defense measures.

Another popular defense strategy is the so-called poison pills and was introduced in 1982 with the name of warrant dividend plan. In its most common forms, it resembles the “flip-over” pill and the “flip-in” pill.

With a poison pill, the target company attempts to make its stock less attractive to the acquirer, by using different approaches. A “flip-in,” for example, allows existing shareholders (except the acquirer) to buy more shares at a discount.

The investors get extra profits by buying more shares cheaply, and, most important, they can dilute the shares held by the acquiring company. This makes the takeover attempt more difficult and more expensive.

Poison pill is one of the most used and controversial defense strategies used, which is derived from the domain of espionage. In particular, it relates to the practice of secret agents being captured who swallow a poisoned pill in order to die and not be interrogated.

Poison pills have many names and are therefore also described as, shareholders’ rights, preferred shares, stock warrants, or options, which the target company offers and issues to its shareholders (Pearce and Robinson 2004).

The board of directors is the only in power to redeem any poison pill once implemented. Rights become valid when they are triggered, not before (Ruback 1988). Triggering usually happens in the event of an unwanted takeover.

In case an unwanted shareholder acquires a pre-specified amount of the outstanding stocks which has been agreed on by the board of directors, it is still possible to stop the escalation by activating the poison pills.

There are usually pre-specified thresholds or ranges to be matched by an acquisition before implementing the defense. The range is usually set as 15–20% of the stock for a single shareholder who has not been in contact with the board of directors.

The threshold becomes higher in case a tender offer is put in place by the acquiring company. A tender offer is defined as targeting more than around the 30% of the targeted company (Pearce and Robinson 2004).

The implementation of a poison pill is such that it dilutes the capital of the targeted company. This action results in the bidder not being able to achieve control on an important part of the company without the consensus of the board, with a loss of time and money.

Another way to use poison pill is to create more time to analyze the actual bid and possibly enter negotiations with the acquirer. The process may lead the two companies to agree on some other terms.

Moreover, such a defense strategy is an efficient way to put pressure on the bidder to raise its premium offer even further thus serving the shareholders’ wealth (Bebchuk et al. 2002).

The preventive nature of a poison pill has been underlined by many researchers, like Theobald (2006), for example. In fact, he argues that the poison pill is a preventive measure even though it is implemented after a bid has been presented.

A flip-in pill, for example, allows the firm to issue preferred stocks that can be bought only by existing shareholders. When they decide to exercise their rights, they get the opportunity to buy additional shares in the company for a price below the market value of that share.

The acquiring company cannot participate in the exercise of those rights, therefore leaving its capital diluted, due to the impossibility to compete with shareholders who can buy them for a discounted price.

The benefit in time delaying is evident. The bidder's operation is slowed down, and the targeted company gets the time to reflect on the bidding situation and manage to give incentives to not sell to the uncertain shareholders (Pearce and Robinson 2004).

In a flip-over pill, the existing shareholders are offered rights rather than preferred stocks. These rights are only triggered and set in motion when the total amount of shares of the target company has been acquired by the bidder.

In particular, with a flip-over pill the shareholders of the targeted company acquire the right to buy the shares of the acquirer at a discounted price, in case a total merger or acquisition is put in place.

The main effect of exercising such rights is the negative impact on the balance sheet. In the event of exercise, in fact the direct consequence is an increase of debt for all the shareholders of the acquiring firm.

Increasing the debt means raising the risk of the company's financial leverage and is thus seen as very unattractive for the acquirer who has to inherit these debts. The disadvantage of such a strategy however is that it is only viable after 100% of the target company is acquired.

The important decisions in a company, including those about a merger or takeover approval, are usually taken with consent of the majority of the votes, represented by the voting rights attached to the shares.

However, it is possible to amend the corporate chart in order to dispose a supermajority for important decision-making. That means any important aspect including a potential takeover could be approved if a large shares of votes are obtained.

Such a defense measure can raise the specific quota needed to approve large majority decisions. The percentage needed for approval normally increases to somewhere between 67 and 90%.

The amendment to the companies' chart can only be implemented by the shareholders, and not by the board. However, it is the board that has the power to decide whether to activate it or not.

In this case, the bidder can complete the merger without actually owning the shares, but it only has to present a merger proposal which then the shareholders have to vote on but now have to acquire a larger acceptance (Ruback 1988).

The supermajority amendments are sometimes activated for all mergers, and sometimes only for the hostile takeovers, or when a large shareholder is involved in the operation.

These amendments are often accompanied by escape clauses which allow the board of directors to redeem the amendment if the board decides to change its attitude toward the bid and enter negotiations, thus not restraining the boards' managerial power.

This type of defense, like the others, is not very efficient if not combined with other strategies. As for the poison pills, it is an efficient method to stop the bidder from immediately gaining control over the company and thus making the deal costly and unattractive. Moreover, the targeted company gains more time to reflect over the bid and perhaps negotiate with the bidder about a higher premium offer (Ruback 1988).

Another popular defense strategy is called golden parachute. It consists in a lump-sum payment to the board of the targeted company, meant at making the hostile takeover more costly and complicated.

The strategy is put in place after the takeover has started, and it usually starts at an acquisition percentage of 25%, with an average registered in the industry, of 26.6% according to public statistics.

Similar to other strategies, the golden parachute is usually implemented in combination with other tools, and on average, as from the findings of Lambert and Larcker, implementing golden parachutes as a line in defense measurements against hostile takeovers, on an average increased the wealth of the shareholders by 3%.

The golden parachute basically aligns the interests and incentives between shareholders and executives of the target company, in order to avoid agency problems, since generally managers are concerned about losing their job and may decide to oppose a hostile takeover, even when the latter has a positive impact on the value for the shareholders.

As it has been proven by Walking and Long in a study of 1984, the rationale behind golden parachute is that the probability of executives opposing a takeover bid is directly related to the takeover's effect on their personal wealth.

Also, it is argued that if receiving a large payment in the event of a job loss, the managers will be less inclined to block any takeover attempts and therefore evaluating the best decision for the shareholders (Harris 1990).

Summarizing, the golden parachute strategy makes more cumbersome and expensive any hostile takeover but only to a certain degree. Payments as a cause of the golden parachute strategy are a drop in the ocean compared to what the acquirement as a whole would cost. Therefore, it can be argued that the real effectiveness of the golden parachute strategy is limited.

Another class of possible defense strategies is the reactive defense measures. One way to implement it, for example, is to attack the logic of the bid, by trying to persuade the shareholders that a fusion will have a harmful outcome on both the company and the stock price.

As a strategy, it is quite effortless and cost-efficient, in case a hostile bid is placed. The board tries to convince the shareholders using arguments like the bid is too low or not adequate to the real value of the firm.

The board can also try to discourage their shareholders by accusing the acquiring company of being incompetent and only trying to acquire the firm's assets. However, such an action risks the result of shareholders thinking that the board is only interested in keeping their positions and not taking the shareholders' wealth into consideration (Weston et al. 2004).

Sometimes the board of the targeted company tends to share important information with other potential acquirers, in order to encourage them to compete in the bidding. This would of course create competition, resulting in a higher bid price overall. Management could also raise doubts about the merger and encourage shareholders to hold their shares for additional bids (Turk 1992).

In the so-called white knight defense strategy, the board tries to involve a third-party friendly company (white knight) in the acquisition. The management of the targeted company can negotiate several deals that do not have to include a full takeover of the firm and risk losing their positions.

The boards can opt for a white knight because of friendly relationship and intentions, believing that the chosen company will be a better fit for the corporate strategies, and to make sure there will be no major changes in employment.

It can also be used to play the other two parties against each other to further sweeten the bid. The most common result of applying a white knight strategy is that the target firm eventually gets overtaken by the white knight.

The independency of the target company is therefore never guaranteed, and the strategy seems to be useful just in preventing a hostile bidder from implementing measures which would result in a greater restructuring of the firm.

The white squire is a variation of the white knight, based on a third-party company acquiring a minority stake of the target company. It is a smaller portion but enough to hinder the hostile bidder from acquiring a majority stake.

In case an investment fund is raised through financial advisors, this can be used to purchase a small portion of the target company, therefore not requiring the identification of an external company. Here, hedge funds and banks are suitable white squires due to their ability to move large amounts of capital on short notice (Weston 2001).

The greenmail strategy is targeted for those cases in which a bidder is more interested in short-term profitability rather than long-term corporate control. It involves the repurchase of a block of shares held by a single shareholder, offering a premium in exchange of the so-called standstill agreement.

Such an agreement states that the bidder will no longer be able to buy more shares for a period of time, often longer than 5 years. The hostile attack will then be over, and more important, such a covenant is proven effective toward short-termed profits seeking bidders, who are given incentives to keep the shares instead of selling.

The greenmail measure is still used because there are some bidders who act intentionally in such a way so to receive a greenmail offer from the targeted company (Pearce and Robinson 2004).

Another type of reactive defense named crown jewel allows the company to sell out its valuable assets when facing a hostile bid, in order to decrease its own value for the bidder and stop the takeover.

Another way of implementing this type of defense strategy is to combine it with the white knight, by selling its assets to another friendly company and later on, when and if the acquiring company withdraws its offer, buy back the assets (Weston 2001).

The risk in implementing the crown jewel strategy is to put the whole operation of the firm at high risk. Moreover, the sale of assets to a white knight should be accompanied by an agreement guaranteeing the possibility to purchase the assets back.

Another downside of the strategy is that the sale of assets in some cases may generate a high volume of incoming cash flows, making the target company even more appealing to the hostile bidder, with an opposite outcome of the strategy (Weston 2001).

The final and last resource is to act against a hostile bid through litigation, by pursuing legal injunction, filing antitrust litigations, restraining orders, or filing a lawsuit against the bidding company.

The aim is to put the bidder under the pressure of gathering information to prove the takeover is legitimate. The institutions handling these types of matters are usually accessed by the bidder in order to file all the documents proving it.

Besides being inefficient against long-term takeovers, the strategy allows the target company to gain space and time to implement further defense strategies in the time the bidder is preparing and presenting its legal preferences.

Another important function of litigation includes the propaganda made by the board against the bidder, so as to put the shareholder in doubt regarding the convenience of the bid.

Moreover, the strategy puts pressure on the bidding company, forcing it to publicly reveal the business plan and strategy to be adopted for the target company after the acquisition is eventually completed (Pearce and Robinson 2004).

14.2 Corporate Governance

Learning Outcomes

- Define and explain management failures.
- Understand remuneration and incentive systems.
- Learn about postcrisis perspectives.

14.2.1 Management Failures

A big role in financial crises has been played for sure by the failures in corporate finance, together with other factors. Many points of weakness are raised in the structure of companies that were considered very strong, making governance tools inadequate in the time the pressure started getting high.

The large lacks in governance skills caused many big financial institutions and large industrial corporations to suffer, and the focus of international initiatives and regulatory bodies was on how to elaborate an effective response to it.

At an international level, the standards for corporate governance are set by the Organization for Economic Cooperation and Development (OECD) and amended by other institutions like the BIS and the World Bank.

The OECD is represented for governance matters by the Corporate Governance Committee (CGC) which is responsible for the principles governing the field and for the development of innovations for the future.

The committee has done an extensive analysis of the state of art of governance before and during the years of crisis, coming to the conclusion that the most severe driving factor of governance failure was the wrong implementation of existing standards.

Moreover, it was clear from confrontation with other organizations that the need of new regulation was not so imminent, while focusing more on the application of existing rules appeared to be a more urgent task.

Based on the principles of regulation recommended by the central authorities, an action plan based on two pillars was developed by the committee. The first pillar relates to the implementation of basic principles in specific areas of actions, while the second one aims at developing dialogue and feedback in view of an effective monitoring and prompt identification of issues.

The range of areas involved in the new regulation includes governance of remuneration, effectiveness of risk management, and board and shareholders' rights and practices. The main lessons from the crisis have been convoluted into conclusions to be published by the organization.

The issue of remuneration has been considered of primary importance in the development of the crisis. In particular, incentive systems have failed in fostering efficiency, because they were not based on performance-related criteria.

In most cases the necessary link between compensation and performance was proven to be weak and difficult to observe. Moreover, compensation schemes were often complicated and obscure.

According to new principles, the remuneration packages should in fact be determined taking conflicts of interest into consideration and with the help of professional advisors. Moreover, the shareholders should also be involved in the approval of compensation schemes.

Long-term performance should be the basis for designing compensation schemes for the management, by linking some part of the remuneration to the corporate results, leaving it variable and adjusted to the related risks.

A natural consequence of the above consideration is the shift of compensation packages toward floating components, to be linked to performance, and with a limit in the fixed part. Transparency is a crucial point as well, in that information on remuneration should be available.

It also turned out that in most cases the boards were not aware of the riskiness of daily corporate life. Risk management practices were so far considered as marginal and were not integrated with the general corporate strategy.

All corporate strategies were centered on profitability, with risk management department left isolated and subordinated, so to make the disclosure of foreseeable risk very inefficient.

The CGC analyzed the situation issuing a main message from its work: the need of a major involvement of the corporate boards in establishing the risk management practices, with risk managers being independent from profit centers.

Many boards have been found not capable to objectively judge the risk profile of their corporation, therefore shifting the focus on the independence of board members rather than on competence and skills.

Phenomena like the interconnection between boards sharing the same members (interlocking directorate) and the tenure of some members have magnified the effect of conflicting interests.

The OECD principles are widely known as the leading force of corporate governance regulation. Supervisory and regulatory authorities should be properly equipped to face the weaknesses of corporate governance.

Several cases of bad governance have come to public knowledge at the beginning of the twenty-first century. Among those, the most famous is probably the Enron Corp. collapse. An American energy, commodities, and services company based in Houston, Texas, Enron was, at the time of collapse, one of the largest energy companies in the world.

In 2001 evidence of fraudulent practices, including institutionalized, systematic, and creative accounting fraud, was discovered, making the company an example of the worst type of corporate fraud and corruption even today.

Back to some years before, at the end of the century, the price of Enron's shares was high, due to the effect of the expectations generated by the fraudulent accounting figures communicated by the management to the public.

Ambitious industrial plans that seemed to be feasible in light of the financial statements were instead totally unreasonable given the true state of finances of the corporation.

The size of Enron was so big that the company turned into a sort of clearing house for the market of energy derivatives, claiming the ability to price them correctly, due to its combination of an underlying utility business with a market trading overlay.

The management of Enron at that time claimed to be pioneers of a new corporate model. Large income was generated by the company, with consequential high compensations paid to the top managers and high fees paid to third-party consultants.

But the rising share price was used to finance off-balance sheet transactions, aimed at inflating further the share price by overestimating the company's earnings. That strategy could not survive the general stock market fall which began in early 2000.

The price inflating strategy was then followed by other companies attracted by the above strategy. Apparently the failure of Enron did not work as a lesson for other companies.

The crash of the big American corporation had a huge impact on financial markets, with many hedge funds having invested large sums in the Enron stocks. The impact was limited for big investors with well-diversified portfolios, but it was huge for other investors and employees who suffered most of the burden, losing their jobs and also their retirement incomes.

The case of Enron was the spy of a global scenario of frauds and conflicts of interest perpetrated widely in the corporate sector. Without the fall of the big company, these would otherwise have been undiscovered.

Legislation in the United States needed to be renewed and updated and the authorities issued the Sarbanes-Oxley Act in 2002, in order to improve the regulation, based on the conviction that the Enron case was in fact just an example of governance fraud.

The act introduced several innovations, for example, increasing the frequency of reporting due by corporations and by widening the range of obligations upon governance.

The main idea behind such a severe approach was that the board of directors of the companies should be chosen in light of independency, from external agents. However, there was no evidence, at that time, of a link between ineffective monitoring and the lack of independence of directors, either at Enron or elsewhere.

In the case of Europe, the most relevant example of corporate governance failure relates to the crack of the British bank and former building society, Northern Rock, in the United Kingdom, as a consequence of the financial crisis.

The British Companies Acts has served in the United Kingdom as the main form of regulation of corporate issues, for many years. After the world global crises of 1929, the regulation had to be changed in view of modern regulation of corporate governance.

The focus is again on interlocking directorate issues, leaving the floor to criticisms about the effectiveness of the Acts as a countermeasure against the issue of multiple boards of companies of the same group interacting with each other.

The Acts basically fails in requiring big groups to publish consolidated balance sheet and income account or an interlocking company to publish details of its holdings. This type of provision may serve as an incentive for transparency.

14.2.2 Remuneration and Incentive Systems

Management compensation and incentive structure have played a crucial role in the effectiveness of corporate governance, even in the years before the financial crisis fully exploded (see Fig. 14.2). The problem of remuneration was recognized at that time, as one of the unsolved problems in many countries.

The main focus is on the American economy, but of course the problem has never been confined to American companies, raising concerns also in Europe (Germany and Sweden in particular) and in the rest of the world, including (somehow surprisingly) China.

From the early 1990s on, the remuneration of managers, especially CEOs, has rapidly increased, making the difference with average workers' salary very big. This phenomenon happened proportionally to the increase in size of the corporations.

The increase in compensations however is not fully explained by the growth in the size of companies. A complementary issue that magnified the phenomenon is the practice of giving performance-related packages, which became very popular in the early 1990s due to salary caps introduced by the regulator.

In Europe, if one takes into consideration the period of years right before the crisis, normal salaries were growing at an average rate of 2%, while management compensations had a growth rate of about 14%. In the following years, the difference

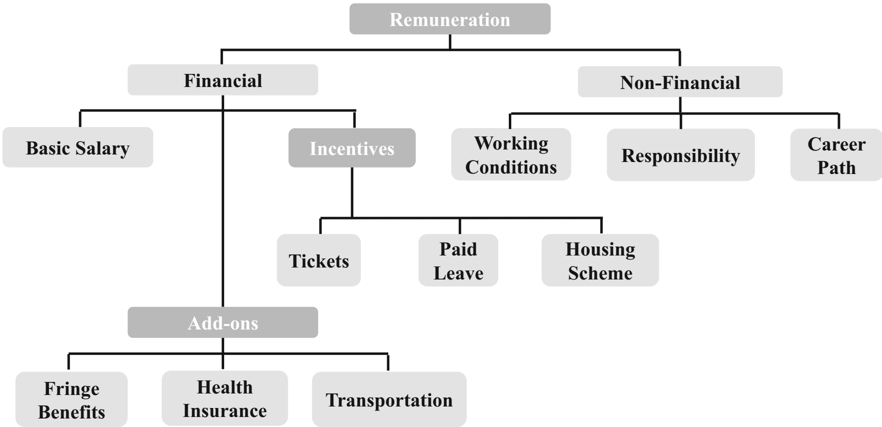

Fig. 14.2 An example of a typical remuneration package for top employees and managers

between the two classes of remuneration has grown more and more with a rapid increase of the ratio of CEO to average worker pay.

Basically a distressed situation was already in place, and the financial crisis contributed to make it even worse. It was in fact very common to reward top managers even in cases of failure or to assign them long lasting pensions and the continued use of company resources.

The presence of performance-related compensation was however insufficient to reach efficiency, given that the linkage criteria between compensation and performance were in practice very weak.

The compensation of top managers was more related to specific targets sometimes disconnected with executive performance, like the level of a company's share price, without any reference to the relative position of the company.

Executives were often compensated with shares of the company, therefore holding equity positions and sharing risk with the shareholders. As a result, the managers were facing a high risk.

However, as opposed to shareholders, managers could rely on the large compensation and bonuses attached to the performance, so to offset the losses on equity holdings and so to remove any incentive for the managers to limit the risk exposure of the corporation.

As a result, a very dangerous risk-taking behavior had spread among managers of major companies, leading to agency problems and highly risky situations. Only some of the managers, facing the pressure of public anger, gave up on their bonuses after their disasters were clearly observable.

A large number of corporations in the past years faced situations where structures of easy career advancement have led to excessive risk taking. As a consequence, dangerous behaviors have been put in place.

These behaviors include, but are not limited to, breaches of compliance obligations at all levels with serious consequences for the whole company, and it was evident that strong incentives had to be complemented by adequate risk management systems.

Another issue faced by companies was the relative lack of consideration of the need for risk adjustments when measuring the performance-related criteria for compensation. Given that below the top management, employees were not remunerated with stocks; this became a major issue.

When risk adjustment is not taken into consideration, there is a conflict between the interest of the employees and that of the shareholders, so that the company is often faced with paying its employees much more than their contribution is, in the mid-long term.

Theories on the functioning of remuneration and management reward are controversial. A part of the literature states that reaching a decision on remuneration through bargaining necessarily leads to optimality and equilibrium, with the regulators left only to guarantee the transparency of the process.

Another point of view believes that managers normally benefit from asymmetric information, which gives them a high bargaining power. It follows that the contracts are never optimal for the corporation but biased by the asymmetry.

Information asymmetry and bargaining power are important issues, magnifying the effect of lack of guidance in developing trustable performance metrics and hindering the construction of an efficient remuneration system.

The OECD principles were revised in the years 2003–2004, strengthening them in terms of information disclosure and the consideration of the long-term interest of the corporation in light of business plan.

Due to the above principles, the board of directors is in charge of developing and disclosing clear and effective remuneration policies, including the compensation due to the board itself and the top management.

There should be a clear turn toward a long-term relationship between remuneration schemes and the interest of the company. Also, external advisors and nonexecutive board members should be put in charge of giving independent judgment about the financial integrity and potential conflicts of interest.

14.2.3 Postcrisis Perspectives

The future of corporate governance regulation is to be built on specific pillars. First of all, the new framework will have to address governance issues to respond to the future developments of financial markets in a prompt and efficient manner and include share ownership and shifts in investment strategy.

Many companies suffered for the menace of the failure of corporate governance, and the lack of core values, integrity, trust, and others has turned down the reputational level of the corporate sector in the world.

Proper and ethical behavior must arise as a resource, a valuable asset for the company, and a company with a reputation for ethical behavior in today’s marketplace attracts both customer and employee loyalty.

The combination of good principles and ethical practices can lead to effective corporate governance. Fairness and honesty can be incorporated in the way companies conduct their affairs.

The legitimate goal of profitability must be bounded somehow by ethical consideration. Corporate image and reputation can be built on policies like environmental protection, ethical training, and compliance mechanisms.

Future corporate governance must face the challenge of the trade-off between the external regulatory provisions embedded in the system and the internal self-regulation, in order to balance the two.

The future of regulation goes through embracing all the aspects of modern corporate governance. Many experts around the world have observed there should be a new approach in terms of regulatory aspects, self-regulation, and the linkage to the market.

All the above considerations have an example in the Anglo-Saxon countries, usually characterized by well-balanced systems.

So the choice is between a self-regulation-based system, and a heavily regulated approach, and market based, where the latter could better serve the interests of the shareholders, in the opinion of the aforementioned scientists.

14.3 Risk Management

Learning Outcomes

- Define and explain the forms of corporate risk.
- Understand the risk management process.
- Elaborate response strategies.

14.3.1 Corporate Risk

A business in its daily life normally faces various types of risk, mostly identifiable in two broad categories, namely, business risk and financial risk. Contrary to what some people think, they are, in fact, not the same.

The difference between them is very important and must be known in order to appropriately handle both of them. The company profitability in fact is put at risk by both types of risk independently and simultaneously.

There is also an investor relationship perspective that is very important and relies on the understanding of the differences between risks. Financial investors, institutions, and other agents having interest in a company need to know the details.

As a broad category, business risk involves all the forms of risk related to business decisions, independently of the financial decisions. Examples of business risks are the risk to enter a new market, entering into a partnership with another business.

A rapid assessment of business risk is done internally. The efficiency and convenience of a project are assessed by experts in the company together with the calculation of the impact of such a project on the value of the company.

Business risk can be affected by several factors like the fluctuating demand for goods and services, changes in competition from other firms in the same industry, and introduction in the market of good alternatives to products and services. Other internal factors include the change of profitability ratios (gross margin, net margin, profit margin) as potential sources of business risk.

Other factors are disconnected from the actual business of the company and relate more to regulatory issues, with new intervention of authorities. Also, changes in the preference of customers can have a negative impact on the business.

Financial risk on the other hand relates to the capital structure of the company, with the focus on the corporate debt, which is the primary source of that type of risk. Companies with huge amounts of debt in fact are considered to be very risky, even if the assessment must also take into account the profitability generated by the investments financed by that debt.

As for business risk, there are several factors impacting on financial risk, including those relating to features and structure of the financial market which is domestic to the main activities of the corporation.

In case of a poor economy, the macroeconomic factors like undeveloped financial markets, high level of interest rates, and underrated currency may result in high risk, independently of the company structure and operations.

As a consequence, the leverage ratio of the company, given by the ratio of debt over equity, can be assumed as a good proxy for the corporate risk. Companies with high leverage usually entail higher levels of financial risks.

Whatever is the type of risk threatening the company, and whatever are the factors driving it, the risk management process is made of very precise steps, which can be summarized as:

- Identification.
- Measurement.
- Mitigation and/or transfer.

This section deals with general methods of risk management in general, and specifically for financial institutions, which are more involved in financial risk. In general, corporate stockholders are ready to accept that success for the business relies on taking appropriate risks (see Fig. 14.3).

As a general rule in finance, risk is such that any investor is compensated to bear it. For any level of risk, there is an adequate rate of return that compensates. So why should a company try to manage it?

The decision of hedging corporate financial risk comes from multiple perspectives. The main issue for a business is the riskiness of its debt. If the company defaults in facing its obligations, the consequence would involve direct and indirect costs that a firm should try to avoid.

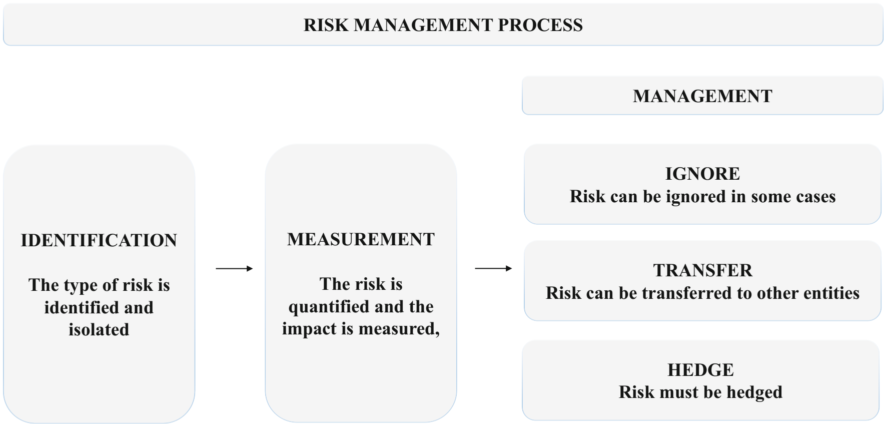

Fig. 14.3 An example of risk management process for a typical corporation

Direct costs of default are those that relate directly to the default of the company, when it happens. The category includes the costs for business reorganization through the bankruptcy process.

On the other hand, indirect costs involve all negative effects that the default can have as consequences. For example, there would be more difficulties in attracting new customers after facing a bankruptcy, as well as in entering new contracts.

In order to understand how costly such a process is, one should consider that often there are opportunity costs arising for a company, even if the default never occurs. The riskiness itself generates costs and that's why it is optimal to hedge risk most of the times.

Some managers may decide to engage in unethical behavior when facing a bankruptcy. It happens quite often that managers, on behalf of the shareholders, make inefficient decisions, just to drain most of the equity before the situation deteriorates.

Examples of inefficient and suboptimal decisions are underinvesting in profitable projects, even if they are useful to increase the value of the company, or maybe investing in negative NPV projects, thus carrying the risks only on the debt holders.

Findings of classical theory have suggested in past decades how to reduce the likelihood of default, through reduction of leverage. On the other hand, there is a tax shield effect which inevitably makes debt attractive to keep in the capital structure until some fair amount.

The tax shield effect is positive provided that the interest payments made on the debt are lower than the earnings associated with it. Generally, the benefits can be allocated in the balance sheet over the following years. Anyway in general the process involves loss of value, making it optimal to hedge risk in order to reduce the likelihood of losses.

Managers are risk averse due to the negative impact on their job that corporate distress may have. Their compensation is tied to the corporate profitability, and also if the company vanishes, they lose their positions.

This makes the interest of both the company and the managers converge toward hedging risk at all levels. The company will also pay less aggregate compensation to managers, due to the fact that risk-averse employees are willing to accept lower compensation if the whole employment package is less risky.

The impact on risk on the financials of the firm is evident as well. The value of corporate shares on the market strictly depends on the economy of the firm. When there is uncertainty about the ability of the firm to meet planned results, share prices tend to go down.

Investors in fact react to the news about the profitability of a company, and the uncertainty about the ability of the firm to meet its earnings and cash flow targets generates reactions on financial markets. The consequence is a high volatility of stock price, beyond market fluctuations.

This is why hedging risk is very important, in order to ensure some stability in the trend of stock prices, with a general positive effect of the new information hitting the market. This is also important in light of the different responses of investors to signals.

Another reason to ensure stability of revenues and cash inflows is that it is then possible to raise equity at a cheaper cost through profits retention. When a company needs capitals for new projects, the external sources are usually quite expensive.

Both corporations and financial institutions must undertake adequate process of their risk exposure, to maintain profitability and investor protection. The social implication of risk and distress is also important, especially for banks.

The following analysis is limited to banks, as an example of financial institutions that manage risk daily. There are several types of financial risk that are faced by a bank in its daily activities, with the main categories being:

- Credit risk.
- Market risk.
- Operational risk.

All banks must assess the three types of risk continuously in order to succeed. The profitability and social protection of liquidity are strongly dependent on the successful management of financial risk, at all levels.

Credit risk is the risk that a counterparty will not meet its financial obligation, leaving the bank with unpaid credits and missing liquidity. Losses on a portfolio, due to credit risk, can be divided into expected and unexpected losses.

Market risk refers to the loss of value due to market fluctuations, and operational risk is defined as the risk of losses due to failures in processes. They are easier to calculate than credit risk since there are more data available.

When all the above risk categories are assessed internally, the next step is aggregation. In order to manage risk properly, in fact, the combined effect of all risks must be taken into account.

On the other hand, the correlation among counterparties and among risks also plays a crucial role in order to assess the right amount of aggregate risk for each bank and optimize the risk management process.

The aggregation process involves the aggregation of the various risks and also the integration of the risks in the single departments of the bank. The management has then the chance to grasp the overall risk of a complex system and use the information to plan future activities and business.

The risk management process goes through several steps. From the identification of the risk to the management of it, good risk management must ensure that all investments undertaken add value to the corporation.

14.3.2 From Identification to Mitigation

The process of risk management is as important for a company as a good business plan. It is divided into steps, first one being the identification of the type of risk and its presence into an investment.

To analyze financial risk in the corporation, there are several methodologies. The most widely used is fundamental financial analysis. The tool is based on information about the firm, where the information mainly comes from the financial statement of the company.

The first look at risk comes from the proper observation of the income statement and cash flow statement. Using data from the financial statement in fact allows analysts to identify the assets at risk and the sources of riskiness to any specific risk factor.

The financial statement analysis is performed for internal use or external extended use, according to the purpose of the analysis. External use is done in the form of communications and reports to the stakeholders and the public.

The publicity of the corporate risk profile is important in that potential investors in a firm are always concerned about the riskiness of the business. For that reason, the major companies dedicate lots of resources to the issue.

The financial statement is composed of several books. The first one is the balance sheet, a report of the assets and liabilities of the company, together with the equity claims against the former.

Since the riskiness of the company is strictly related to the capital structure and leverage of the company, the balance sheet is a source of information about the capital structure of the company.

Starting from the balance sheet, the primary risk analysis is focused on the calculation of indices and ratios from asset and liabilities lined. The most important are:

- Debt on equity ratio shows the financial leverage of the firm. The higher the ratio, the higher the risk.
- Debt on asset ratio shows what part of the firm's capital is financed through debt.

- Return on asset (ROA) describes how much of capital is financed by the profits, therefore independently of debt sources and recapitalizations.
- Long-term debt on total debt describes the structure of company liabilities.

The information given by both long-term and short-term financial entries is very important when translated into ratios, because it generates risk awareness. All companies should manage to at least finance the fixed assets using fixed capital.

Doing that, the risk can be kept at reasonable levels, so that the company can benefit from appropriate management of resources.

Some companies operate internal risk management departments, but nowadays it is quite common for some companies to outsource the risk management tasks and engage outside consultants.

The expertise of external consultants allows for rapid risk identification, in the forms that are not evident to the insiders. An external eye is sometimes required to get a broader view of the company.

Time and resources involved in the risk management process are not small. There are some periods of corporate life in which the analysis of risk becomes very cumbersome for the management and employees. External consultants are therefore also useful in helping to assess risk with a consistent efficiency throughout the whole life of the firm.

As the first step in the risk management process, risk assessment involves the analysis of the likelihood and consequences of possible risk events and prioritizing those that must receive proper treatment. The criteria to evaluate the risk likelihood can be summarized in Table 14.1.

Once the likelihood of a risk event is established, it is time to assess the severity of the impact of such an event on the corporation. The strength of impact can thus be represented like in Table 14.2.

The information from the two tables can then be merged in order to get an aggregate representation of likelihood and impact of risk. This gives the complete information about riskiness in terms of the potential impact on the profitability of the firm. Such an outcome is usually represented in the risk priority Table 14.3.

When identification is done, the following step is the appropriate management of the risk. A good risk mitigation program can address the responsibility for the monitoring of the various risks.

Table 14.1 An example of how the likelihood of a risk event can be classified. Several levels of likelihood correspond to different probabilities for an event to happen

Level	Likelihood of event
5	**Almost certain**: The event will occur with a probability very close to 100%
4	**Very likely**: High probability that the event will occur in the short-medium term
3	**Possible**: Reasonable likelihood that the event will occur in the medium-long term
2	**Unlikely**: Low probability of an event, even in the medium-long term
1	**Rare**: The event will occur with a probability very close to 0%

Table 14.2 An example of the impact a shock may have, given that the event has occurred

Level	Impact
5	**Disaster**: The desired outcome will not be achieved at all
4	**Severe**: The outcome is heavily affected by the shock, with big losses
3	**Moderate**: The business is partially affected, with moderate losses
2	**Minor:** The impact of the shock does not play a major role on the outcome
1	**Negligible:** The impact is so small that can be ignored

Several levels of impact severity correspond to higher damages to the business

Table 14.3 The likelihood and impact scales can be combined to get a table of the different combinations of risk

	Catastrophe	Major	Moderate	Severe	Disaster
Almost certain	Extreme	Extreme	Major	Major	Medium
Very likely	Extreme	Extreme	Major	Medium	Minor
Possible	Extreme	Major	Major	Medium	Minor
Unlikely	Major	Major	Medium	Minor	Minor
Rare	Medium	Medium	Minor	Minor	Minor

A complete program includes the management and fixing of any potential material or virtual weakness in the corporate processes. The company board should implement recommendation from identification stage in order to reduce the risk exposure.

Effective signaling and vigilance of governance inappropriateness should be put in place and effective. Board members should give formality to decisions regarding risk management, with transparent reporting, in order to properly translate into clear and accessible reports of the firm's financial situation and exposure.

14.3.3 Response Strategies

After risk has been identified and measured, risk process terminates with the application of an appropriate response strategy to it. To do this, all risks must be first quantified and ranked in terms of impact and severity.

First of all, all risks are dealt with according to the risk tolerance of the agent. In this case the company must set its own level of risk tolerance and respond accordingly to it, so as to hedge the part of risk that is really unwanted.

When the level of risk is below tolerance, two possible reactions are avoidance and acceptance. In case tolerance is overtaken and an active strategy is needed, the two possible strategies are mitigation and transfer.

There are several factors to be considered when choosing the right response to implement. One example is the eventual dependence of the target task to other connected tasks, with riskiness being potentially transmittable. In this case, if severity is measured only on the first task, it could be underrated compared to the cumulative risk.

The first level of response is acceptance, which implies that risk [I inserted the word risk but I am not sure. You were saying that something is low but the word was missing] is so low that it is not worthwhile to do anything until the risk event occurs. The level of risk is therefore lower than the level of tolerance, and intervention is minimal.

It is in fact true that acceptance of risk does not mean absolute immobility. The event is not ignored, but no preventing action is put in place until the actual event occurs. Many insignificant risks fall into this category, and they are not the focus of our discussion.

Acceptance can have an active or a passive form. Active acceptance involves the tolerance of risk, accompanied by a detailed plan about actions to take in case the event occurs, as to be ready in advance.

On the other hand, passive acceptance is defined as the situation when no planning is made at all. In fact, the risk is so negligible that it cannot generate a real concern, and further actions are taken in real time when needed, and not planned before.

Transfer is another response method, involving the externalization of the responsibility for the risk or the risk itself. Risk can be transferred in several ways, for example, by rejecting parts of a project which are too risky.

It is also possible to act on prices to transfer risks, through demand of goods and services at a fixed price. Such a strategy implies that the contractor working on the project bears the risk of extra costs on the project realization.

The position of vendors is to set prices that are usually slightly higher than fair ones, and keep them fixed. The risk quote is therefore included in the prices, according to what a normal compensation for that type of risk would normally be.

Risk premium is a concept widely used in finance, which is useful to understand how proper compensation for risk is determined, on financial markets and business transactions.

The most popular way to transfer risk is however through an insurance, which involves payment of an affordable periodical premium in exchange of coverage against the target risk, which is transferred on the insurance company. The premium amount is a significantly small cost of the risk itself and the insurance pays if the risk event happens.

Risk avoidance is an extreme form of response to risk. It consists in implementing actions to fully and completely eliminate every possibility of an adverse event. Risk can be eliminated at the beginning, from any project, or dealt with after.

The obvious consequence of an avoidance approach is that all possibilities of an extra revenue as compensation of it are lost. It is therefore not so obvious to implement as a straightforward strategy.

A proper assessment is required before deciding to give up on all possible risk and, consequently, on potential extra compensation. In project design, risk can be sometimes avoided just by designing the project around it.

A much less extreme and therefore more popular form of risk response is mitigation, consisting in reducing unacceptable risk below tolerance level. Such a result is usually obtained through a proper reduction of the probability of the event to happen. This reduces the expected loss associated to the risk and therefore requires less capital to be put on it.

The general way of implementing the strategy is to invest a little amount of extra money in the project in order to reduce its risk profile and avoid potential future large losses and extra expenses.

All of the above strategies come at a cost, including avoidance, where no money is allocated for risk, but expenses are required in order to redesign the investment. Acceptance is also costly in that money must be allocated for contingencies, even if no initial cost is incurred.

Risk mitigation involves setting up a budget in order to face a risk event but also requires some initial amount of money in order to finance the mitigation activities. Risk transfer requires a small amount of money to be given to the insurance or to the subcontractor.

14.4 Summary

Mergers and acquisitions are important operations put in place when two companies find out that combining effort may reduce the costs by increasing the economies of scale and therefore proceed with merging.

There are costs and benefits associated to acquiring a new firm, and firms choose their strategy based on both economic considerations and legal aspects related to the environment of operation of both firms.

Corporate governance is an important aspect of the firm's management and involves the highest levels of managerial force, impacting on the operations and financial decisions of the firm.

The election of the board of directors with the subsequent election of the CEO, CFO, and other top managers is the key to success. A specific aspect is the remuneration policy, with a focus on the aspects related to the shift from contracts with high fixed compensation to contracts heavily based on performance.

Risk management is about reducing and managing the risk associated with daily operations of the firm and protecting the corporate financials from the effects of external or internal shocks.

Modern risk management is based on well-planned processes that go from the identification of the source of risk, to the assessment of the danger associated to the risk and the management of it.

Credit risk, market risk, and operational risk are the three major types of risk faced by institutions, and the management of such risks involves the development of appropriate response strategies.

Problems

1. What are the main reasons for a merger?
2. What are the forms of speculative rationale?

3. Explain the role of diversification in the firm's economy.
4. Describe how a hostile takeover takes place, and explain the difference between positive and negative takeover.
5. Describe the main types of poison pills, as a tool to prevent takeovers.
6. The issues of concern to those who have worked to develop new explicit mechanisms for the external governance framework include:
   (a) Company probity.
   (b) Executive honesty.
   (c) High audit fees.
   (d) Political influence.
   (e) The politics of profit and executive compensation.
7. Other issues that have not been addressed explicitly by corporate governance reformers yet which are viewed as important by society include:
   (a) Corporate performance.
   (b) Social responsibility.
   (c) Ethics.
   (d) Environmental standards.
   (e) Health and safety.
8. Distinguish between internal and external corporate governance, and explain how they relate.
9. Explain the interdependence between economic development and corporate governance.
10. Describe the foundation of the present system of corporate governance as it applies to limited liability companies and how it developed.
11. What are the forces that shape corporate governance reform efforts and what triggers them?
12. When managing risk you will only be expected to counter risks which your business may reasonably be expected to face while providing its services. However risk management involves a process of steps to be taken in order. This order is:
    (a) Risk identification, risk analysis, risk treatment, risk monitoring and review.
    (b) Risk identification, risk treatment, risk analysis, risk monitoring and review.
    (c) Risk analysis, risk identification, risk treatment, risk monitoring and review.
    (d) Risk identification, risk analysis, risk monitoring and review, risk treatment.
13. What are the factors impacting on business risk?
14. List and explain the various forms of response to risk.
15. Explain in what context it is better for a firm to externalize the risk management function.

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Index

A

Abandonment, 324, 325
Accounting information, 349
Adjusted present value (APV) model
    applying, 457
    complex example, 246, 456
Agency costs, 7, 243, 361, 439
All-equity financed companies, 249
American options, 300, 327, 328, 331
Annual reports, 386
Annuities
    present value, 42, 50, 51, 65
APT, *see* Arbitrage pricing theory (APT)
APV, *see* Adjusted present value (APV) model
Arbitrage pricing theory (APT), 112
Arbitrage proof
    example, 30, 33, 34
Asset earning power, 104
Audit, 349, 388, 393, 493
Average rates of return, 113

B

Bankruptcy costs, 226, 241, 242, 246
Beta value
    adjusted, 116, 265, 272
    estimation, 106
    measurement, 264, 265
    portfolio beta, 262, 264
Bid (selling) price, 189, 191, 193, 219, 451
Binomial model, 250, 324
Black and Scholes model
    assumptions, 314
    dividends, 314
    option valuation, 300, 310, 314–317
    using, 215, 314
Bonds
    coupon bonds, 54, 59, 68, 159, 163, 164, 167
    pricing, 150, 161, 163
    zero coupon bonds, 151, 152, 158, 163, 164, 167, 181, 355
Booming economy, 39
Borrowing
    capital market, 97
    lending, 24, 55, 97, 314, 356
    possibility of, 155
Bretton Woods, 299, 455
Business risk, 463, 484, 485

C

Call options
    combinations, 306, 308, 318
    hedging, 306
    valuation, 215, 314, 317, 324, 327, 335
Capital asset pricing model (CAPM)
    arbitrage pricing theory, 112
    assumptions, 102–106
    beta value, 106
    cost of equity, 195, 231–233
    dividend valuation model, 197
    empirical evidence, 111
    expression, 107
    security market line, 106–109
    summary, 110–112
    validity, 113
Capital cost of, *see* Cost of capital
Capital market borrowing, 97
Capital market line (CML)
    assumptions, 102
    market portfolio, 97, 103
    market price of risk, 99–102
    separation theorem, 225

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Capital rationing
- concept, 128

Capital structure
- changing, 231
- constant, 231, 236, 238
- earnings per share, 226, 234, 236, 237
- mixed, 229, 232
- pecking order theory, 241–244, 250
- trade-off theory, 241–244, 250

CAPM, *see* Capital asset pricing model (CAPM)
Caps (salary), 481
Cash flows
- financing, 23, 123, 232, 368, 419
- guiding rules, 132
- investment appraisal, 122
- project cash flows, 125

CML, *see* Capital market line (CML)
Collars, 211, 308
Company valuation
- adjusted present value model, 241
- dividend model, 267, 286, 287, 292
- free cash flow model, 284, 286, 292

Compound interest factors, 55, 156, 157
Constant capital structure, 231, 236, 238
Consumption
- CCAPM, 110
- factor, 110, 260
- level, 110

Contingent exposure, 454
Convertible debt
- advantages, 217
- cost, 218
- definitions, 217

Core activities, 7
Corporate tax
- rate, 151, 230, 231, 247, 250, 253, 266

Corporate taxation, 210, 247
Cost of debt capital
- fixed rate debt, 231
- impact of corporation tax, 179
- opportunity cost, 123, 232
- semi-annual interest payments, 216

Cost of equity capital
- capital asset pricing model (CAPM), 195, 201, 233, 258, 266
- dividends, 201
- market price, 194, 195

Country risk, 442
Coupon bonds
- pricing, 54, 159, 164

Covariance
- between two variables, 80, 81
- calculation, 92, 104

Cum dividend, 195
Currency exchange, 34
Currency finance overseas, 432
Currency swaps
- market, 302

Current market share price, 216

D

DCF, *see* Discounted cash flow (DCF)
Debt
- cost of convertible, 186, 216–218
- floating rate, 334

Debt capacity, 243, 398
Debt capital, *see* Cost of debt capital
Decision rule
- internal rate of return (IRR), 122
- net present value (NPV), 122
- payback period, 126

Decision trees, 130, 132–134, 145, 146, 320, 328
Depreciation
- foreign exchange rates, 447

Directors
- regulation, 481
- relationship with shareholders, 234, 471, 474

Discounted cash flow (DCF)
- assumptions, 293
- discounted payback, 126
- internal rate of return (IRR), 122
- net present value (NPV), 122

Diversification, 82, 87, 93, 102, 103, 109, 118, 263, 268, 269, 343, 432, 439, 440, 456, 460, 463, 468, 493
Dividend growth model, 194, 198, 201, 292
Dividend growth rate
- changing, 194

Dividends
- Black and Scholes model, 215, 324
- patterns, 200
- as residual, 13, 364, 367
- as signals, 236, 399
- valuation model and, 197, 199

Dividend valuation model (DVM)
- bid and offer prices, 191
- *vs.* CAPM, 102
- cum dividend, 195

dividend patterns and, 200
ex dividend, 235
DVM, *see* Dividend valuation model (DVM)

E

EPS, *see* Earnings per share (EPS)
Equity capital, *see* Cost of equity capital
European options, 300, 331
Exchange rates foreign, *see* Foreign exchange rates
Ex dividend, 194, 195, 235, 314, 328
Exercise prices, 213–215, 321, 324–326, 331
Expected returns
risk and, 96, 100, 102, 104, 108, 112, 115, 260, 441
Expected utility
individual choice amongst risky, 191
investors' behaviour axioms, 191
risk and, 191
utility function, 110

F

FDI, *see* Foreign direct investment (FDI)
Financial decisions
conditions, 43
types of long-term finance, 419
value base, 6
Financial investments, 147, 244, 297
Financial lease
evaluating, 365
Financial markets
market efficiency, 451
share dealing, 344
term structure of interest rates, 58, 485
Financial risks, 114, 156, 298, 299, 484, 485, 487, 488
Fisher effect, 449
Floating rate debt, 334
Floor, 218, 237, 303, 306, 355, 450, 481
Foreign currency options, 455
Foreign direct investment (FDI)
domestic *vs.* foreign projects, 434
economic risk, 434
Foreign exchange (FX) hedging
contingent exposure to FX risk, 454
traded options *vs.* OTC options, 306
transaction risk, 454
translation risk, 455
Foreign exchange (FX) markets
discounts and premiums, 455–457
exchange/delivery of currencies, 450, 454, 455
forward, 451, 453, 455
rates of depreciation and appreciation, 456
spot, 452–454, 460
swap, 451
Foreign exchange (FX) rates
buying and selling, 451, 452
cross rates, 460
determinants, 450
forecasting, 453, 460
inflation, 459
interest rates, 457, 458
Forward FX market
vs. FX futures market, 453, 455
Forward interest rates, 447, 448
Fundamental analysis, 234, 278, 284–289, 443, 488
Futures contracts, 189, 300, 301
FX, *see* Foreign exchange (FX)
FX options contracts
over-the-counter options, 452
traded currency options, 302

G

Gambles, 69, 146, 262, 360
Gamma of a call option, 318
Gearing
up, 457

H

Hedging, *see* Foreign exchange hedging; Transaction risk hedging
Hirshliefer separation theorem, *see* Separation theorem

I

Incremental cash flows
IRR and, 126
Inflation
foreign exchange market, 447
real and market rates of interest, 162
Insiders, 351, 437, 489
Interdependent projects, 326
Interest coverage ratio, 22

Interest rate
  collars, 211
  floors, 355
  foreign exchange market, 445, 447, 457, 458
Interest rate futures
  hedge efficiency, 301
Interest rate risk
  interest rate futures, 301
Internal rate of return (IRR)
  decision rule, 122, 125–126
  incremental cash flows, 125
  model, 60–62, 125–126
  NPV *vs.* IRR, 60
Intrinsic value
  determinants, 206
Investment, 262
  building blocks, 33
  decision, 129, 131, 225, 243, 320–322, 326, 410, 445, 454
  expected returns, 73, 89, 108, 110, 114, 232, 260
  financing decision, 6, 239, 432, 484
  physical, 12, 325
  put options in the shares, 305, 307
  return on investment, 278, 292
  risk, 127, 411
  risk and expected utility, 110, 191
  risk-free, 113, 127, 258, 259, 325, 447
  risk-free bonds, 31, 181
  risk-free portfolio, 97, 98, 103, 441
  shares, 33, 187, 190, 191, 197, 218, 227, 237, 254, 282, 414, 438, 464, 474
Investment appraisal
  adjusted present value (APV), 244–246
  changing capital structure, 224, 225, 227, 242, 243
  financing cash flows, 122
  and inflation, 147, 154
  payback method, 126–127
  return on capital employed, 252
Investors
  behavior, 30, 112, 483
  individual choice, 98, 127, 145, 375, 400, 458

L
Leasing, 353, 360, 364–369
London Stock Exchange, 353
Long-term finance, 339–369, 371, 407, 419
Low coupon bonds, 167

M
Management charges, 146, 301, 388, 393
Margin, 9, 18–21, 26, 38, 81, 110, 111, 143, 188, 190, 207, 220, 226, 230, 241, 259, 264, 280, 303, 383, 387, 397, 456, 457, 479, 485
Marginal rate of return, 111
Market efficiency, *see* Efficient market hypothesis (EMH)
Market portfolio
  assumptions, 103, 441
  risk, 106, 262, 433, 441, 442
Market price
  of risk, 99–102, 113, 310
Market rate of interest, 44, 45, 58, 154, 155, 172, 457
Maturity mismatch, 383
Maturity of option, 79, 153, 212, 316, 321, 325, 330
Mixed capital structure companies, 369
Money markets, 353, 375, 400, 439
Multi-asset portfolios
  three-asset portfolio, 115
Mutually exclusive investments, 124
Mutually exclusive projects, 122, 123, 126, 129

N
Net present value (NPV)
  calculating present values, 59–60
  decision rule, 122–125
  discounting, 125, 329
  NPV *vs.* IRR, 60–62, 125

O
Offer price, 189, 191, 414, 428
Operating lease, 364, 365
Opportunity cost of debt capital, 123, 232
Option contract markets, 79, 321
Option exchange market, 306
Option hedge setting up, 99, 306
Option valuation
  binomial model, 310–313, 324
  Black and Scholes model, 310, 314–317
  building blocks of investment, 321, 322

characteristics, 324
'Greeks', 317–320
intrinsic value, 206
market value, 315
premium, 306
put-call parity theorem, 322
real options, 310–321
summary, 333
terminology, 310–320
time value, 313
using share options, 315
Option writer, 306
OTC, *see* Over-the-counter (OTC)
Overseas currency finance, 432

P
Payback method
advantages, 127
decision criterion, 126, 127
disadvantages, 126
discounted, 127
time value of money, 126
working capital, 126
Pecking order theory
consideration of debt, 242
financing dilemma, 241
market signals, 243
rules, 242
Perpetuities, 41, 42, 48–52, 62–68, 197, 198, 201, 240, 273, 280, 288, 327
Personal taxes, 242, 246–249
Physical investment, 12
Political risk, 438, 460
Pool of funds concept, 368
Portfolio betas, 108, 109, 115
Portfolio theory
capital market line, 97
diversification within companies, 102, 109, 268, 343, 432, 468
multi-asset portfolios, 114
risk-free investment, 96–98
summary, 113
two-asset portfolios, 97
Potential decision maker, 328
PPPT, *see* Purchasing power parity theorem (PPPT)
Present value of annuity, 42, 50, 51, 65
Present value calculation, 42, 49, 59–66
Present value factors, 127
Pricing bonds, 161, 163
Probability, 39, 83–85, 88, 131–133, 145, 146, 169, 191, 231, 303, 310–313, 318, 334, 335, 358, 410, 433, 476, 489, 491
Profits, 2–4, 6, 8, 9, 15, 17–21, 26, 29, 31, 33–35, 37, 54, 55, 67, 74, 112, 118, 121, 127–130, 139, 145–147, 153, 175, 181, 185, 190, 207, 211, 215, 227, 231, 234, 238, 242, 252, 277, 281, 282, 286, 290, 299, 304, 306, 322, 323, 345, 349, 350, 366, 368, 371, 374, 380, 381, 384, 386, 406, 410–413, 415, 416, 431, 434, 437, 440, 445, 451, 457, 464–467, 471, 474, 477, 479, 484, 486, 487, 489
Project cash flows
inflation and IRR rule, 125
investment appraisal, 122
Project risk weighted average cost of capital (WACC), 230, 232, 233
Put options
combining with calls, 308, 313, 316, 331
valuing, 314, 324

Q
Quality spread, 136

R
Random walk, 169
Rate of interest, 172, 301, 363, 381
Real options, 297–333, 461
Real rate of interest, 323, 433, 449
Redemption, 25, 356, 398
Regulations
audit, 349
directors' transactions, 349, 481
ineffective, 481
reports, 281, 282, 303, 366, 481
stock exchange rules, 210
takeovers and mergers, 398, 464, 465
Required return on equity capital
changing, 233, 433
Return on investment, 54, 68, 90, 155, 278, 292
Rho of an option, 317, 319
Risk
abandonment decision, 321, 324, 325, 335
country/political, 56, 154, 259, 438, 440–442, 447, 460

Risk (*cont.*)
- downside, 265, 457, 478
- economic, 57, 150, 252, 440, 453
- expected NPV, 61, 62, 123, 124, 130, 246
- expected return and, 30, 73–88, 92, 96, 100, 104–106, 110, 112, 115, 260, 324, 432, 441
- expected utility model, 93, 110, 191
- financial, 73, 114, 156, 298, 299, 484, 485, 487, 488
- individual choice, 260, 341 (*see also* Interest rate risk)
- investment, 56, 73, 74, 92, 97, 100, 104, 108, 113, 127, 129, 131, 150, 153, 154, 225, 243, 260, 262, 264, 268, 297, 320–322, 325, 326, 333, 341, 342, 366, 410, 445, 454, 468
- investment decision, 74, 129, 131, 225, 243, 320, 322, 326, 410, 445, 454
- market portfolio, 97, 98, 103, 104, 106, 115, 264, 440–442
- market price, 2, 99–102, 113, 310
- overestimating, 262
- return and, 73–113, 117, 119, 137, 264, 342, 383, 399, 440, 441
- sensitivity analysis, 165, 320
- summary, 112, 113
- systematic/unsystematic, 102, 103, 106, 109, 116, 262, 270 (*see also* Transaction risk hedging)
- translation, 454, 455 (*see also* Uncertainty)
- underestimating, 268

Risk-adjusted discount rate
- certainty-equivalents, 197, 252
- risk and return, 127

Risk-adjusted WACC, 229, 230, 232, 254, 273
Risk-averse, 98, 260, 261, 263, 398, 440, 441, 444, 487
Risk-free bonds, 36, 118, 150, 154, 247, 258
Risk-free investment
- borrowing possibility, 97, 98, 104, 151, 154, 188, 259, 314
- combined borrowing/lending, 13, 97, 98, 314
- portfolio, 97, 468
- possibilities, 118
- riskless asset plus risky portfolio, 98
- risky-riskless boundary, 31, 33, 99
- two-asset portfolio, 35, 36, 97, 114

Risk-reduction effect, 381
ROCE, *see* Return on capital employed

S

Security market line (SML)
- and capital market line (CML), 102, 107, 114
- derivation, 107

Segmented markets, 16, 272, 303, 442, 469
Semi-annual interest payments, 216
Shareholder
- and market, 8, 35, 236, 238
- regulation, 211
- relationship with directors, 234, 235, 349, 471, 474
- value, 237
- wealth, 126, 128

Share options
- combining calls and puts, 300, 305
- speculating with calls, 306, 307
- speculating with puts, 306, 307
- writing/selling, 306

Share price volatility, 209
Single-owner firm, 465
Single-period capital rationing
- and IRR, 129
- mutually exclusive investments, 129

Single-period investment-consumption model, *see* Investment-consumption model
SML, *see* Security market line (SML)
Spot FX market, 451, 452
Spot interest rates, 448
Stock exchange rules, 210, 214, 271, 290
Stock market
- analysis, 186, 193
- secondary, 189
- workings of, 186

Supply and demand, 30, 31, 56, 190, 218, 301, 325, 352, 392, 394, 444
Swaps, 12, 153, 300–302, 334, 452, 455

T

Taxation
- capital structure, 223, 225, 226, 228, 236, 238, 242, 246–250, 253, 354, 438
- dividend considerations, 5, 210, 238
- foreign projects, 456, 457
- weighted average cost of capital (WACC) and, 229–231, 233

Technical analysis, 142, 187
Term structure of interest rates
- spot interest rates, 448
- yield curve, 162–164, 169, 178, 179

yield to maturity (YTM), 118, 150, 159–161, 165, 179, 180
Theta of an option, 317, 319
Time horizon, 50, 67, 75, 88, 105, 113, 127, 130, 192, 233, 273, 285, 311
Time to maturity, 152, 165, 213, 317, 319, 324–326, 332
Time value of money (TVM)
discounting process, 44–46, 49, 66, 178, 199, 216, 274, 280, 325, 381
net present value (NPV), 59–60, 67
Time value of option
time to expiry, 213, 300, 317, 331
volatility of share price, 155, 209
Total market value model, 232
Traditional valuation model, 203, 210, 320, 410
Transaction costs, 31, 32, 35, 186, 226, 235, 246, 252, 263, 314, 375, 419, 420, 439
Transaction risk hedging
swaps, 301, 302
Translation FX risk, 450–455
TVM, *see* Time value of money (TVM)
Two-asset portfolios
covariance, 91, 92, 109
risk and expected return, 82, 83
risk-free investment, 97

U
Uncertainty, 373
impact, 130, 321, 373
simple approach (*see* Unsystematic risk)
Utility function
shape, 108

V
Vega of a call option, 319

W
WACC, *see* Weighted average cost of capital (WACC)
WDA, *see* Writing-down allowance (WDA)
Weak efficiency, 35, 37, 380, 471
Wealth
shareholder, 380, 456, 474, 476
Weighted average cost of capital (WACC)
assumptions, 229, 231
project discount rate, 228
project risk and, 229, 230, 233
risk-adjusted, 233
taxation and, 230, 233
Writing (selling) options, 16, 306
Writing-down allowance (WDA), 15

Y
Yield curve, 58, 149, 150, 162–164, 169, 177–179, 211, 219, 220, 231
Yield to maturity (YTM), 53, 54, 68, 118, 150, 159–161, 165, 180

Z
Zero coupon bond, 151, 152, 158, 163, 164, 167, 181, 355
Zero sum games, 191